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Mass generation in chiral fermion systems



QUANTUM
—LIMITS—

What opens the gap - and which symmetry survives?

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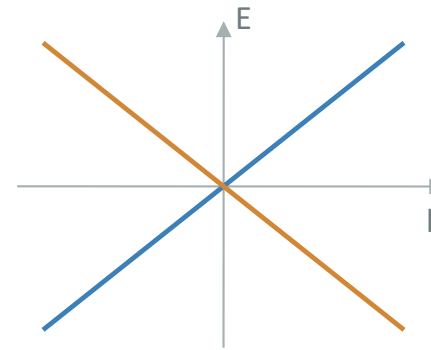
**How do fermions
gain mass?**

Massless Dirac Fermions

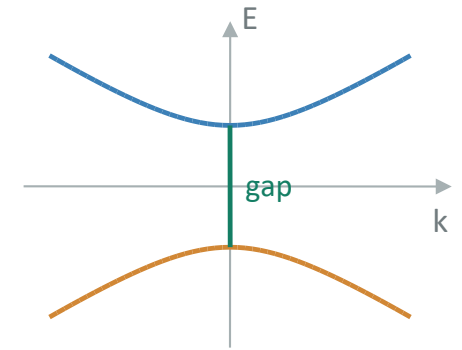
Standard model

Dirac fermions are the basic building blocks

Massless



Massive



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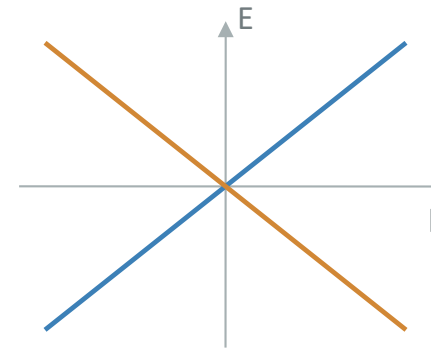
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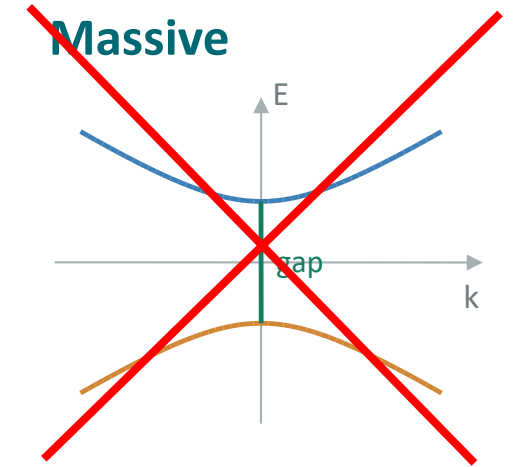
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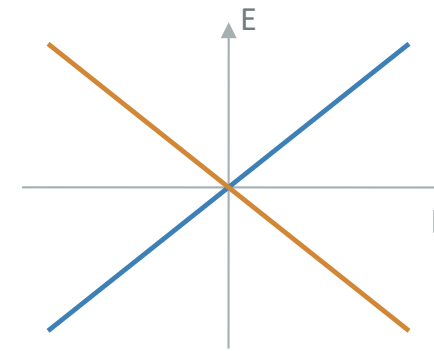
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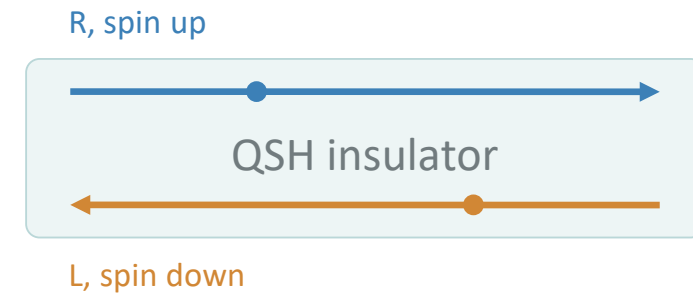
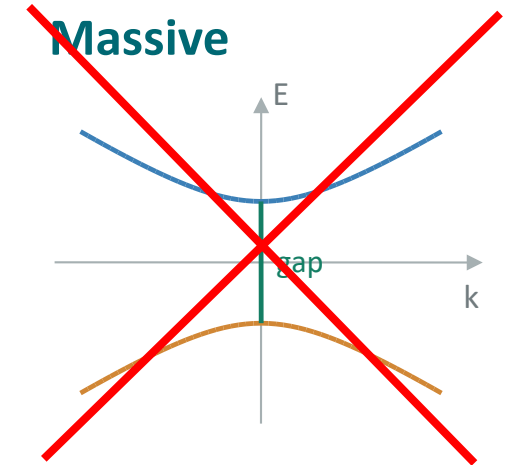
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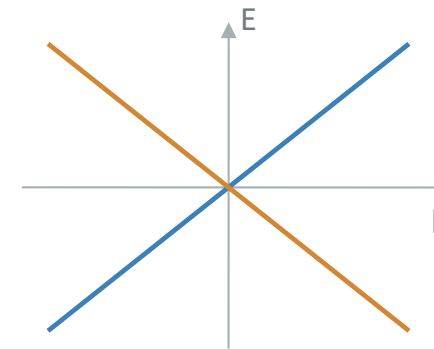
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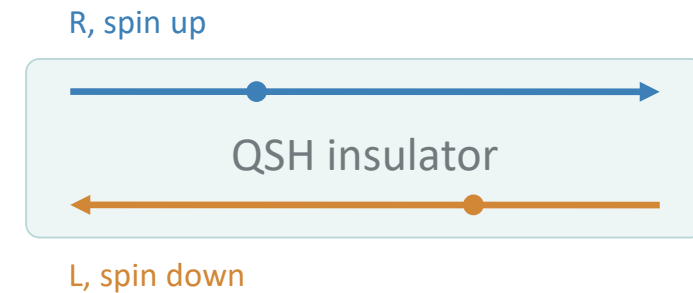
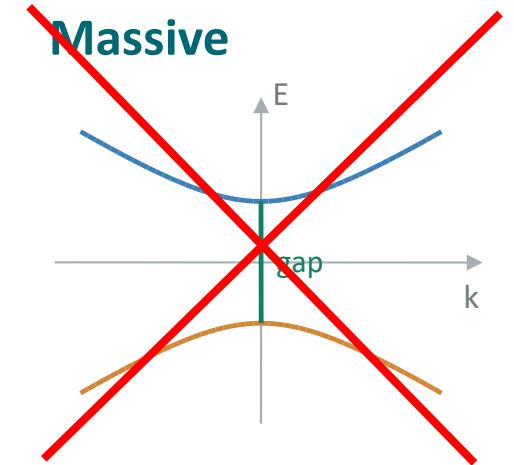
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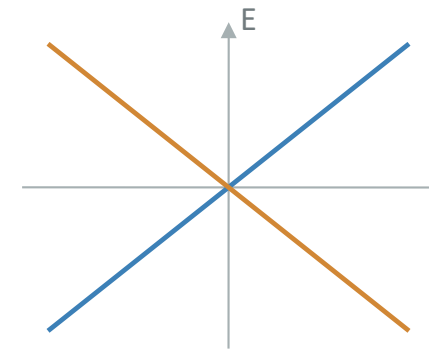
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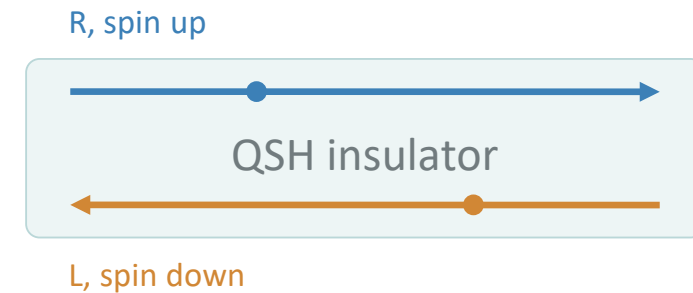
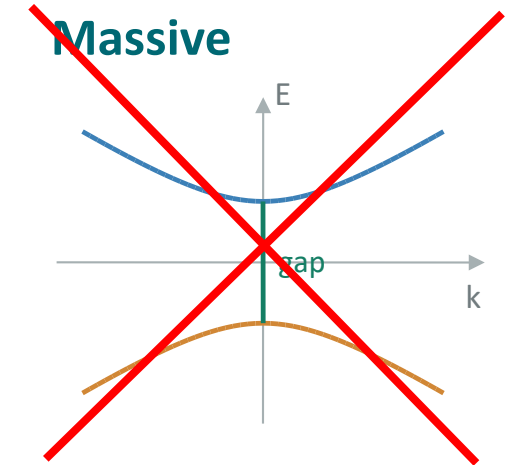
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In single-particle case mass of a chiral fermion is prohibited by symmetry. Can interactions generate it?

