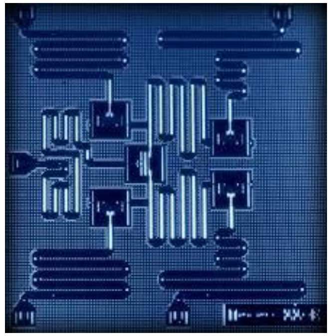
Correcting coherent errors with the surface code

Sergey Bravyi (IBM)
Matthias Englbrecht (Munich)
Robert Koenig (Munich)
Nolan Peard (MIT, Munich)

QIP 2018, Delft

How to preserve quantum information for a long time ?

Build better qubits



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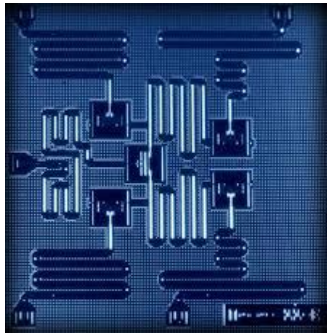

Quantum error correction

- Redundant encoding of quantum states
- Diagnose errors by syndrome measurements
- Syndrome-dependent recovery operations

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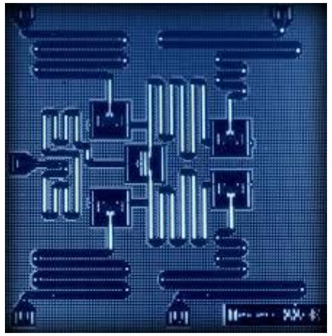

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Classical simulation of quantum error correction circuits with toy noise models provides insights into how well a given quantum code can perform in practice.

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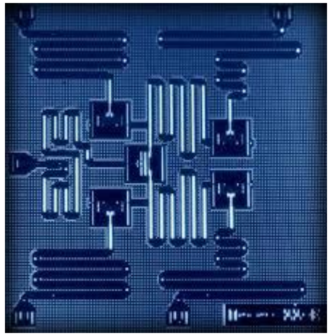

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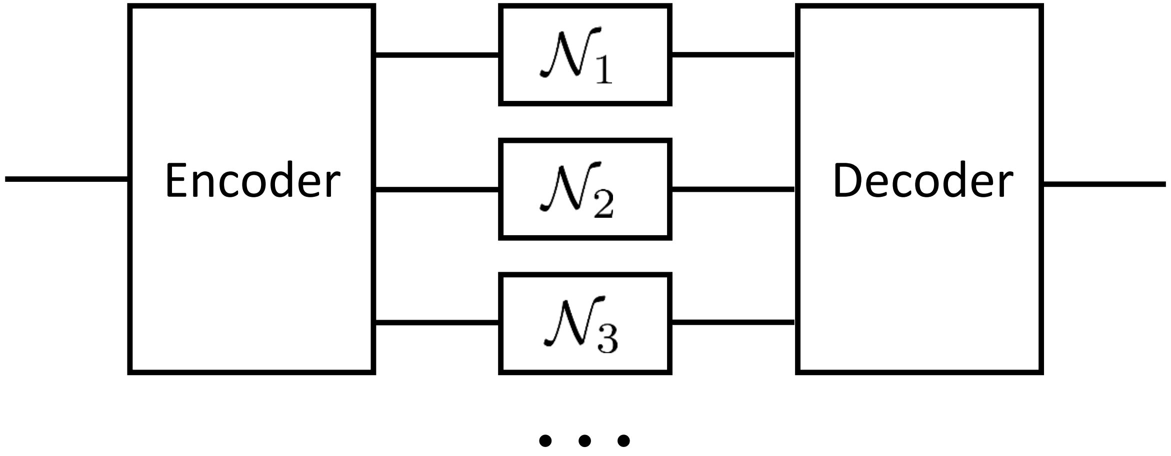
This talk: efficient algorithms for a classical simulation of large-scale QEC circuits

Coherent vs Pauli noise models



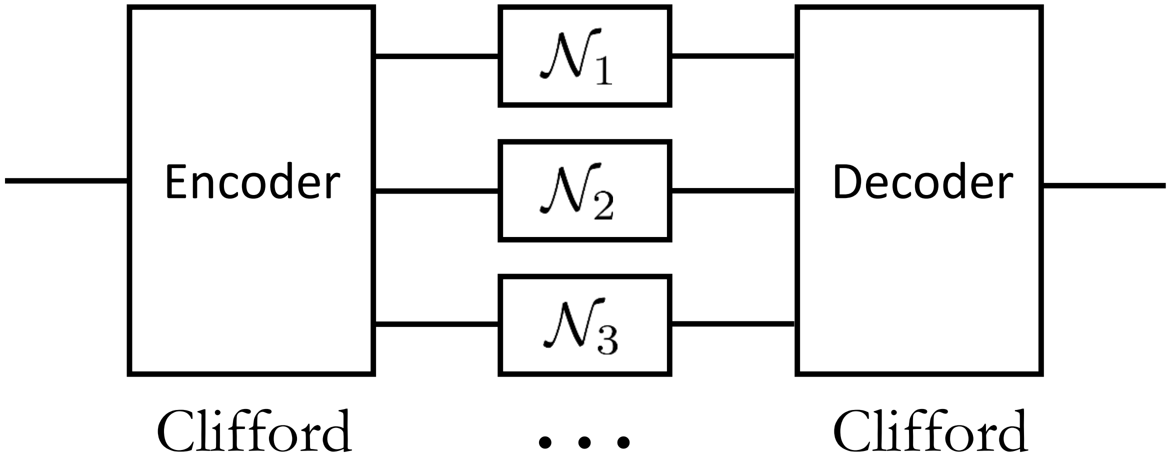
$\mathcal{N}_i$ - trace preserving completely positive maps

Pauli noise: models random errors caused by unwanted interactions with the environment

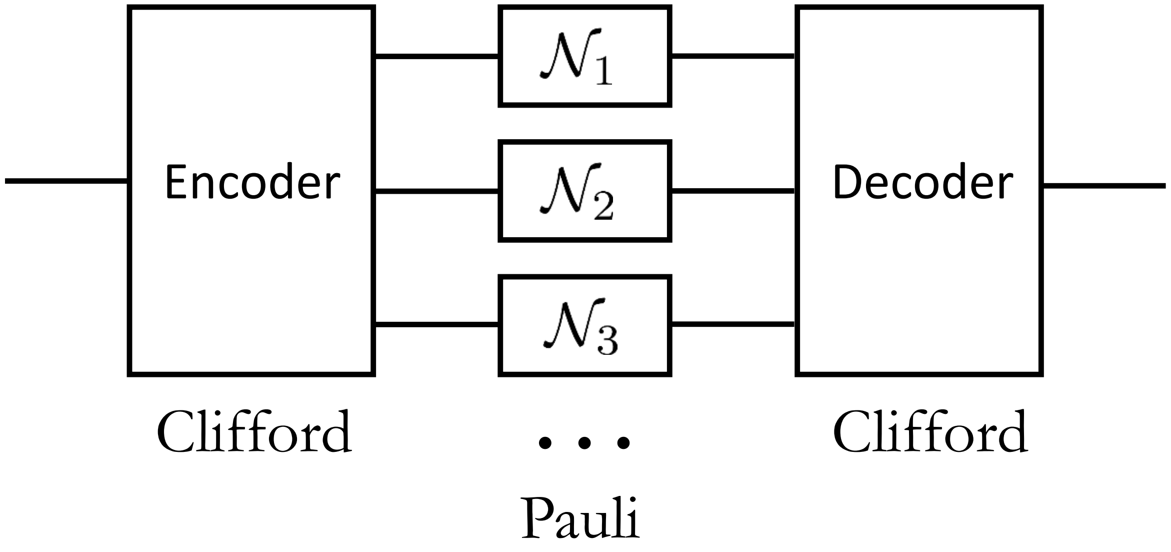
$$\mathcal{N}_i(\rho) = (1-\epsilon)\rho + \epsilon_x X\rho X + \epsilon_y Y\rho Y + \epsilon_z Z\rho Z$$

Coherent noise: models systematic errors caused by imprecision in the classical control

$$\mathcal{N}_i(\rho) = U\rho U^\dagger \qquad\qquad U \in SU(2)$$



Stabilizer-type quantum codes: Clifford encoding/decoding circuits.



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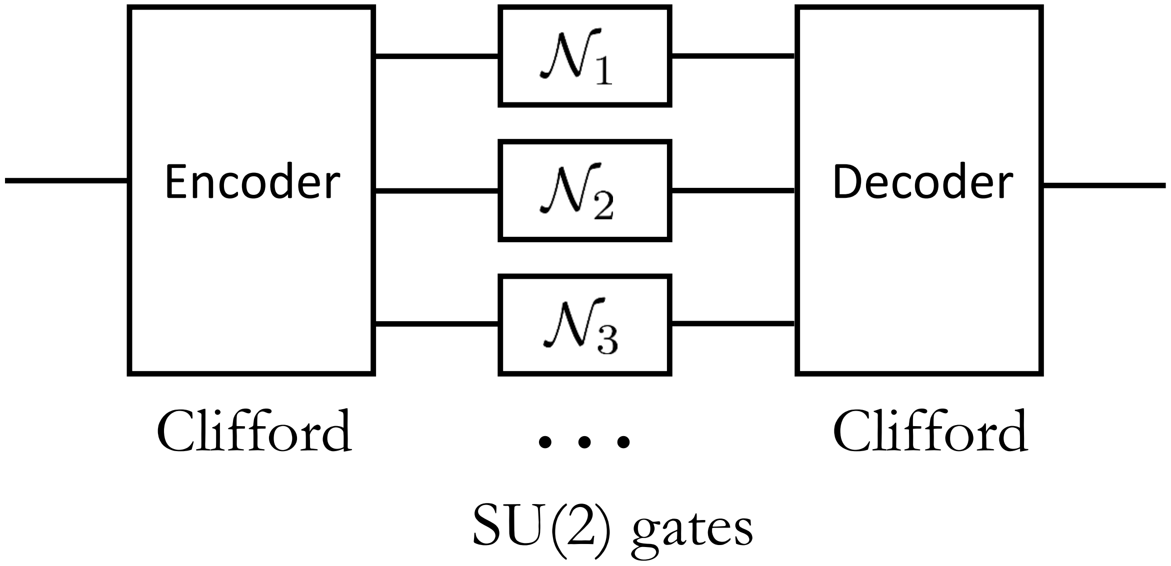
Pauli noise is easy to simulate numerically (use Gottesman-Knill)



Stabilizer-type quantum codes: Clifford encoding/decoding circuits.