Let's try to test it

From a continuum field to a lattice

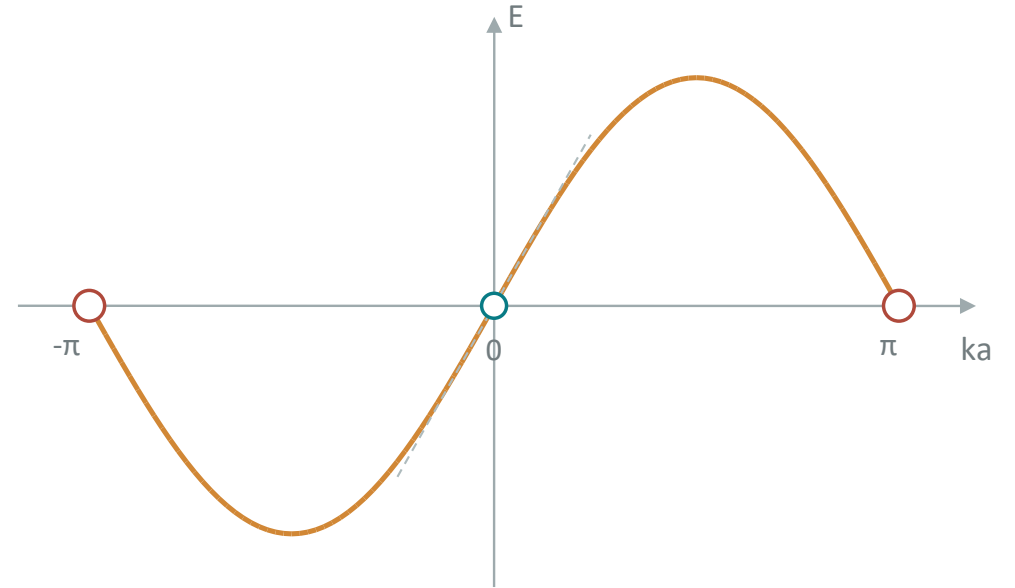
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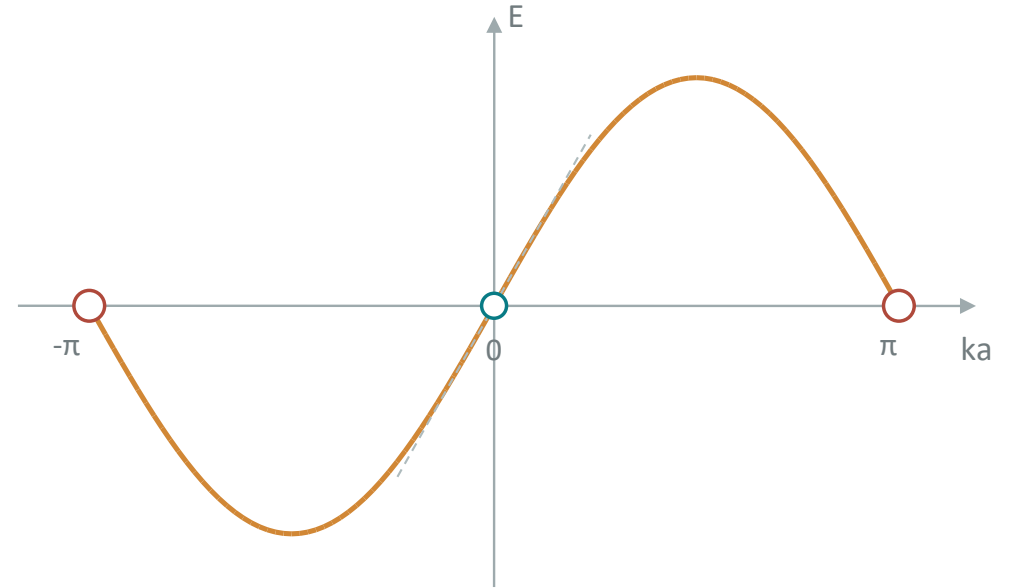
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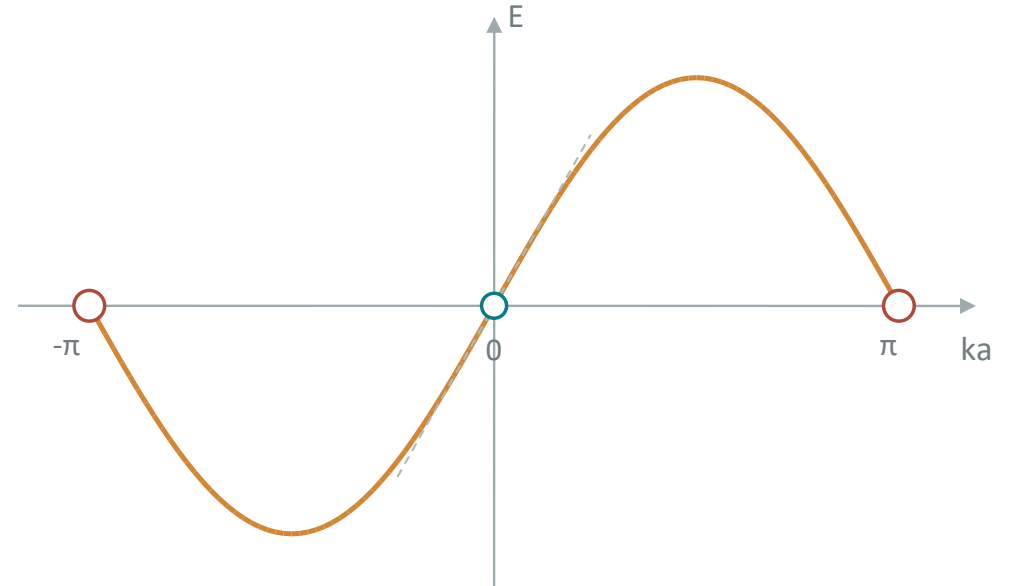
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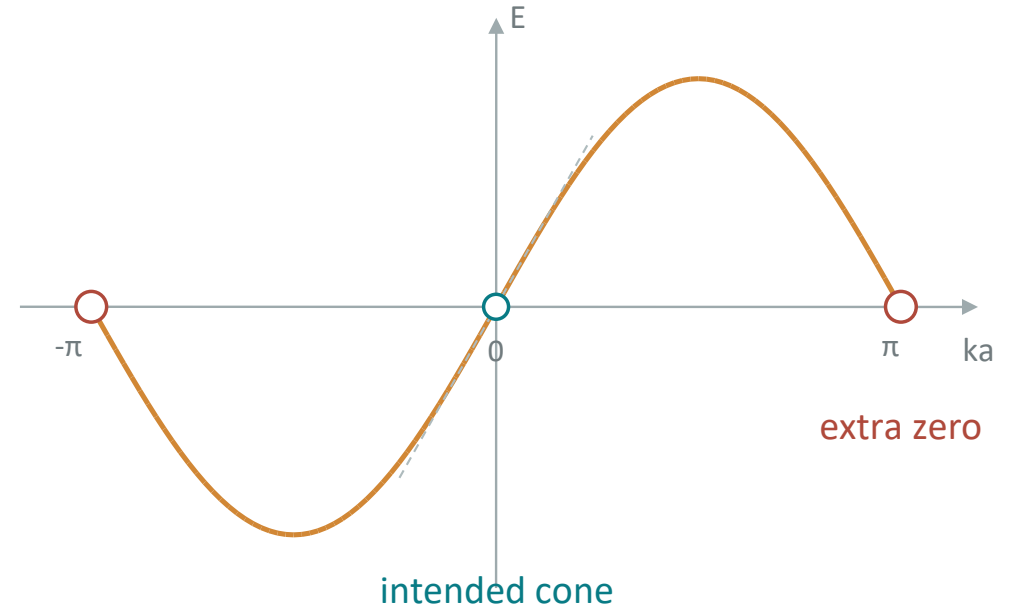
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Interactions couple the doubled low-energy sectors and gap them out.

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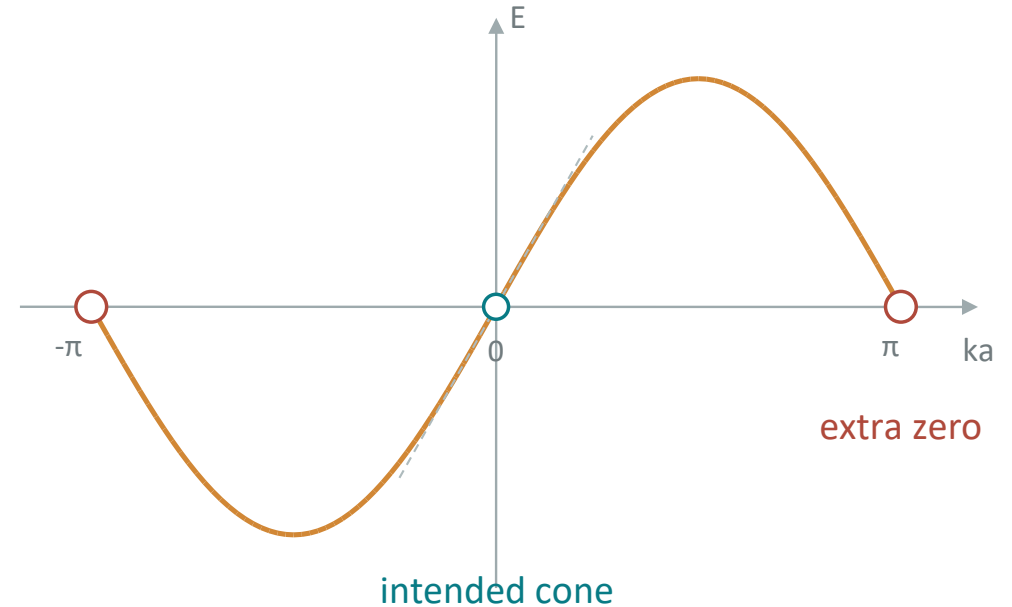
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Wrong low-energy content gives the wrong mass-generation problem.

Tangent fermions

Nielsen-Ninomiya no-go theorem

No doubler + protected chirality: a local lattice hopping is obstructed.