Pauli noise is easy to simulate numerically (use Gottesman-Knill)

Coherent noise is described by Clifford+SU(2) circuits.
Brute-force simulation requires exponential time.

Special case: **surface codes**

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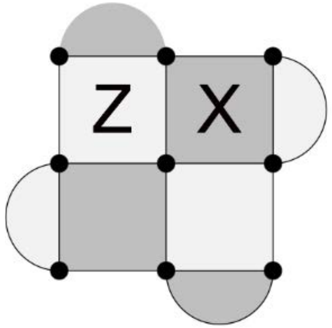
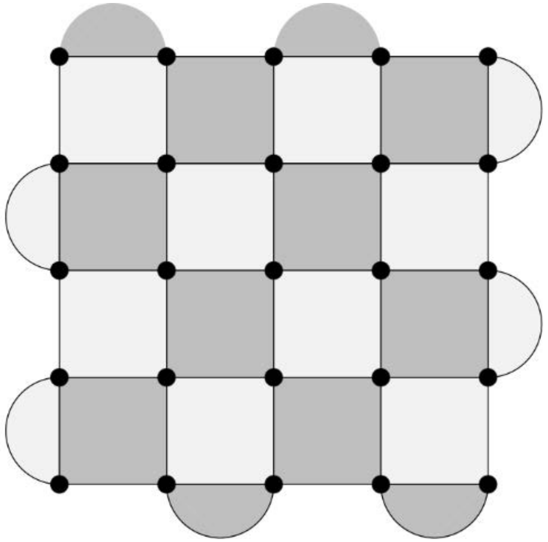
Encodes one logical qubit into $n = d^2$ physical qubits with distance d



$d = 3$



$d = 5$

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Wen, PRL (2003)
Bombin and Martin-Delgado, PRA (2007)

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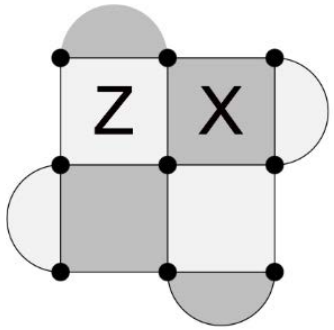

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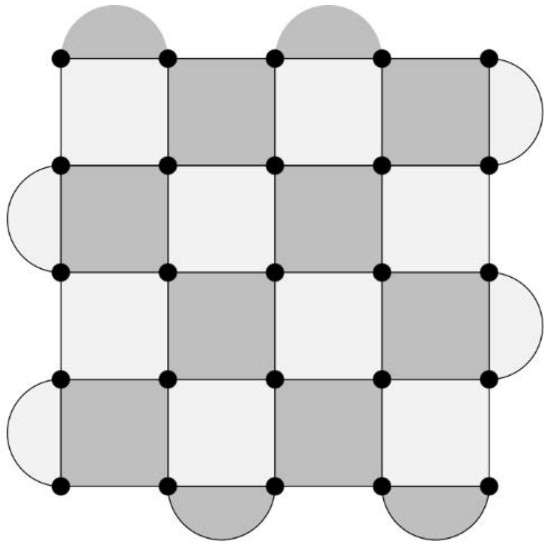

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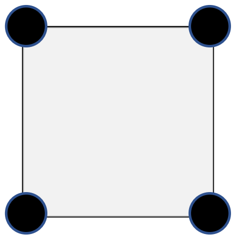
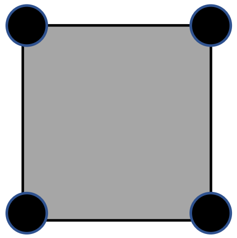
stabilizers (parity checks):



$ZZZZ$



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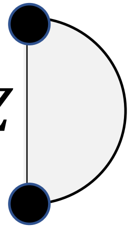
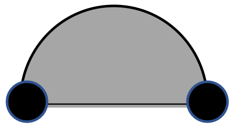
ZZ 



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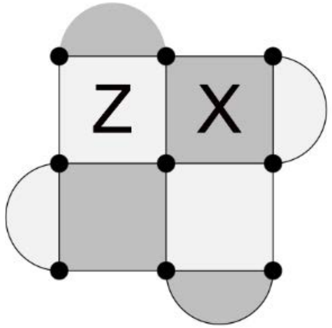

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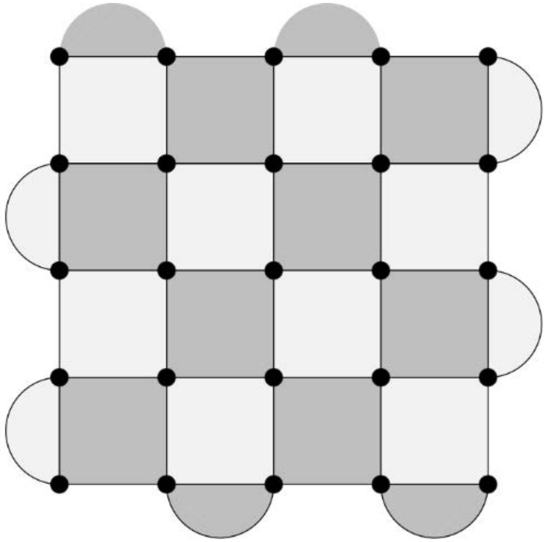

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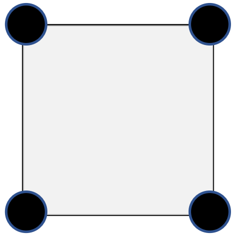

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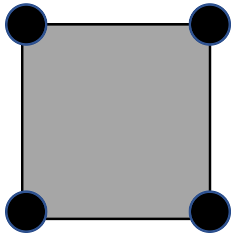

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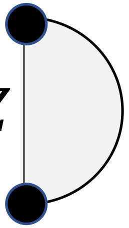

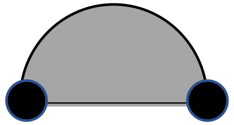

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Logical (encoded) states are defined as +1 eigenvectors of all stabilizers

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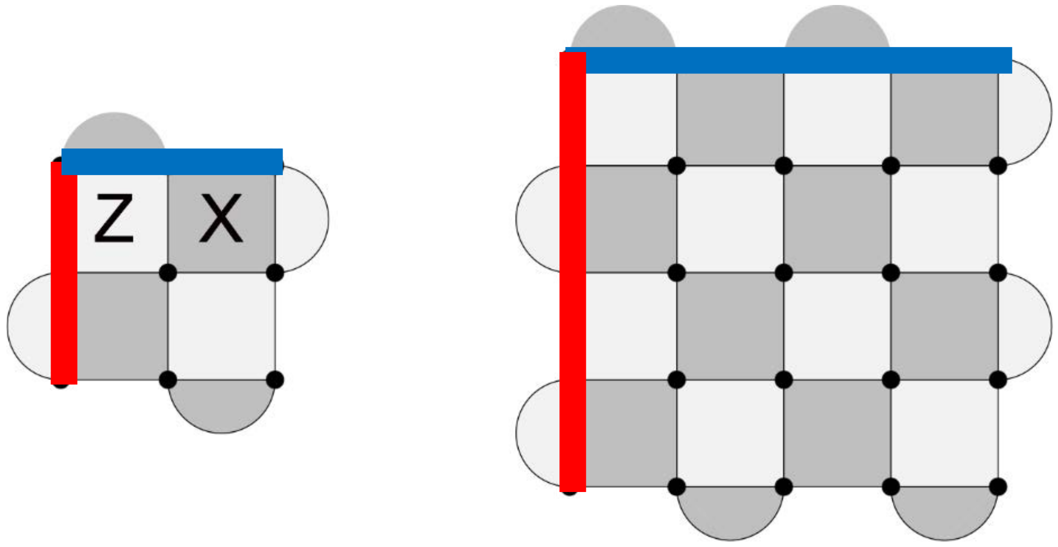

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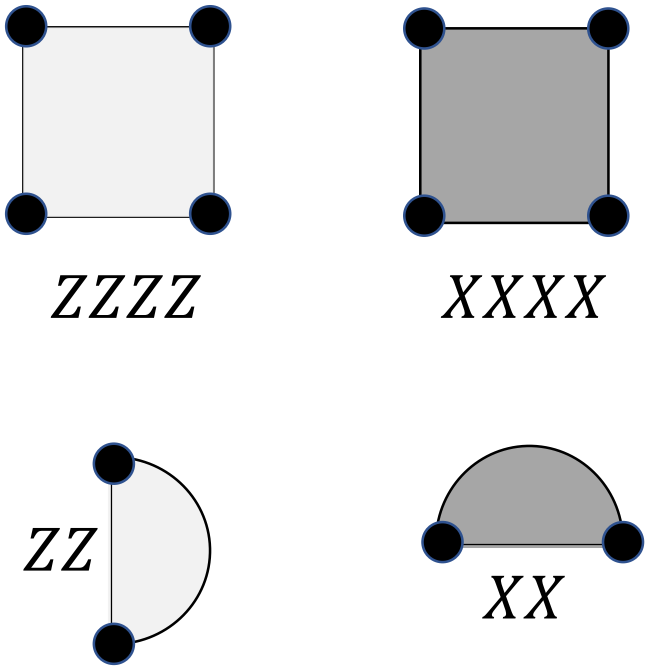
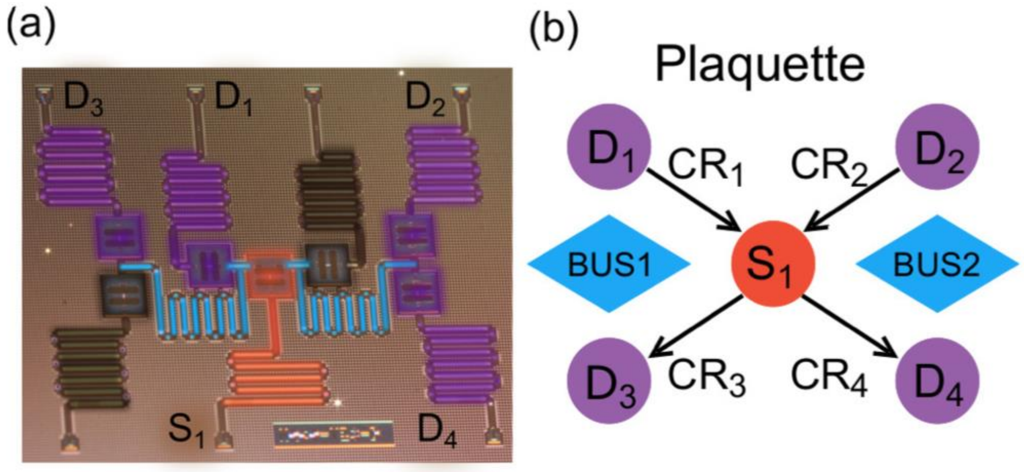


$$\left.\begin{aligned} X^L &= X^{\otimes d} \\ Z^L &= Z^{\otimes d} \end{aligned}\right\} \text{logical Pauli operators}$$

Special case: **surface codes**

High error threshold (above 1%) for the Pauli noise.
Syndrome readout requires only nearest-neighbor gates on a grid.
Fowler et al, PRA (2009)

One of the most attractive candidates for an experimental realization



DiCarlo et al. (TU Delft)
Nature Communications 2015
Physical Review Applied 2017

Takita, Corcoles, et al. (IBM)
Nature Communications 2015, PRL 2016

Barends, Kelly et al. (UCSB) Nature 2014, Nature 2015

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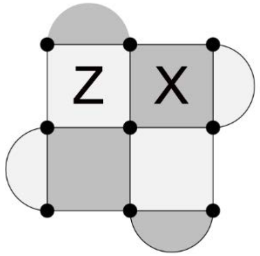
What about coherent noise ?

Our results

- Large-scale simulation of the surface codes subject to coherent errors such as systematic Z-rotations. Simulated systems with up to 2400 qubits.

- Efficient and exact simulation algorithm. Runtime: $O(d^4)$

- Estimates of the error threshold and the effective logical channel.



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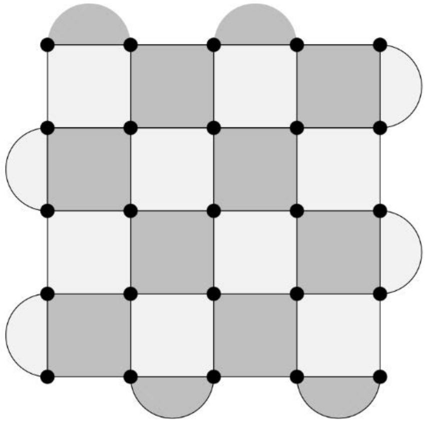

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Result 1: simulating storage of logical states with coherent errors $\boldsymbol{U = \exp(i\theta Z)}$

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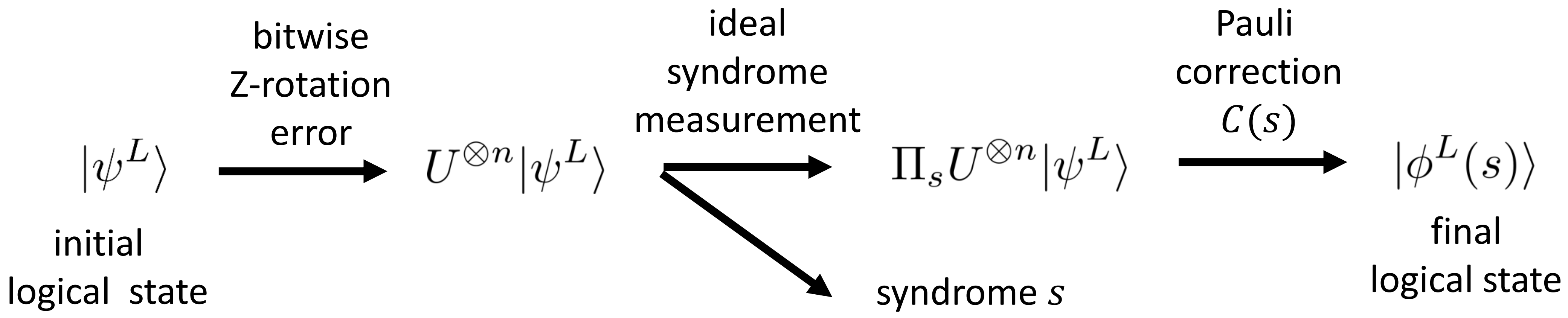

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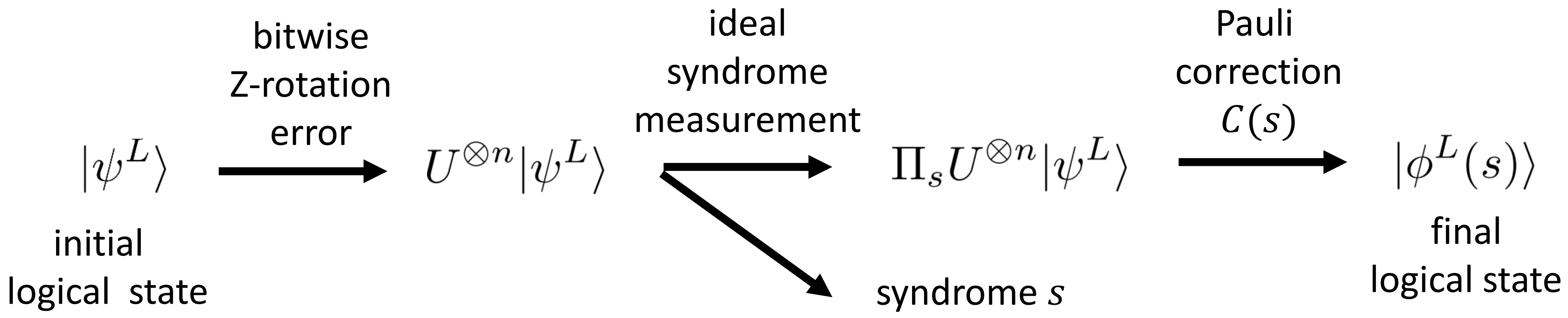

- **Input**: distance d, angle θ, initial state ψ^L
- **Output**: syndrome s, final logical state $\phi^L(s)$

Result 1: simulating storage of logical states with coherent errors $U = \exp(i\theta Z)$

$$|\psi^L\rangle \xrightarrow{\text{bitwise Z-rotation error}} U^{\otimes n}|\psi^L\rangle \xrightarrow{\text{ideal syndrome measurement}} \Pi_s U^{\otimes n}|\psi^L\rangle \xrightarrow{\text{Pauli correction } C(s)} |\phi^L(s)\rangle$$

initial logical state

syndrome s

final logical state

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The Z-rotation angle can be qubit-dependent.

Result 2: simulating logical state preparation with coherent SU(2) errors

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Surface code enables preparation of logical-X (or Z) basis states by initializing each physical qubit in the X-basis, measuring the syndrome, and applying a Pauli correction:

$$\underbrace{|+\rangle^{\otimes n}}_{\text{physical basis state}} \xrightarrow{\text{measure syndrome } s} \quad \xrightarrow{\text{apply Pauli correction } C(s)} \underbrace{|+^L\rangle}_{\text{logical basis state}}$$

Result 2: simulating logical state preparation with coherent SU(2) errors

We simulated a noisy version of this protocol with **errors in the initial state preparation**:

$$\underset{\text{product state}}{|\psi\rangle^{\otimes n}} \xrightarrow{\text{measure syndrome } s} \quad \xrightarrow{\text{apply Pauli correction } C(s)} \underset{\text{final logical state}}{|\phi^L(s)\rangle}$$

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Input: distance d, initial state $|\psi\rangle \in \mathbf{C}^2$

Runtime: $O(d^4)$.

Output: syndrome s, final state $\phi^L(s)$

Our algorithms rely on a mapping of the surface code to a system of Majorana fermions

Wen (2003)

Kitaev (2005)

Terhal, Hassler, DiVincenzo (2012)

VOLUME 90, NUMBER 1 PHYSICAL REVIEW LETTERS week ending 10 JANUARY 2003

Quantum Orders in an Exact Soluble Model

Xiao-Gang Wen

Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

(Received 1 May 2002; published 10 January 2003)

We find all the exact eigenstates and eigenvalues of a spin-1/2 model on square lattice: $H = 16g \sum_i S_i^y S_{i+\hat{x}}^x S_{i+\hat{x}+\hat{y}}^y S_{i+\hat{y}}^x$. We show that the ground states for $g < 0$ and $g > 0$ have different quantum orders described by *Z2A* and *Z2B* projective symmetry groups. The phase transition at $g = 0$ represents a new kind of phase transition that changes quantum orders but not symmetry. Both the *Z2A* and *Z2B* states contain Z_2 lattice gauge theories at low energies. They have robust topologically degenerate ground states and gapless edge excitations.