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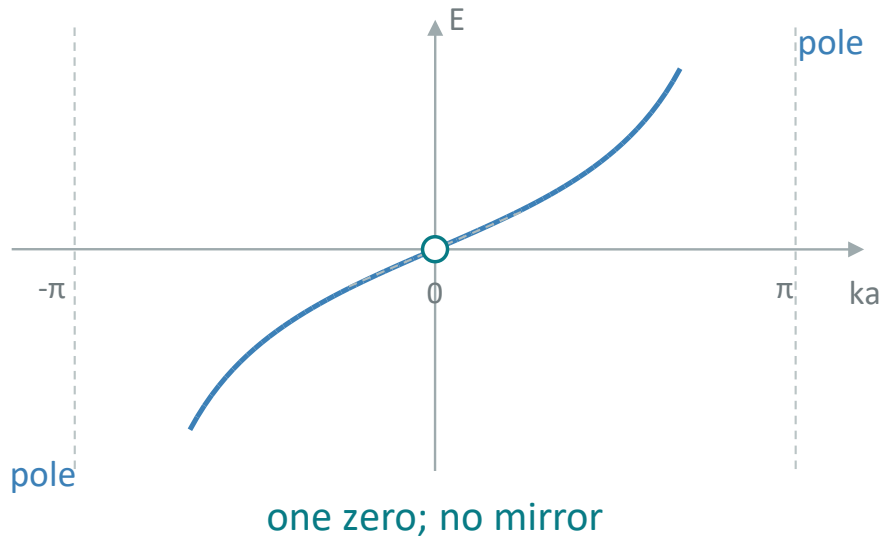
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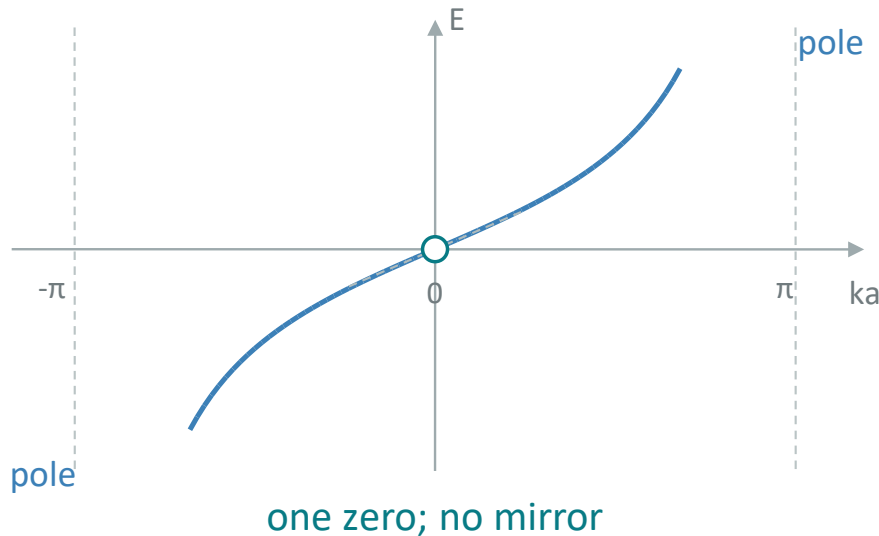
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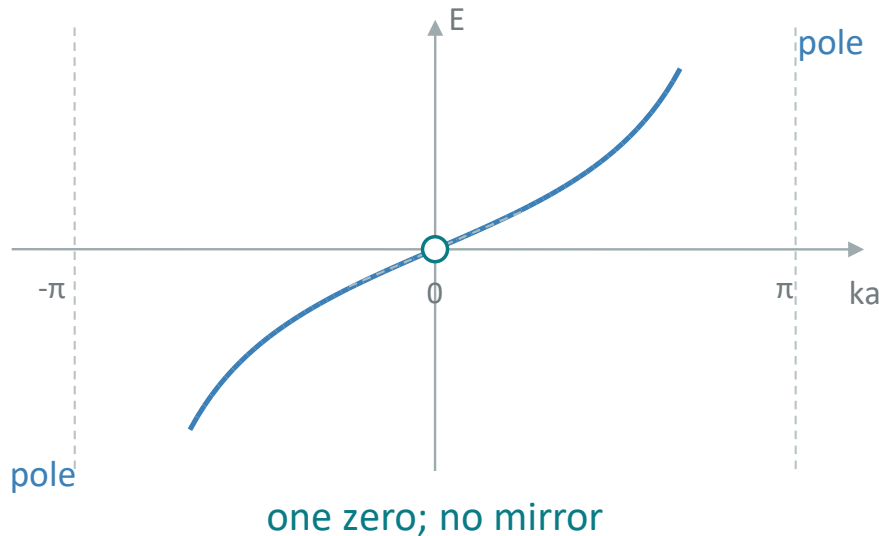
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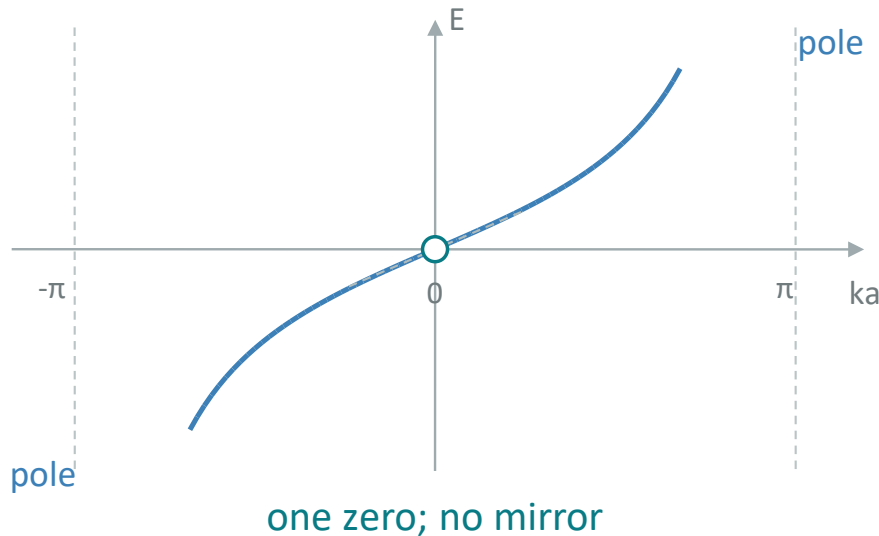
Local action
QMC

Fixed-size MPO
Tensor networks

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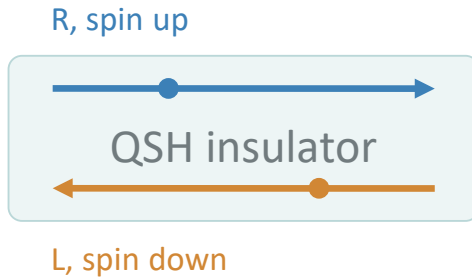
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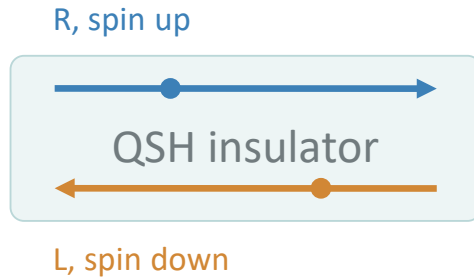
A protected and efficient tool for interacting chiral fermions, directly in 1D.

One spin flip is odd. Two are even.



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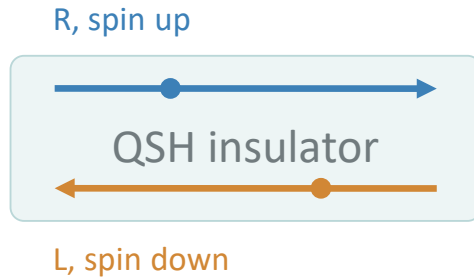
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One flip: a forbidden bilinear

$$B_n = c_{n,R}^\dagger c_{n,L}, \quad B_n \xrightarrow{\mathcal{T}} -B_n^\dagger$$

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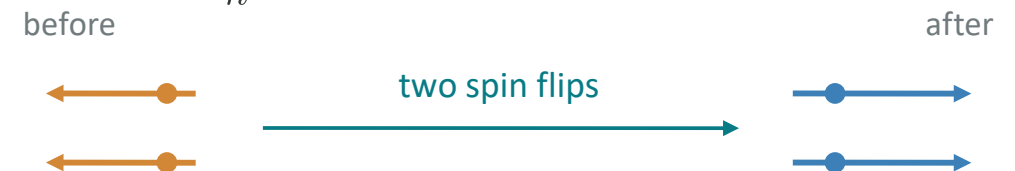
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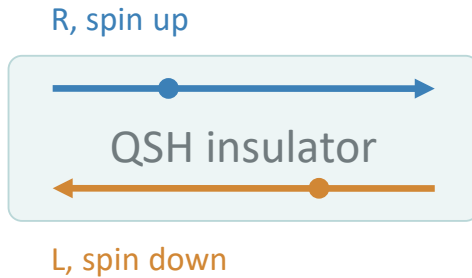
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Two flips: an allowed interaction