DOI: 10.1103/PhysRevLett.90.016803 PACS numbers: 73.43.Nq, 03.65.Fd, 03.67.Lx, 11.15.–q

Previous work on simulation of QEC with coherent errors

- Surface code. Simulation by tensor network algorithms (PEPS).
  Runtime is exponential in the code distance d.
  Works for any single-qubit noise channels
  Darmawan and Poulin, PRL 2017.

- Repetition code (1D surface code). Analytic solution.
  Greenbaum and Dutton, Quant. Sci. Technol. 2018

- Repetition code with the circuit-based noise model (noisy encoding/decoding).
  Simulation by mapping to dynamics of free fermions.
  Suzuki, Fujii, and Koashi, PRL 2017

Outline

- Majorana representation of the surface code
- Sketch of the simulation algorithm
- Numerical results

Majorana fermions

n qubits = $2n$ Majorana modes

$$c_1 = X_1 \qquad c_3 = Z_1 X_2 \qquad \ldots \qquad c_{2n-1} = Z_1 \cdots Z_{n-1} X_n$$

$$c_2 = Y_1 \qquad c_4 = Z_1 Y_2 \qquad \qquad c_{2n} = Z_1 \cdots Z_{n-1} Y_n$$

Commutation rules:

$$c_p c_q = -c_q c_p \quad \text{if} \quad p \neq q \qquad\qquad c_p^2 = I$$

Products of Majorana operators c_p form an operator basis of n qubits

Suppose $|\phi\rangle$ is a normalized n-qubit state. Define a covariance matrix

$$M_{p,q} = \begin{cases} \langle\phi|ic_p c_q|\phi\rangle & \text{if} \quad p \neq q \\ 0 & \text{if} \quad p = q \end{cases}$$

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We say that $|\phi\rangle$ is a **Gaussian state** if it obeys Wick's theorem:

$$-\langle \phi | c_p c_q c_r c_s | \phi \rangle = M_{p,q} M_{r,s} - M_{p,r} M_{q,s} + M_{p,s} M_{q,r}$$

(and similar formulas for the higher-order correlators)

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A Gaussian state is fully specified by its covariance matrix M.
This requires only $O(n^2)$ real parameters.

Fermionic Linear Optics (Knill 2001, DiVincenzo and Terhal 2002)

- Operations that map Gaussian states to Gaussian states.
- Can be efficiently simulated by keeping track of the covariance matrix

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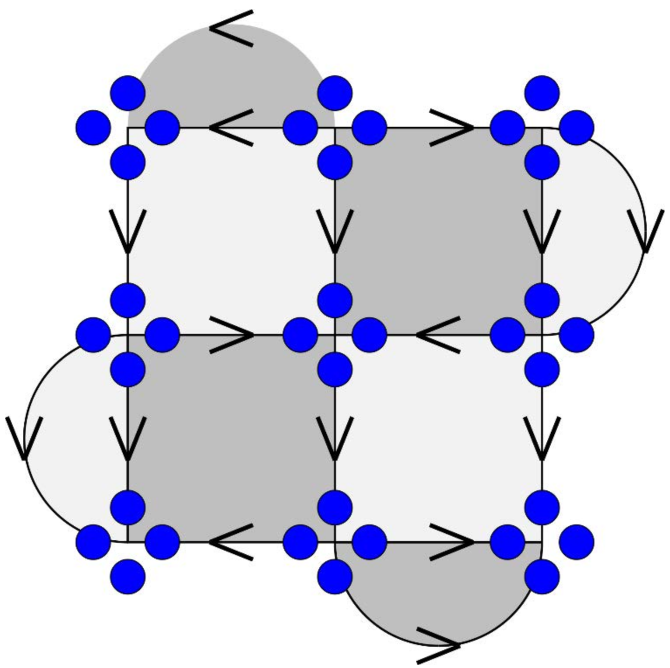
two-mode parity projector	$\frac{1}{2}(I + i c_p c_q)$	$O(n^2)$
two-mode initialization	$i c_p c_q \lvert 0\rangle = \lvert 0\rangle$	$O(n)$

Majorana representation of the surface code Wen (2003)



$4n$ Majorana fermions $c_1, \dots, c_{4n}$ (blue dots)

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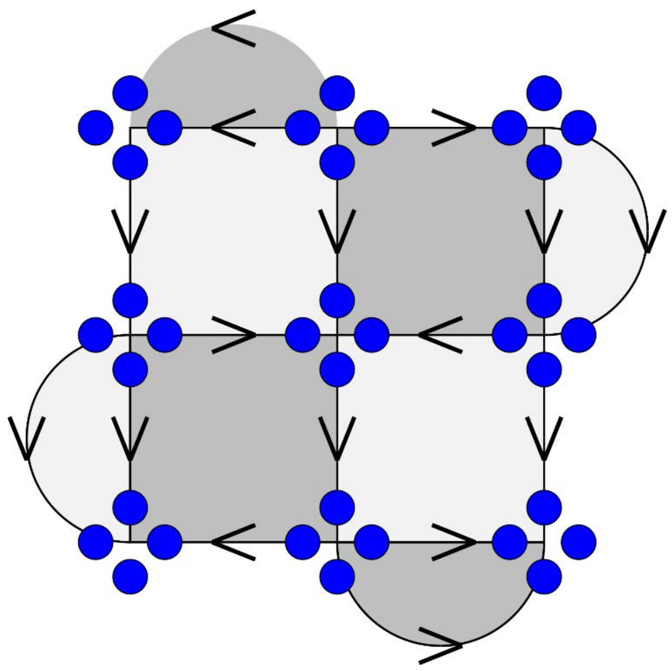

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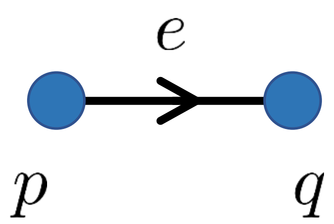
Edge operators:



$$A_e = ic_p c_q$$

Majorana commutation rules imply $A_e A_{e'} = A_{e'} A_e$

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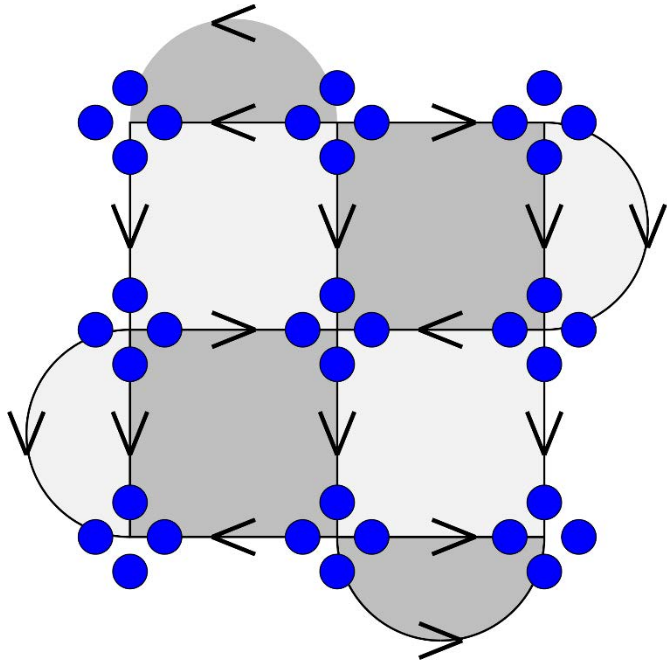

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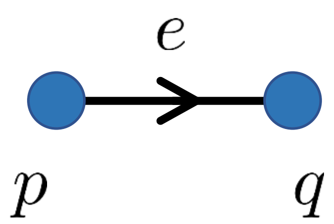


$$A_e = ic_pc_q$$

Majorana commutation rules imply $\quad A_eA_{e'} = A_{e'}A_e$

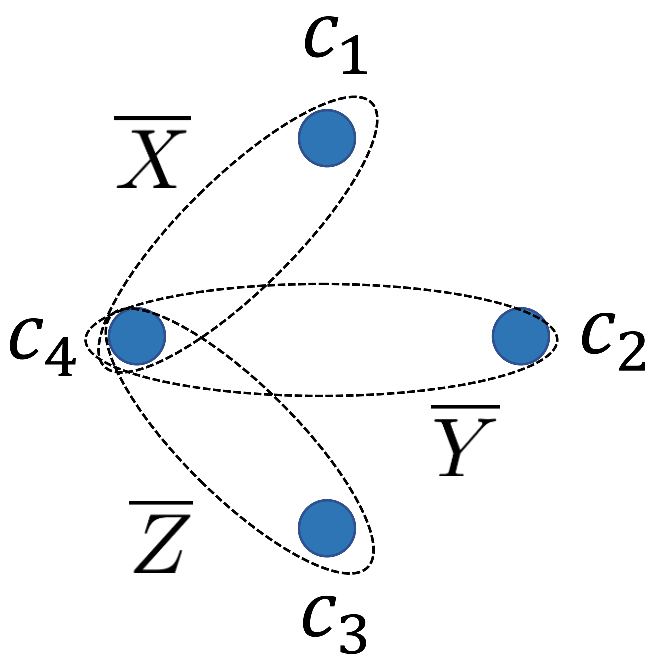
Strategy for simulating syndrome measurements:

Express each plaquette stabilizer as a product of edge operators A_e.

Measure syndromes of the edge operators (two-mode parity measurements).

Each plaquette syndrome is a product of the edge syndromes.

Majorana C4-code:



stabilizer: $S = -c_1 c_2 c_3 c_4$

$$\overline{X} = i c_1 c_4 = i c_2 c_3$$

$$\overline{Y} = i c_2 c_4 = -i c_1 c_3$$

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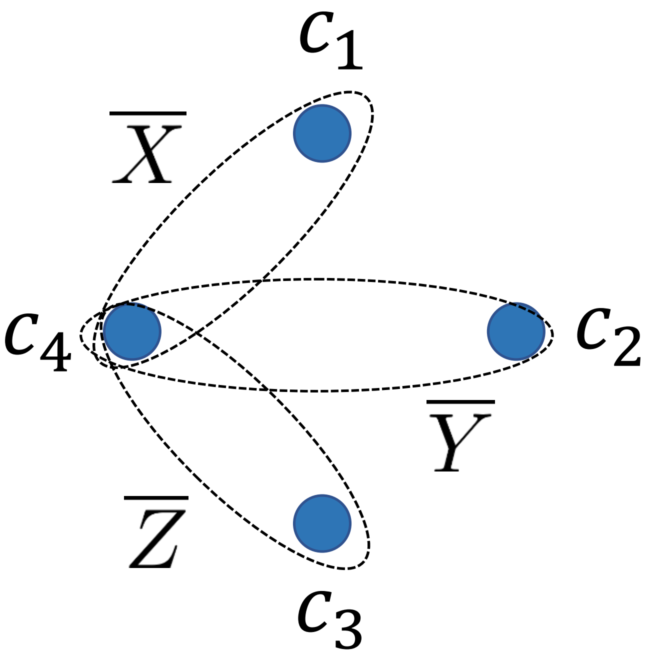

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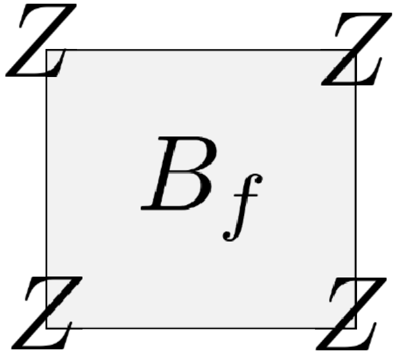
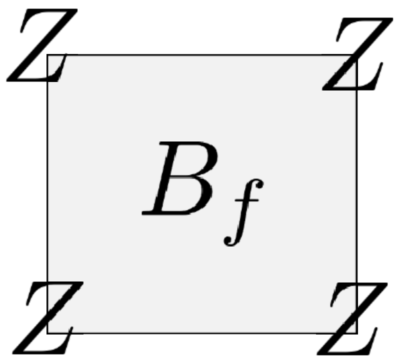
- Any logical state is Gaussian
- Any logical (non-Pauli) operator can be realized by Fermionic Linear Optics



plaquette
operator



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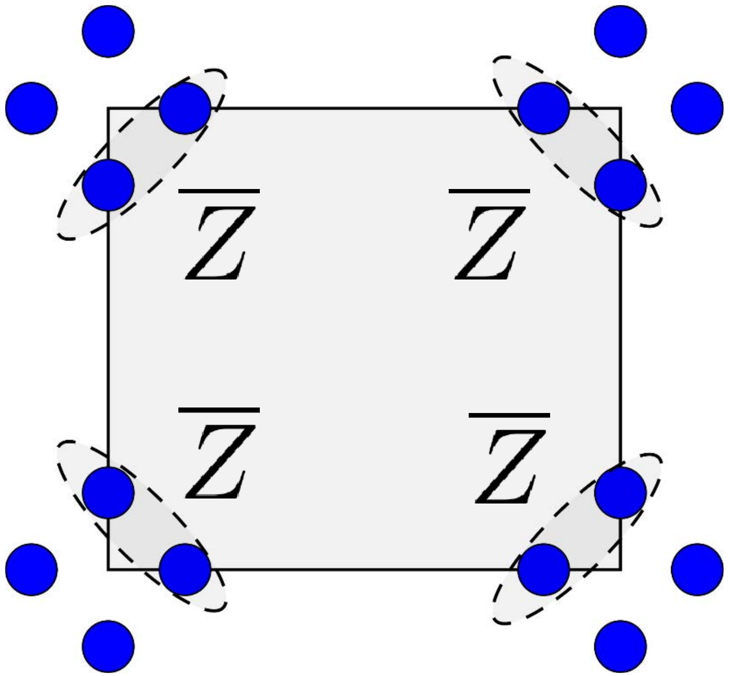
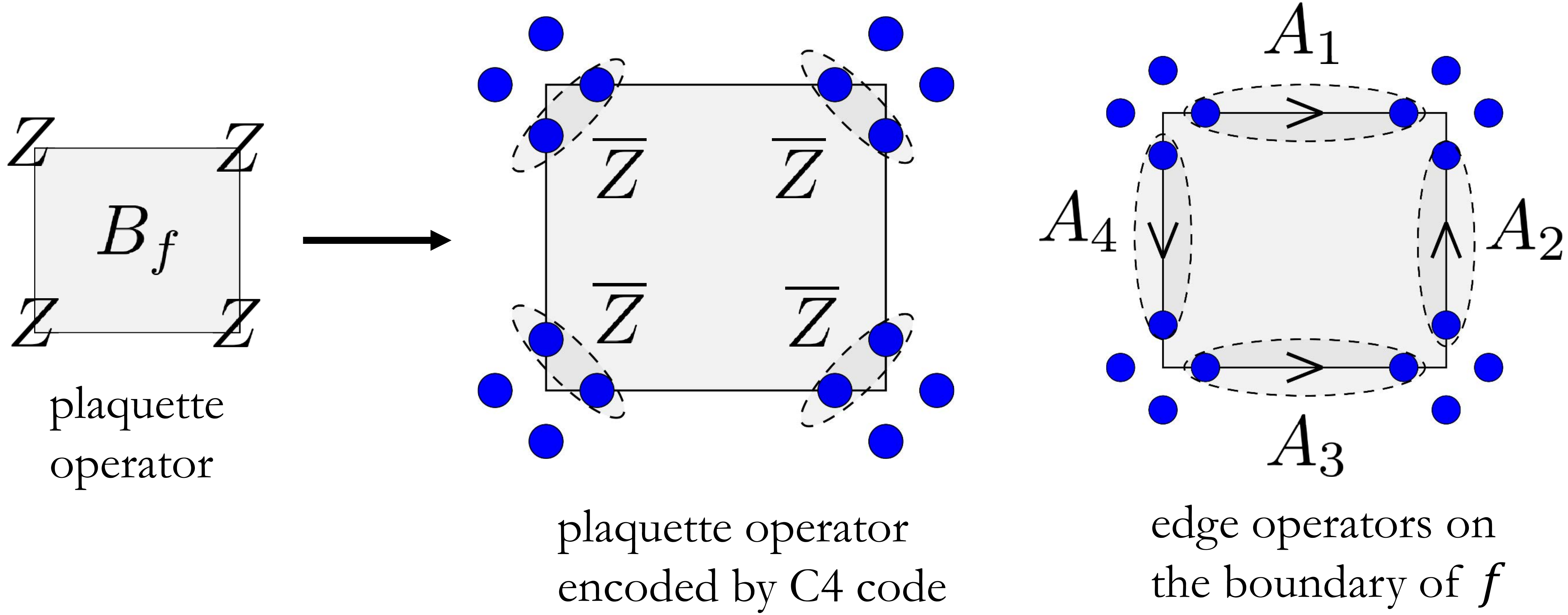