$$H_2 = g_2 \sum_n (B_n B_{n+1} + \text{H.c.})$$



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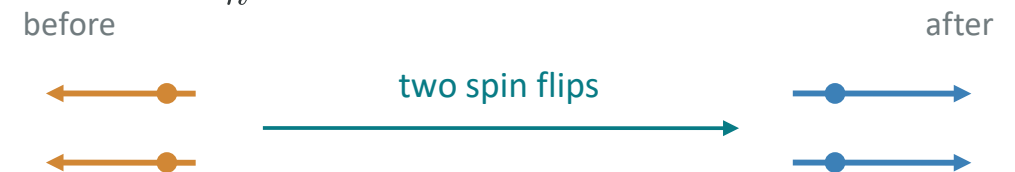
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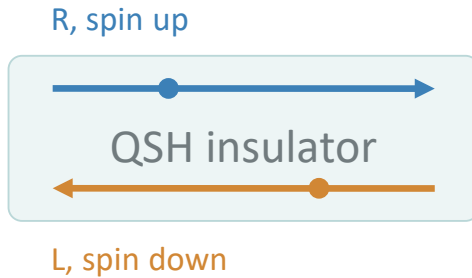
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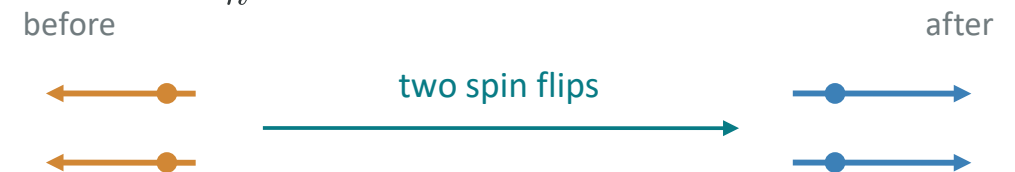
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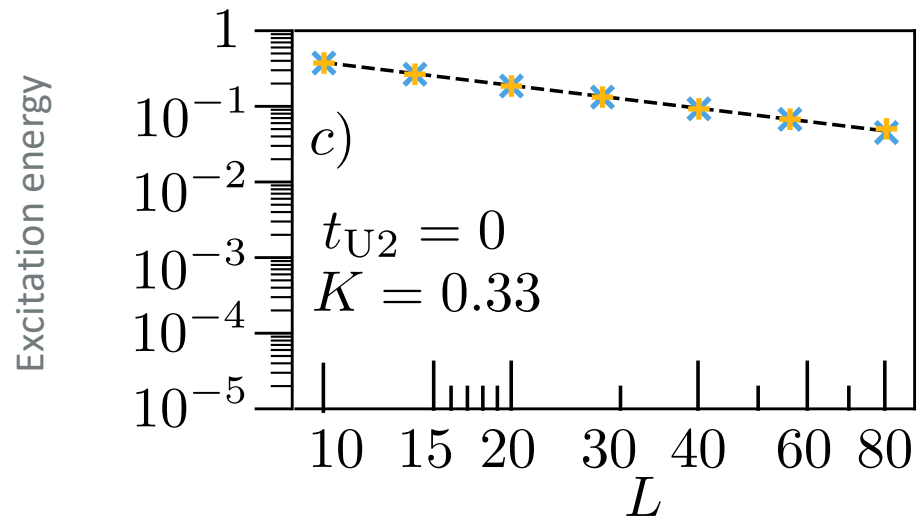
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The two-flip process is allowed; the density interaction makes it relevant.

A gap opens - and a ground-state partner appears

No two-particle backscattering

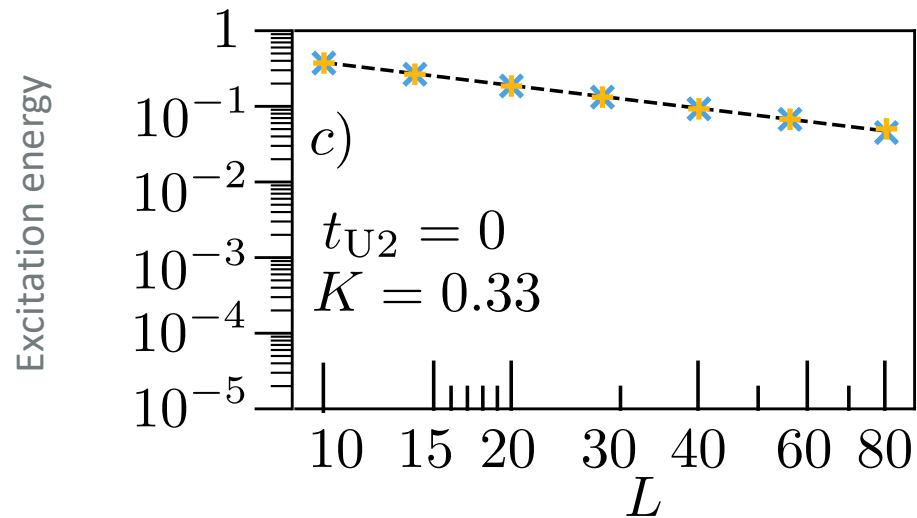


Reference: finite-size spacing falls as $1/L$

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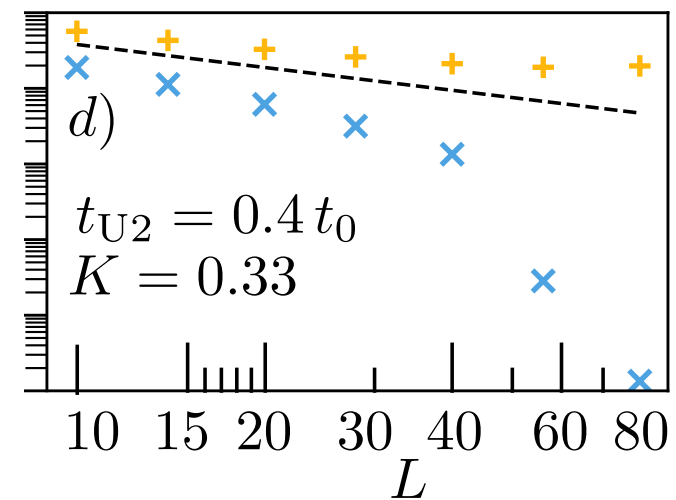
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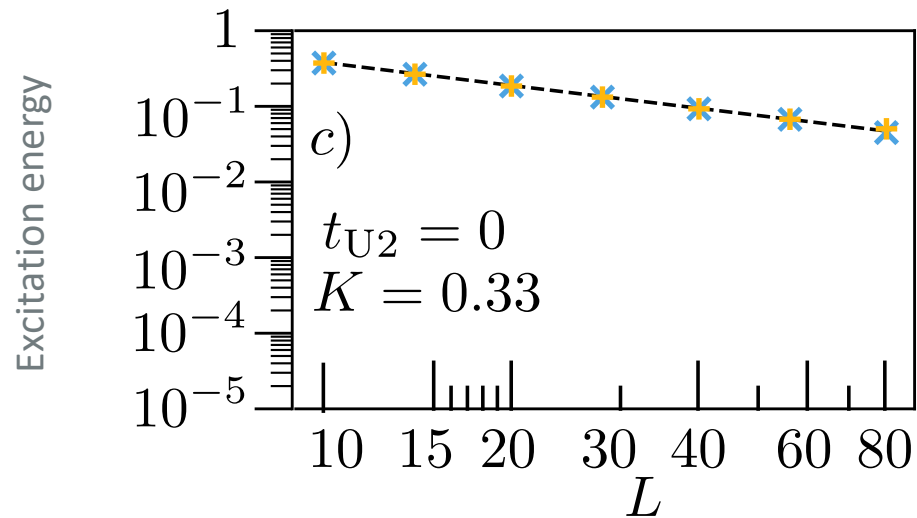
With two-particle backscattering



Yellow: finite excitation gap
Blue: a second ground state

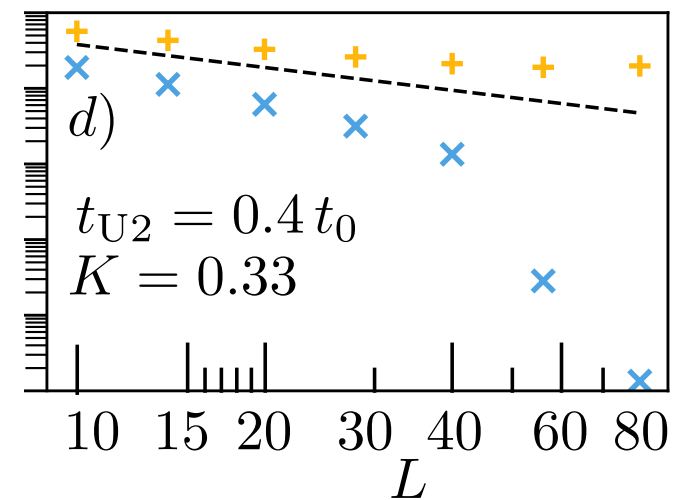
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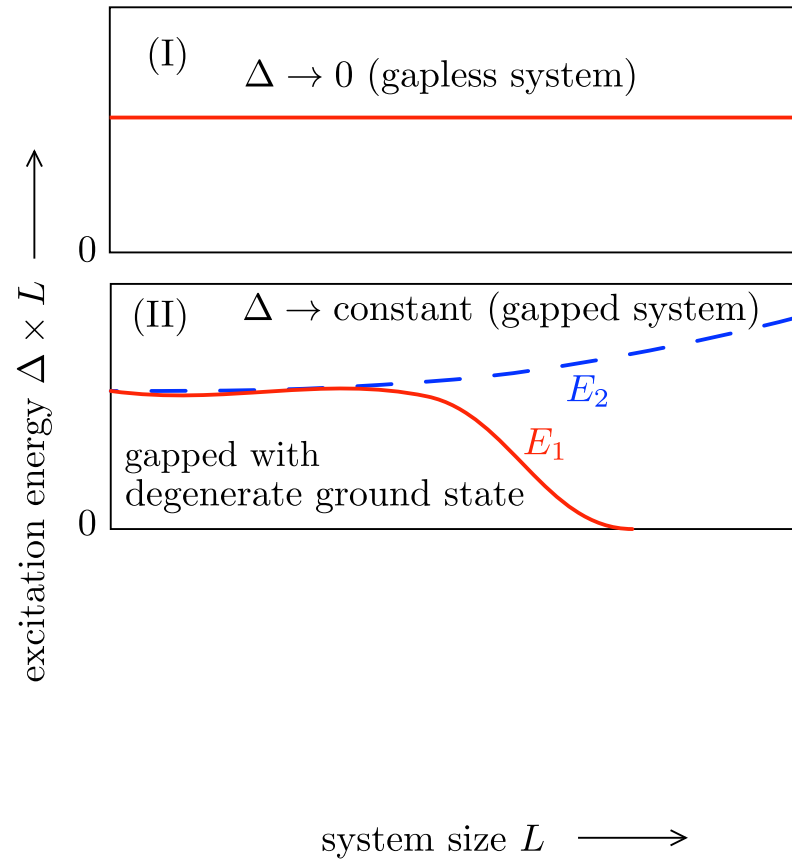


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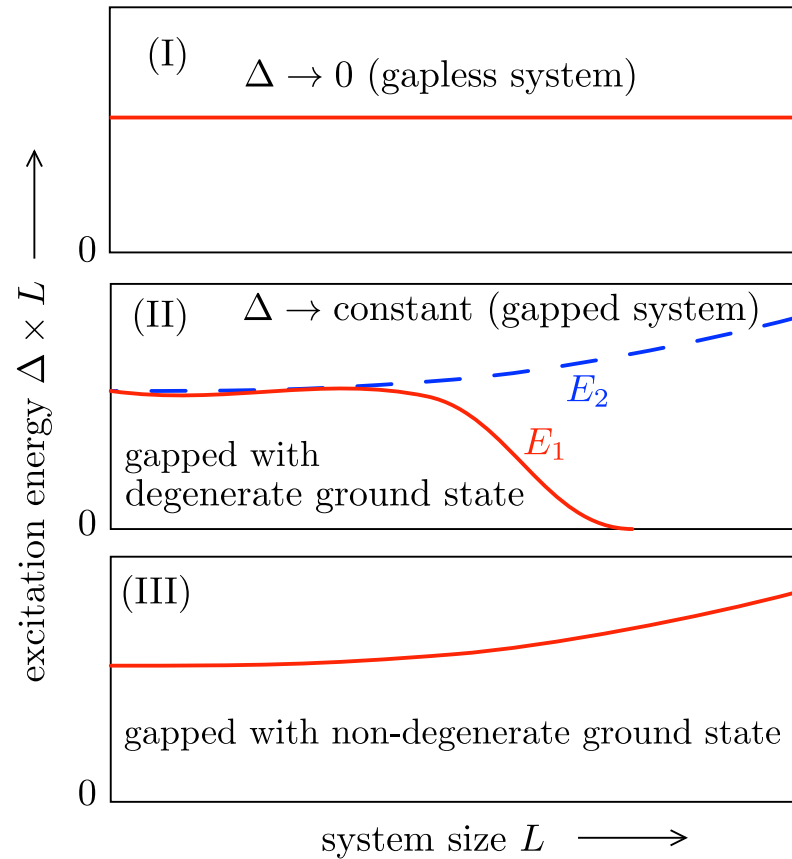
The Hamiltonian preserves TRS, the ground states breaks the symmetry spontaneously.

$$H_2 \sim g_2(B^2 + B^{\dagger 2}), \quad B = \psi_R^\dagger \psi_L \quad \langle B \rangle^2 \neq 0 \quad \Rightarrow \quad \langle B \rangle_\pm = m_\pm = \pm m_0$$

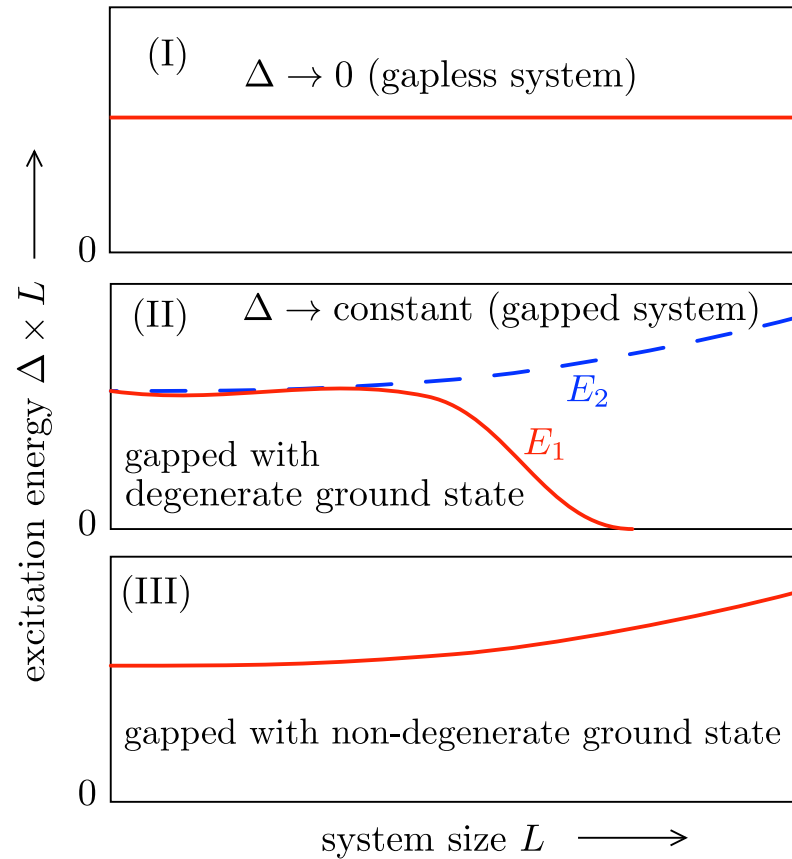
Can there be mass generation without a mass?



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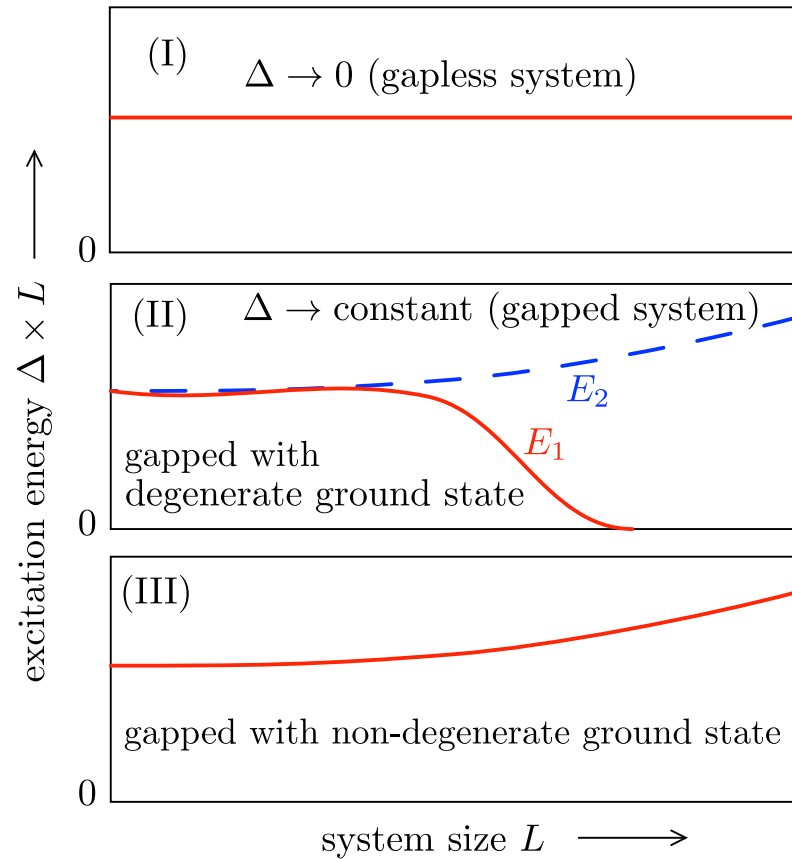
Can there be mass generation without a mass?



**Mass as an excitation gap,
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$$\Delta > 0, \quad \langle \psi_R^\dagger \psi_L \rangle = 0$$

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**Mass as an excitation gap,
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**Drop the TRS requirement.
Charge conservation can still forbid every
bilinear mass.**