plaquette operator
encoded by C4 code



Z
Z
Z
Z
B_f
plaquette operator
$\overline{Z}$
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$\overline{Z}$
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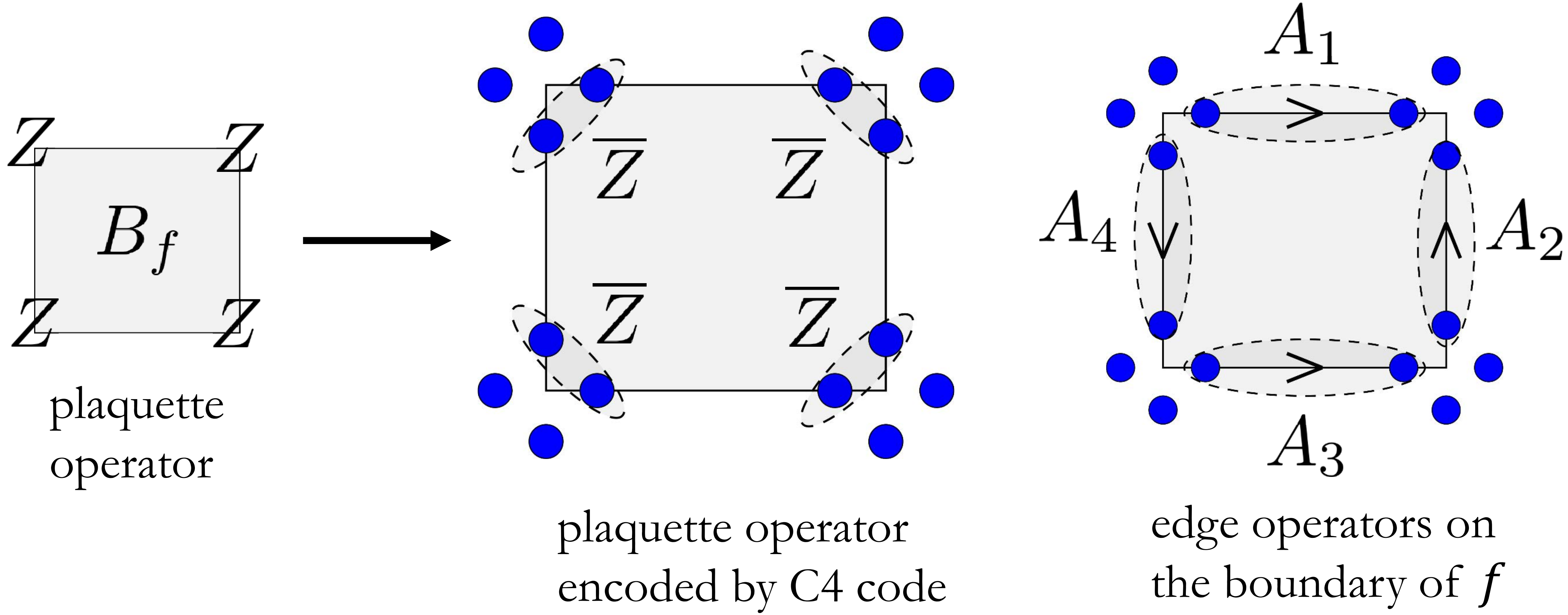
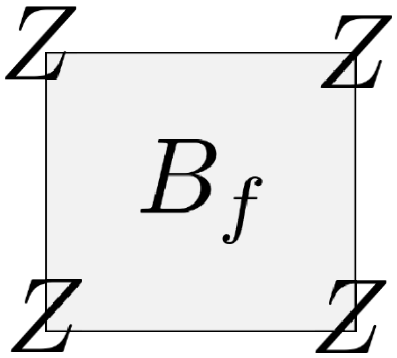
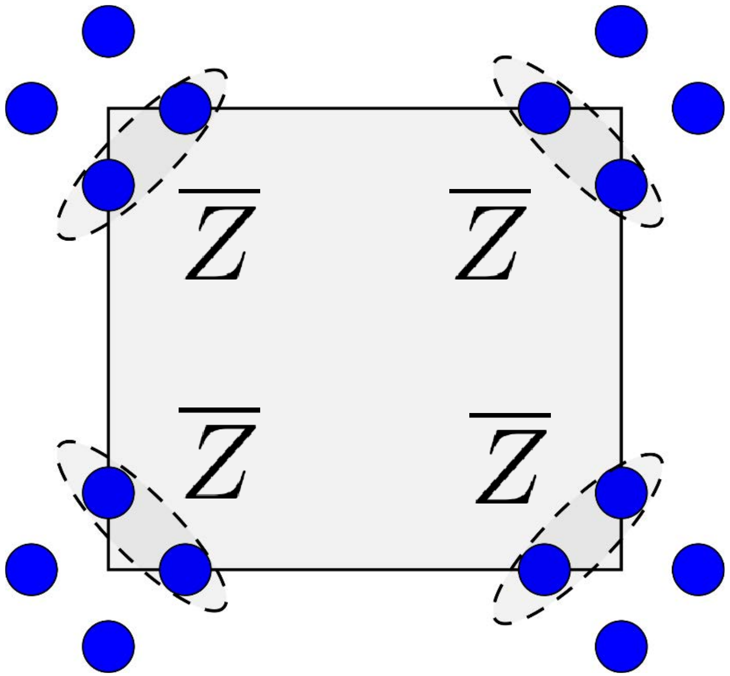
plaquette operator encoded by C4 code
A_1
A_2
A_3
A_4
edge operators on the boundary of f



$$\overline{B}_f = A_1 A_2 A_3 A_4$$



plaquette
operator



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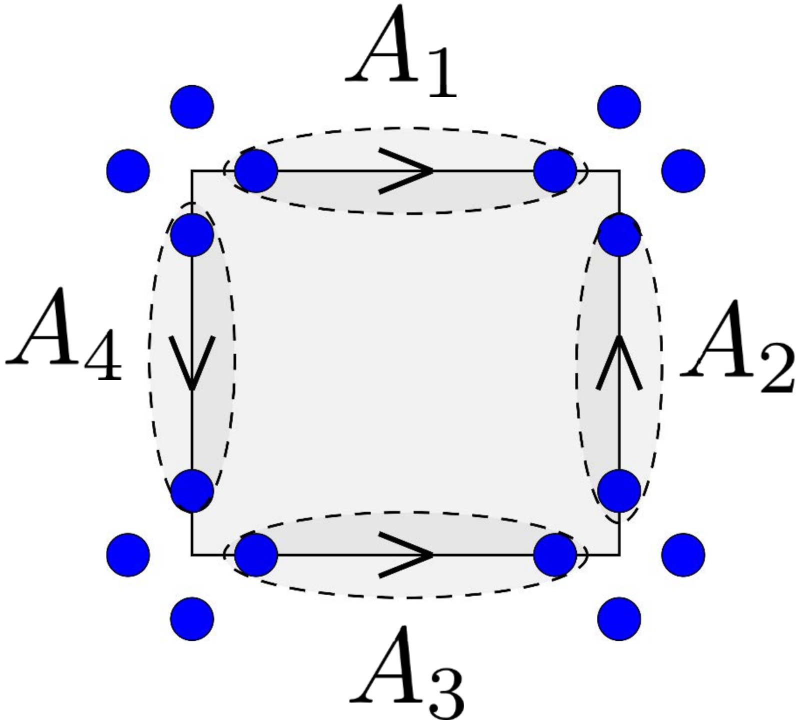

edge operators on
the boundary of f

$$\overline{B}_f = A_1 A_2 A_3 A_4$$

The same formula applies to X-type plaquette operators

Simulating syndrome measurements: overview

Goal: measure syndromes of all plaquette operators B_f on a product state $|\psi^{\otimes n}\rangle$.

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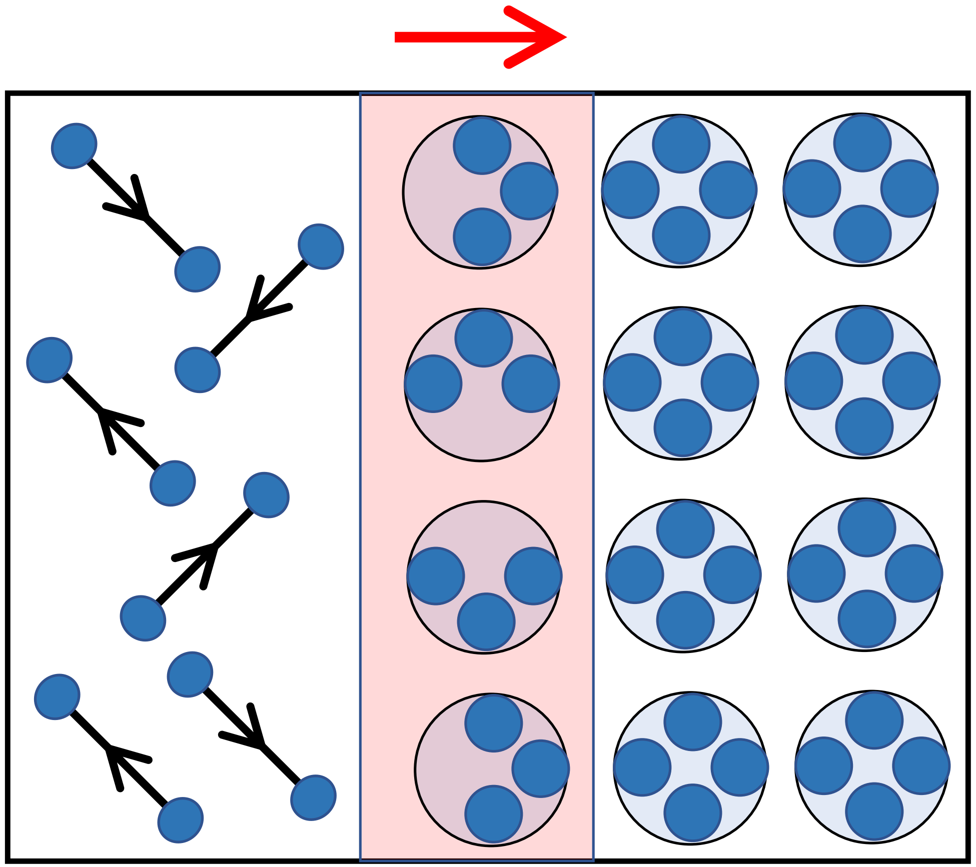
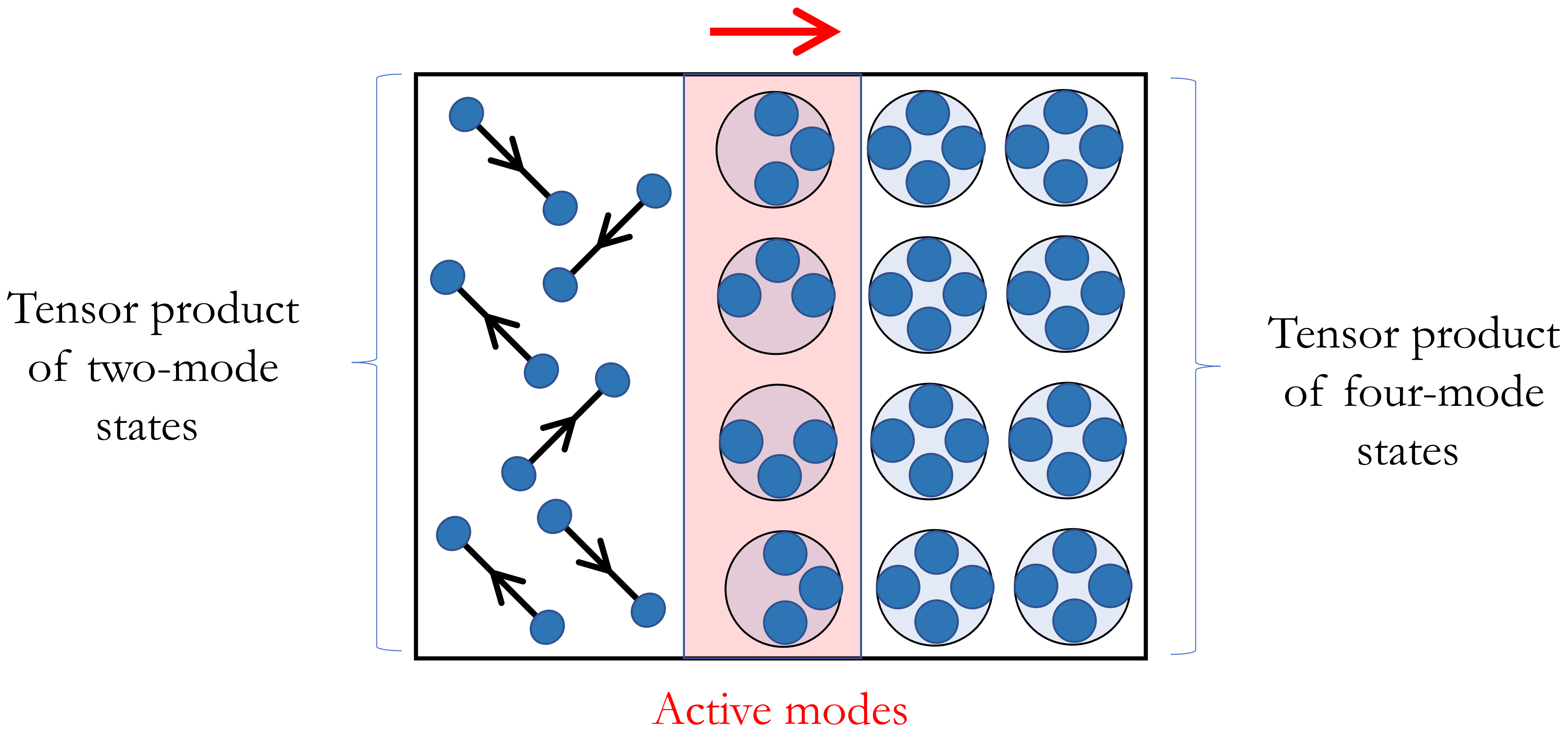
- The state $|\bar{\psi}^{\otimes n}\rangle$ is Gaussian since any logical state of C4 is Gaussian.
- Simulating two-mode parity measurements on a Gaussian state is easy. Runtime $O(n^2)$ per measurement. Overall runtime is $O(n^3)$.