right movers

→ qR1

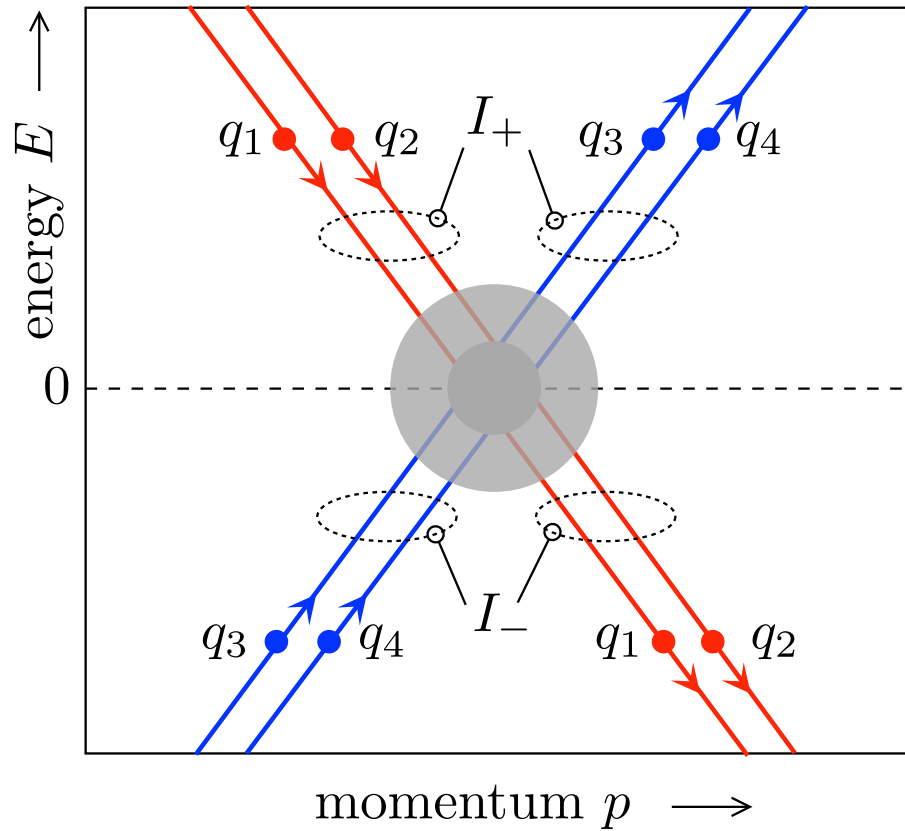
→ qR2

left movers

← qL1

← qL2

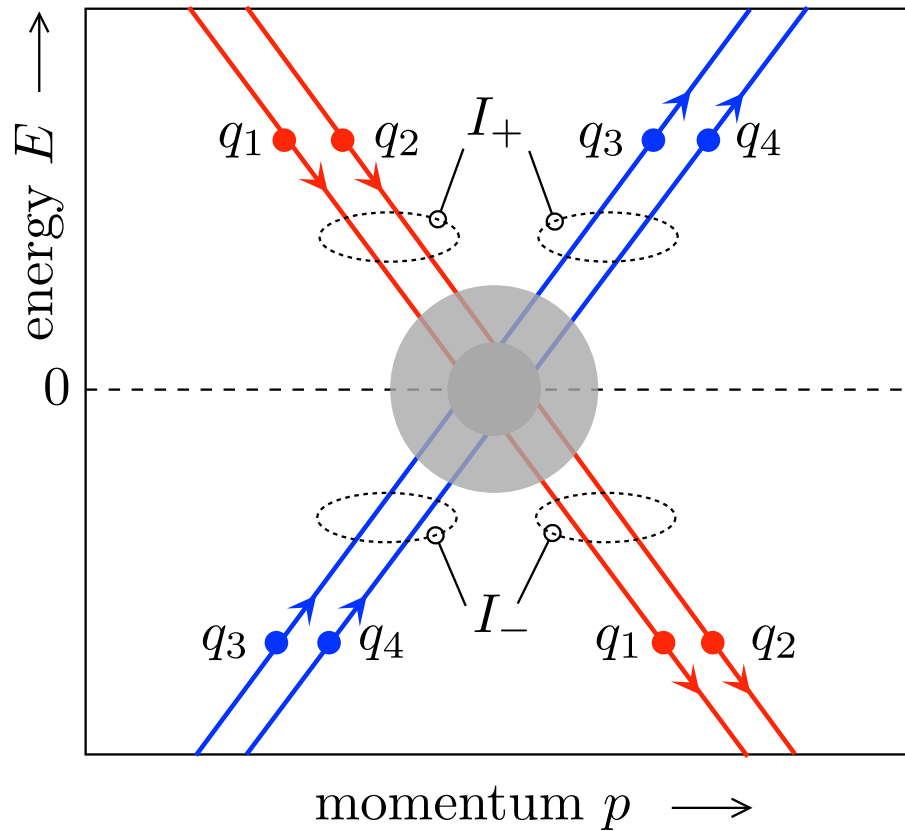
Which charges can be symmetrically gapped?



An electric field creates a current.

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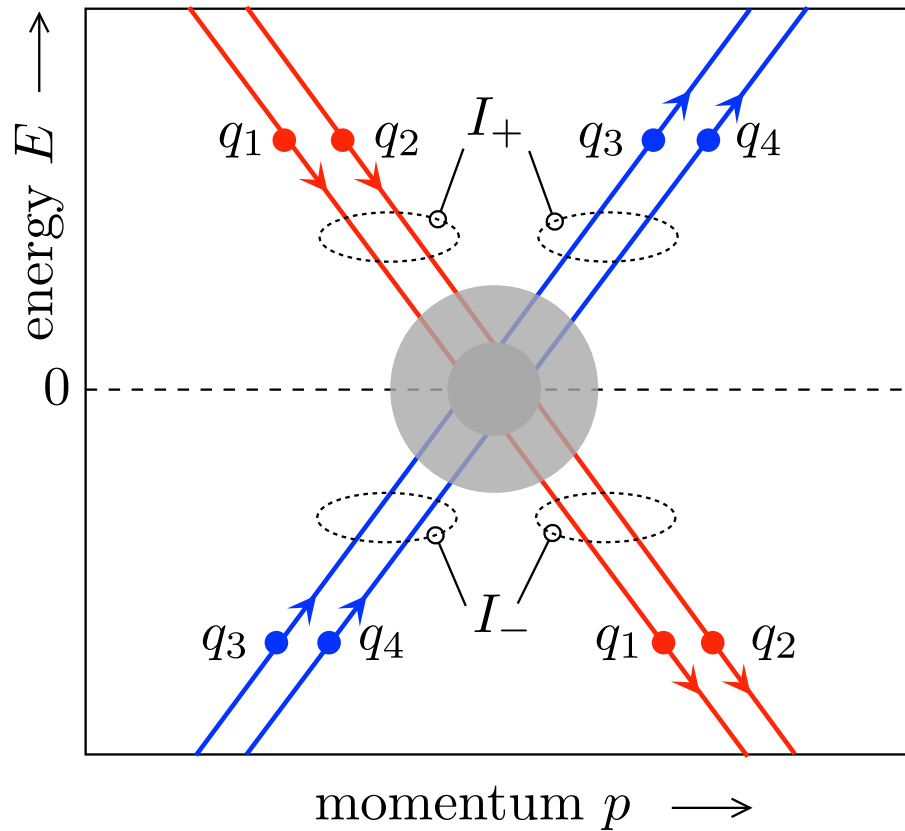
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Each state carries that charge again.

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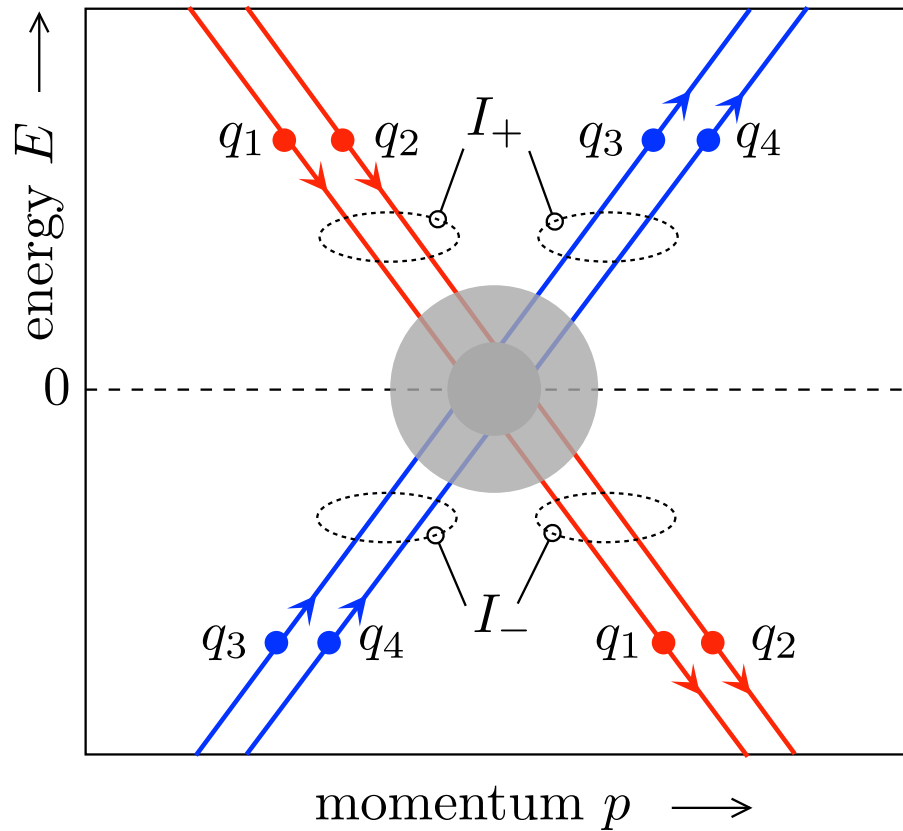
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Right movers: 3 and 4. Left movers: 5 and 0. The anomaly cancels.

$$3^2 + 4^2 = 5^2 + 0^2$$

Construction of the symmetric mass generation Hamiltonian

Minimal compatible interaction is six-fermion

$$H_{\text{SMG}} = g_1 \sum_n c_{n,3} c_{n,4}^\dagger c_{n+1,4}^\dagger c_{n,5} c_{n,0} c_{n+1,0} + \text{H.c.}$$

$$\underbrace{3 + 5 + 0 + 0}_{\text{annihilated}} = \underbrace{4 + 4}_{\text{created}}, \quad \Delta Q = 0, \quad \Delta N = \pm 2 \quad \text{Charge is conserved. Particle number is not.}$$

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- At the free point: highly irrelevant

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- Original proposal requires ultra strong coupling

Construction of the symmetric mass generation Hamiltonian

Minimal compatible interaction is six-fermion

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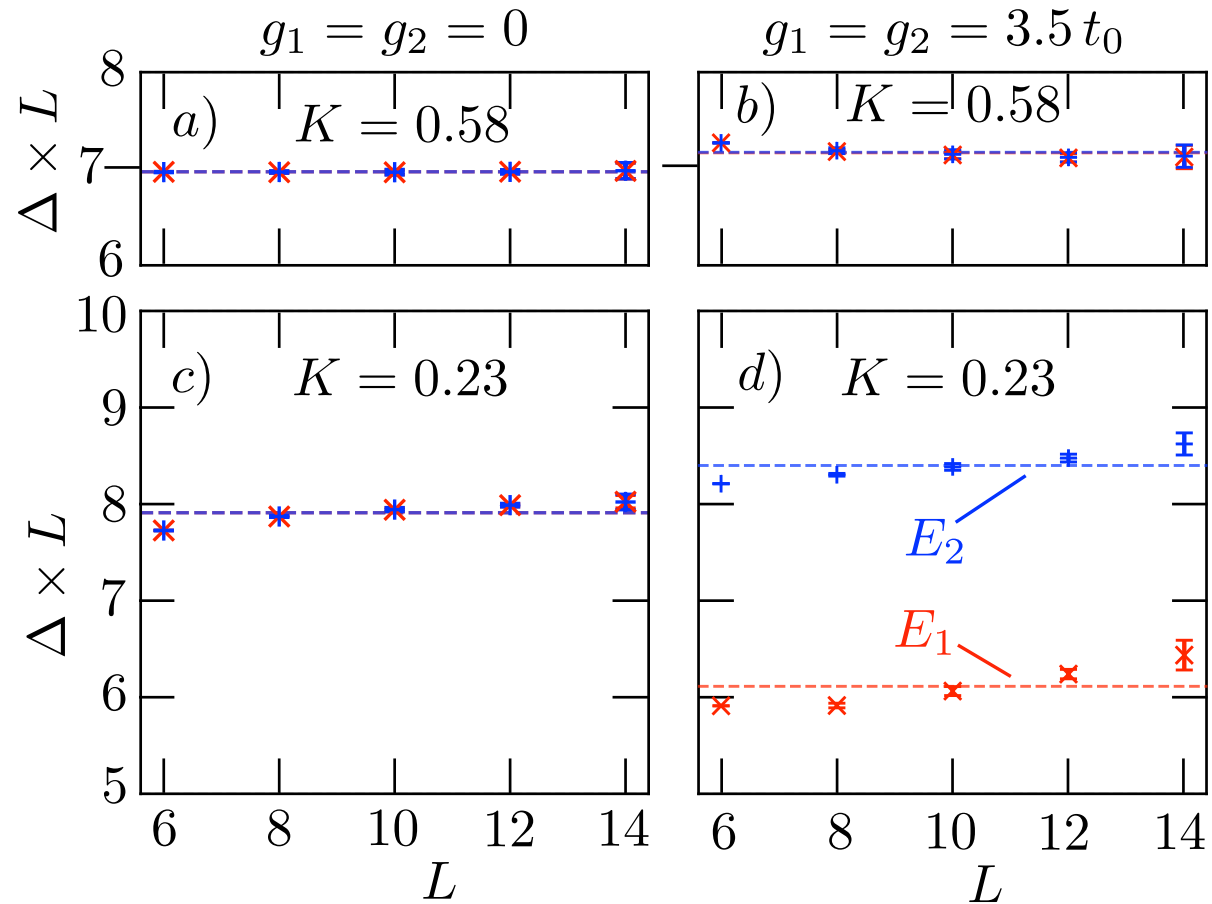
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**Our construction: using analogy with QSH case –
add a specially weighted density-density interaction allowing for controlled numerics
numerics**

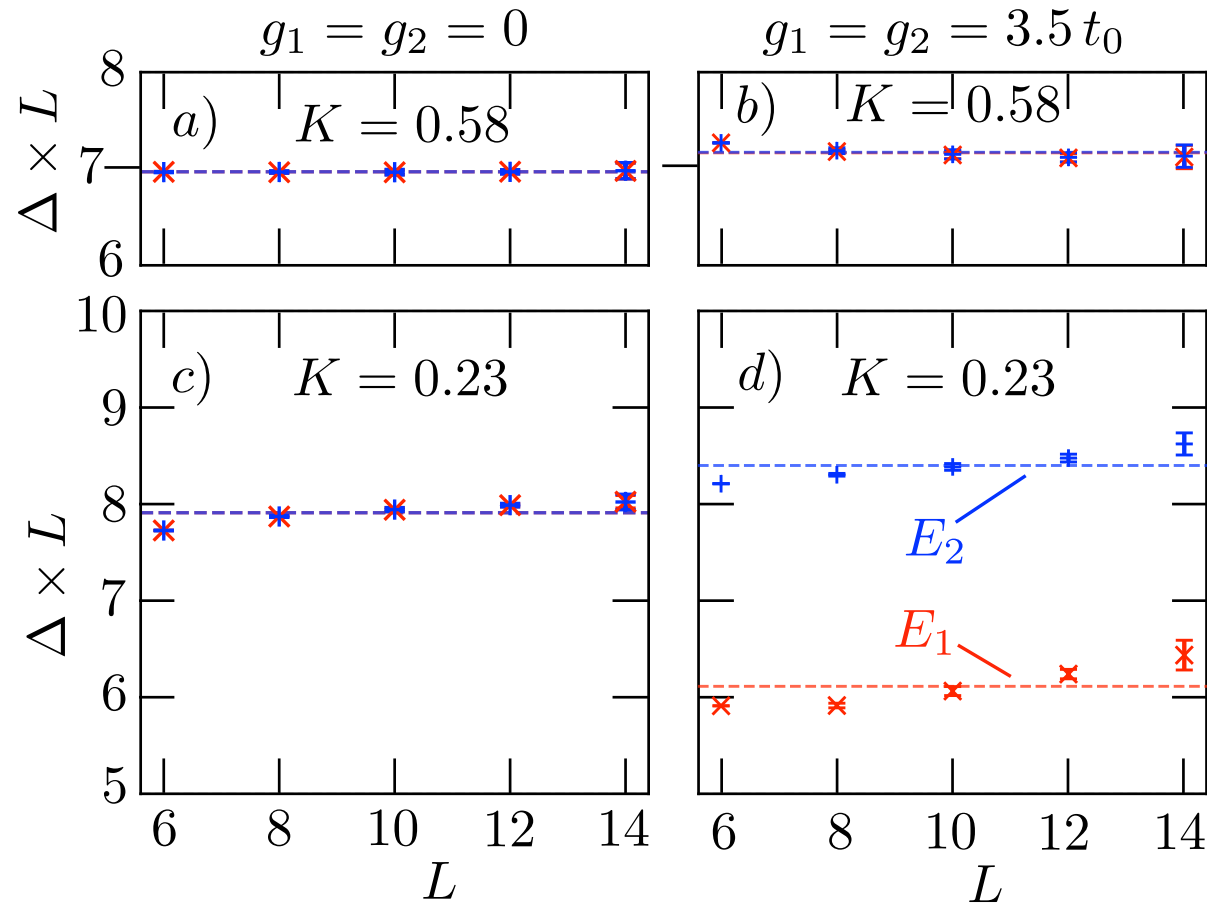
Two ingredients. One symmetric gap.



The density interaction alone

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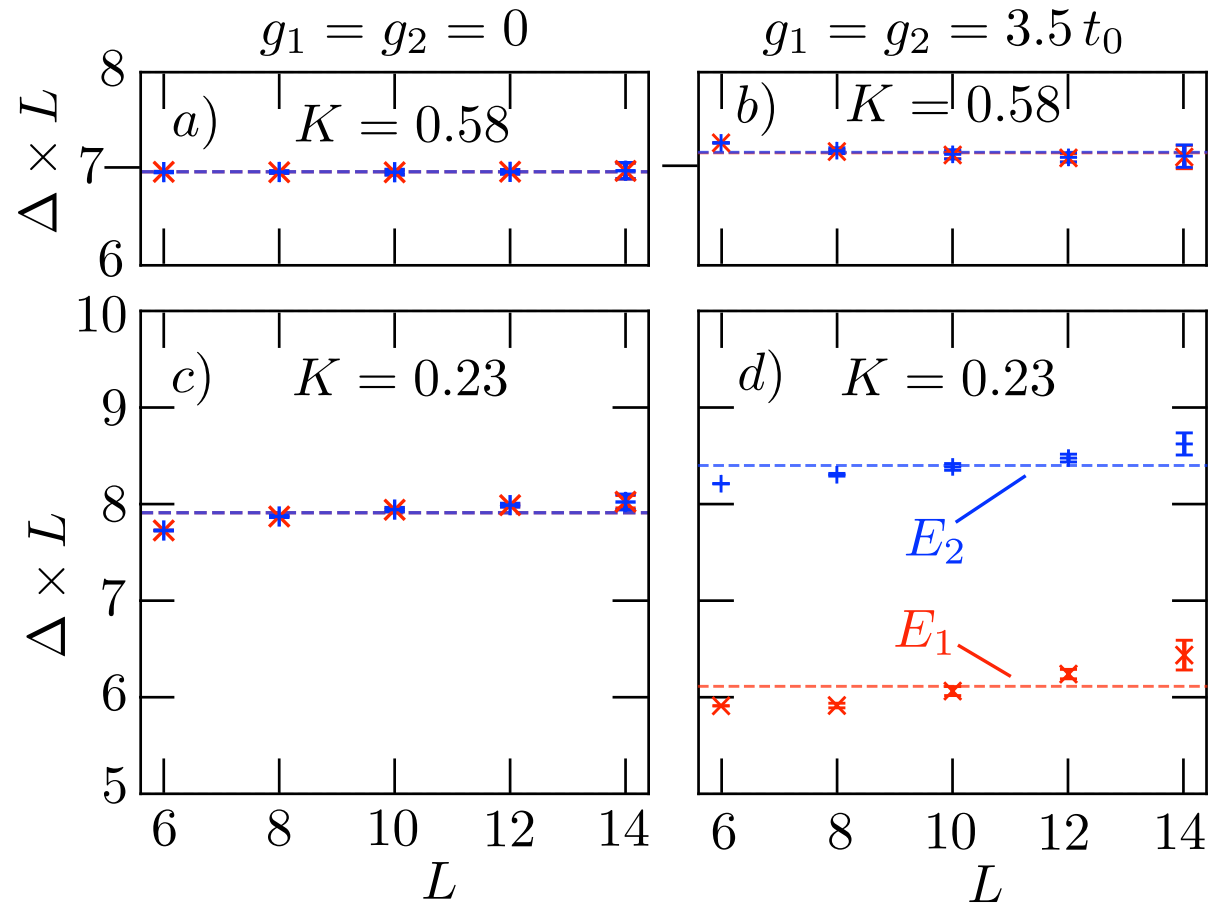
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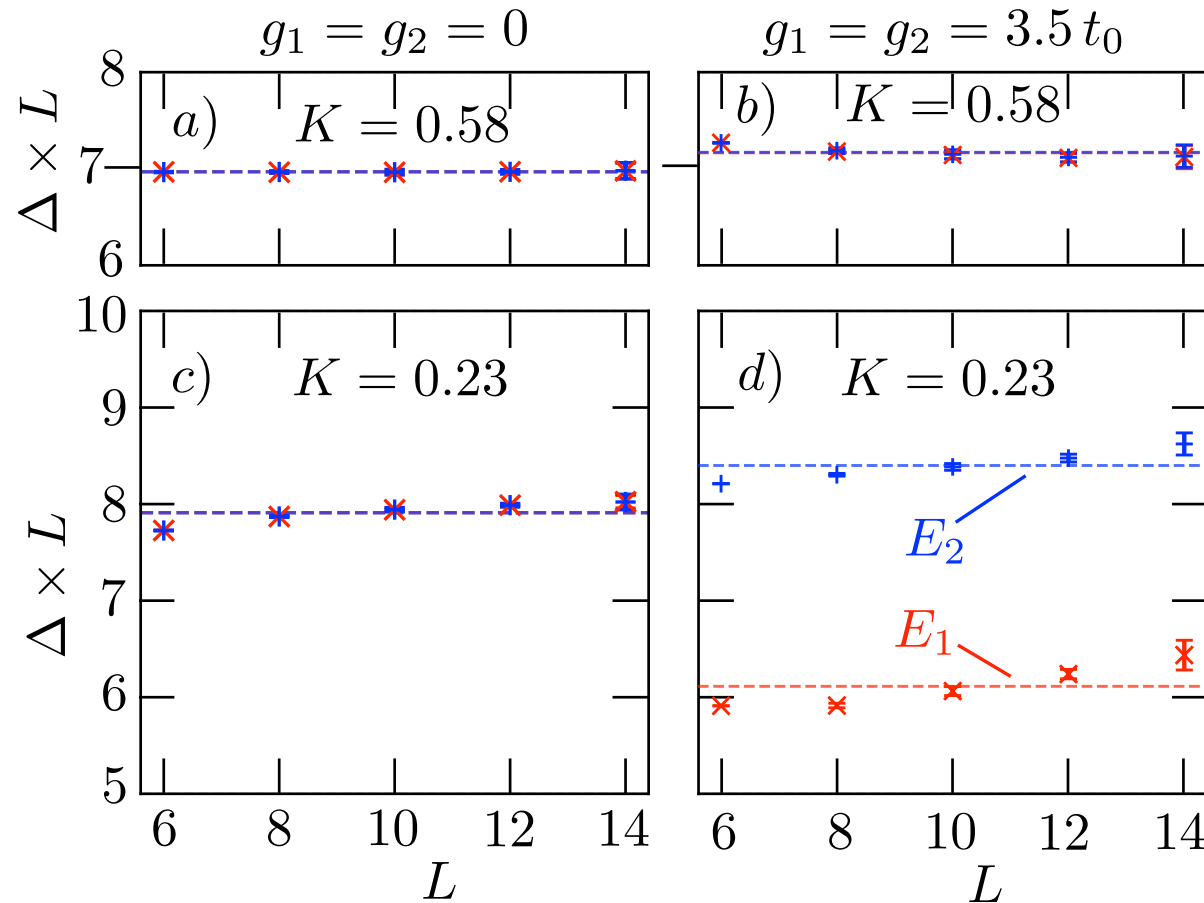
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Both ingredients together

A gapping signature, without a new ground-state partner.

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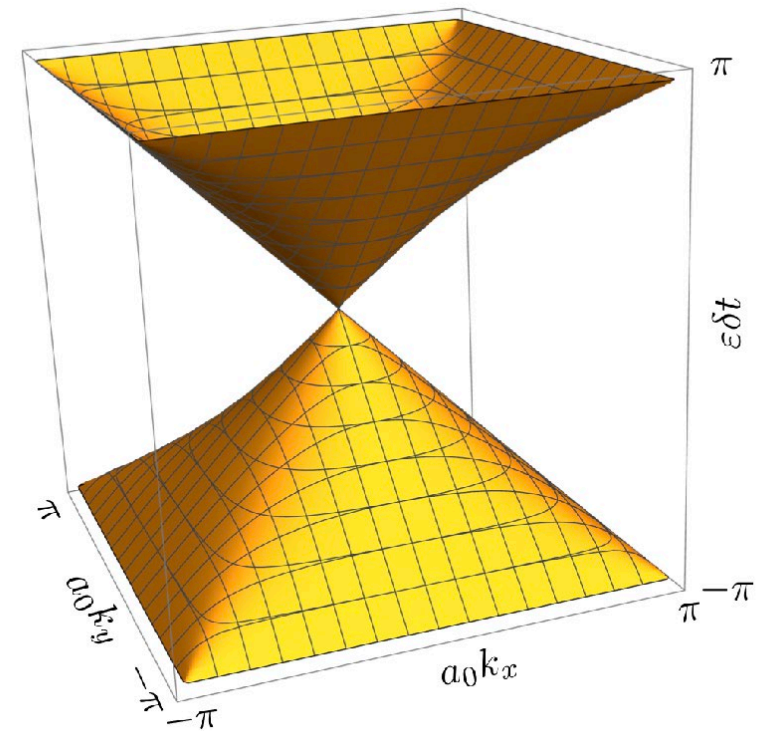
a gapping signature!

A gap without a symmetry-breaking bilinear mass: symmetric mass generation.

What we have learned about mass

1 Interactions can generate a bilinear mass

even when the Hamiltonian forbids it:
the ground state breaks the symmetry.



2D space-time / Floquet tangent dispersion

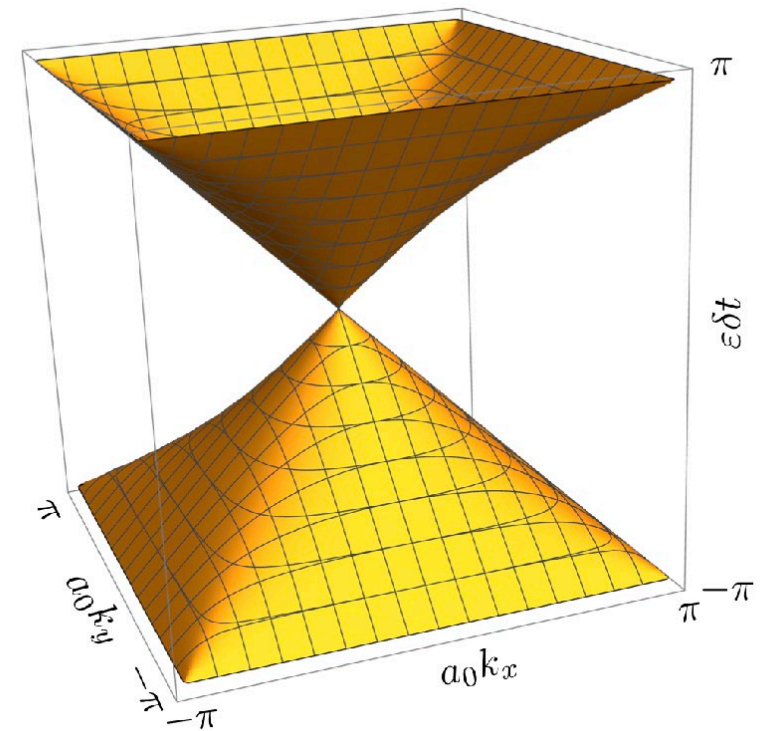
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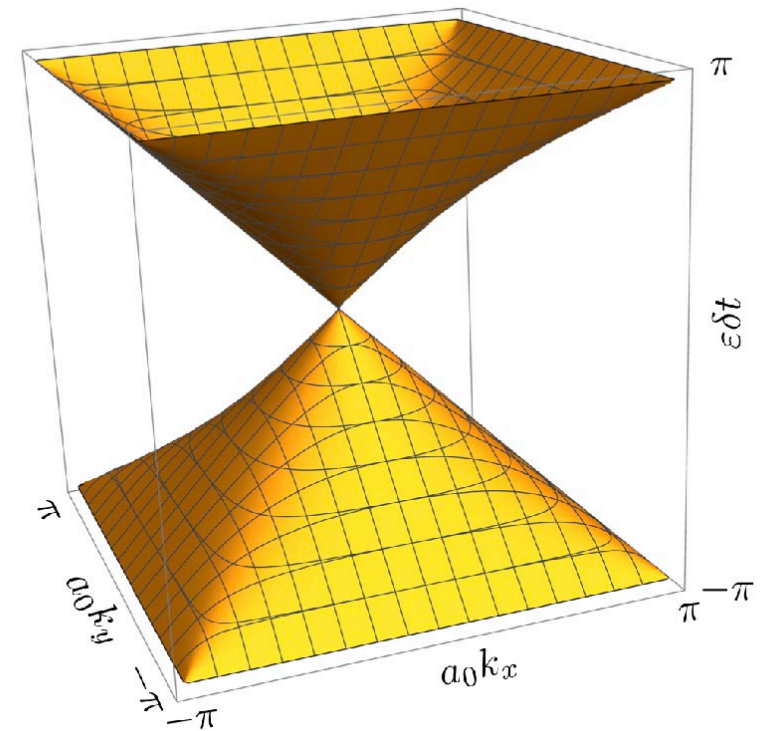
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3 Correct numerical tools are essential

Tangent fermions retain the protected
interacting problem for chiral fermions directly in 1D.



2D space-time / Floquet tangent dispersion