Runtime can be improved by measuring edge operators in a specific order:



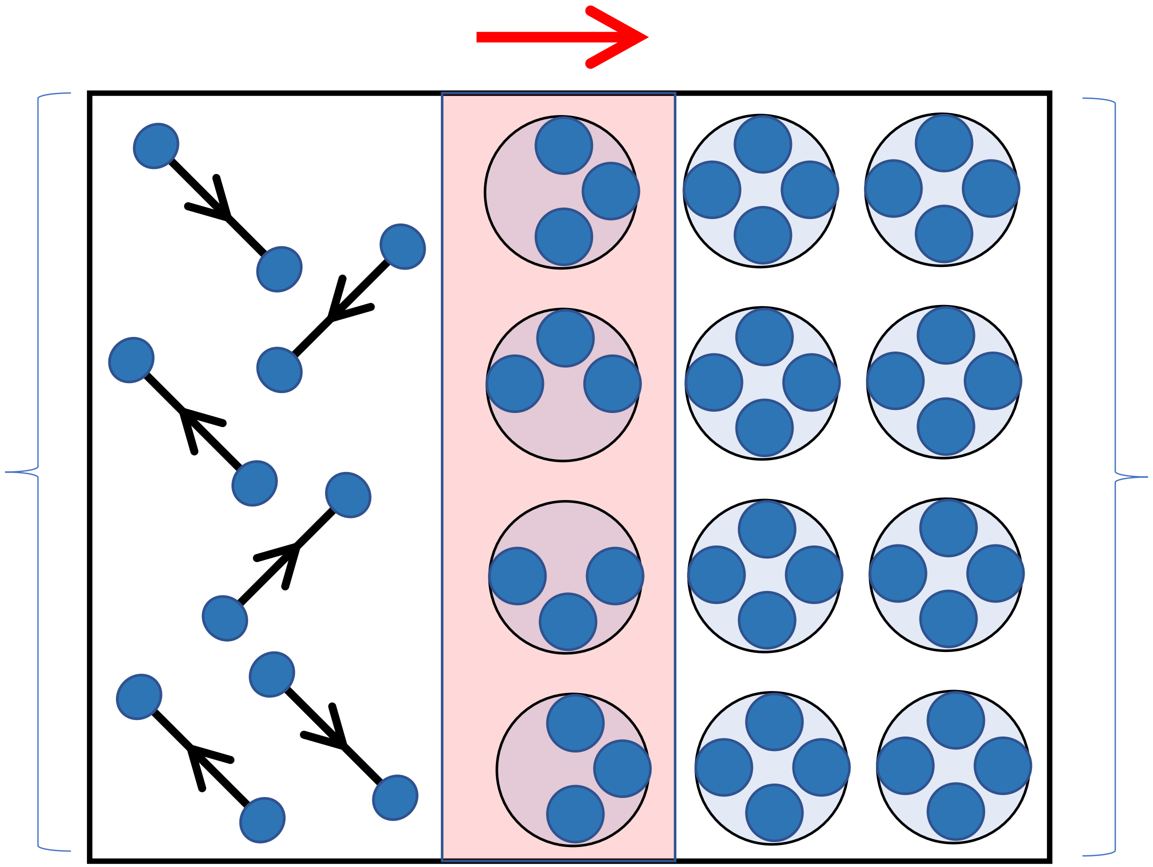
Active modes

Runtime can be improved by measuring edge operators in a specific order:



Runtime can be improved by measuring edge operators in a specific order:

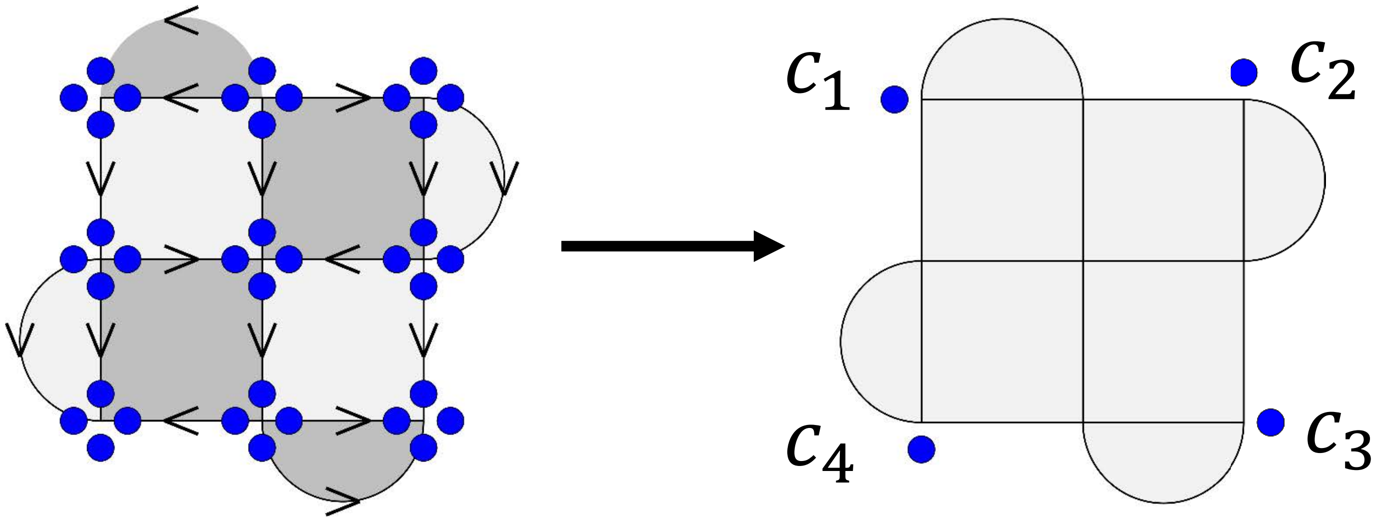
Tensor product of two-mode states



Tensor product of four-mode states

At each time step all except for $O(\sqrt{n})$ "active" modes are in a product state.
Inactive modes can be removed from the simulator. This reduces the runtime to $O(n^2)$

How to measure final logical state:



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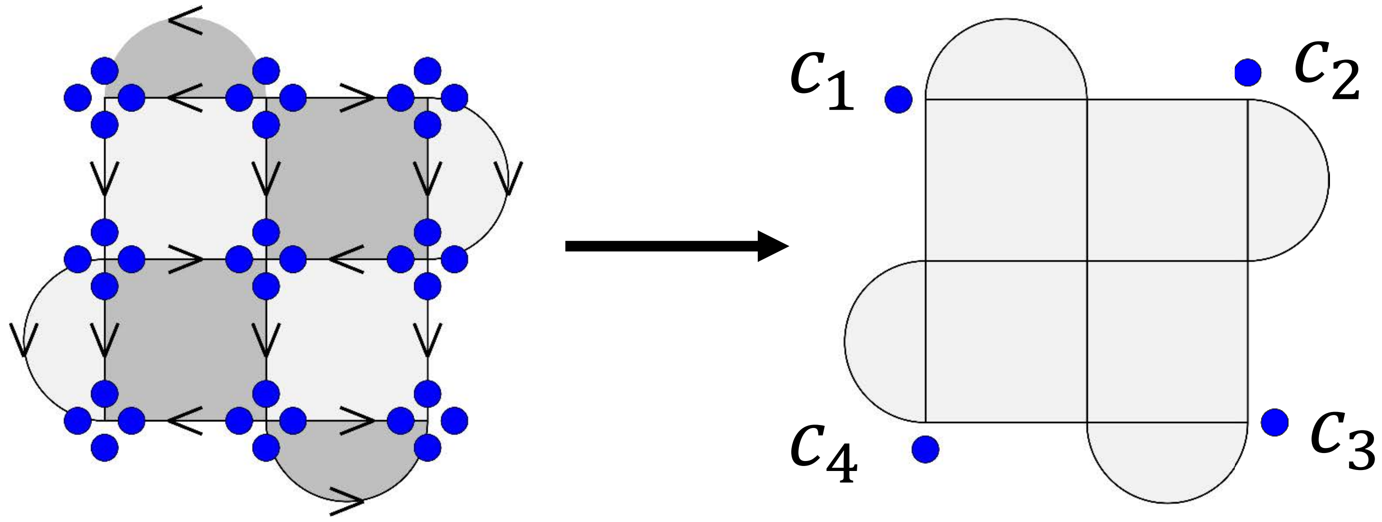
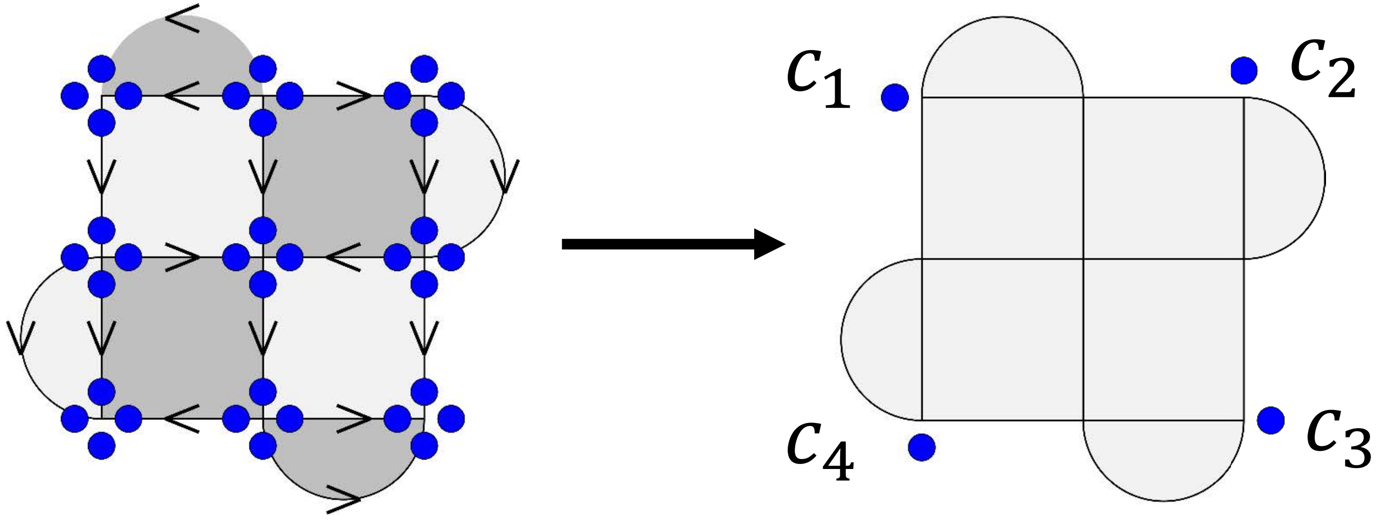


Claim: once all edge operators have been measured, the reduced state of the four corner modes is the final logical state encoded by the C4-code

How to measure final logical state:



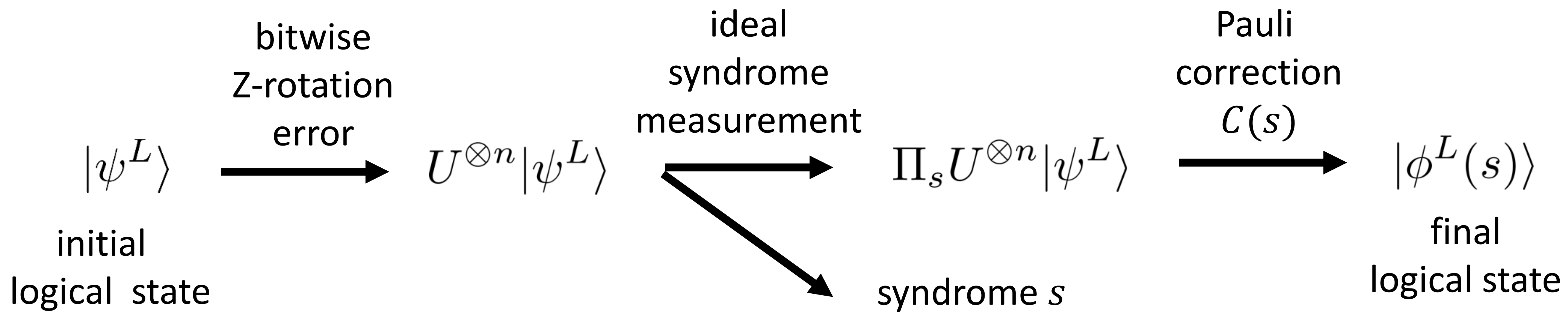
Claim: once all edge operators have been measured, the reduced state of the four corner modes is the final logical state encoded by the C4-code

The logical Bloch vector is determined by expectation values of the "logical" edge operators $\bar{X} = ic_1c_4, \quad \bar{Y} = ic_2c_4, \quad \bar{Z} = ic_3c_4$

Simulating storage of a logical state: sketch of main ideas

$$
\begin{tikzcd}
|\psi^L\rangle \arrow[r, "\text{bitwise Z-rotation error}"] & U^{\otimes n}|\psi^L\rangle \arrow[r, "\text{ideal syndrome measurement}"] \arrow[dr] & \Pi_s U^{\otimes n}|\psi^L\rangle \arrow[r, "\text{Pauli correction } C(s)"] & |\phi^L(s)\rangle \\
\text{initial logical state} & & \text{syndrome } s & \text{final logical state}
\end{tikzcd}
$$

Simulating storage of a logical state: sketch of main ideas



Encode each qubit of the initial logical state by C4. This results in a non-Gaussian state.

Measure syndrome of X-stabilizers by bitwise X-measurements.

Construct a sequence of Gaussian states that results in the same measurement statistics for bitwise X-measurements.

Structure of the logical channel

We shall only consider Z-rotation errors $U = \exp(i\theta Z)$

Syndrome probabilities: $p(s) = \|\Pi_s U^{\otimes n}|\psi^L\rangle\|^2$

syndrome projector

initial logical state

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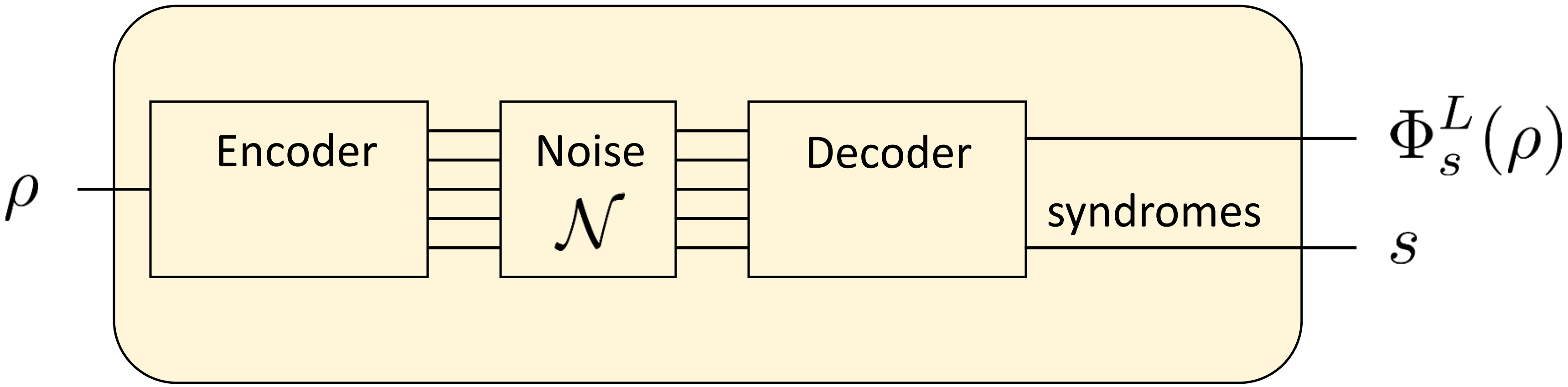
Lemma (logical rotation angle)

The syndrome probability distribution $p(s)$ does not depend on the initial logical state.
The logical qubit undergoes a Z-rotation by some angle $\theta(s)$.

Structure of the logical channel



logical channel

$$\Phi_s^L(\rho) = e^{i\theta(s)\overline{Z}}\rho e^{-i\theta(s)\overline{Z}}$$

Logical error rate: $$P^L = 2\sum_s p(s)|\sin\theta(s)|$$

average diamond-norm distance between the logical channel Φ_s^L and the identity

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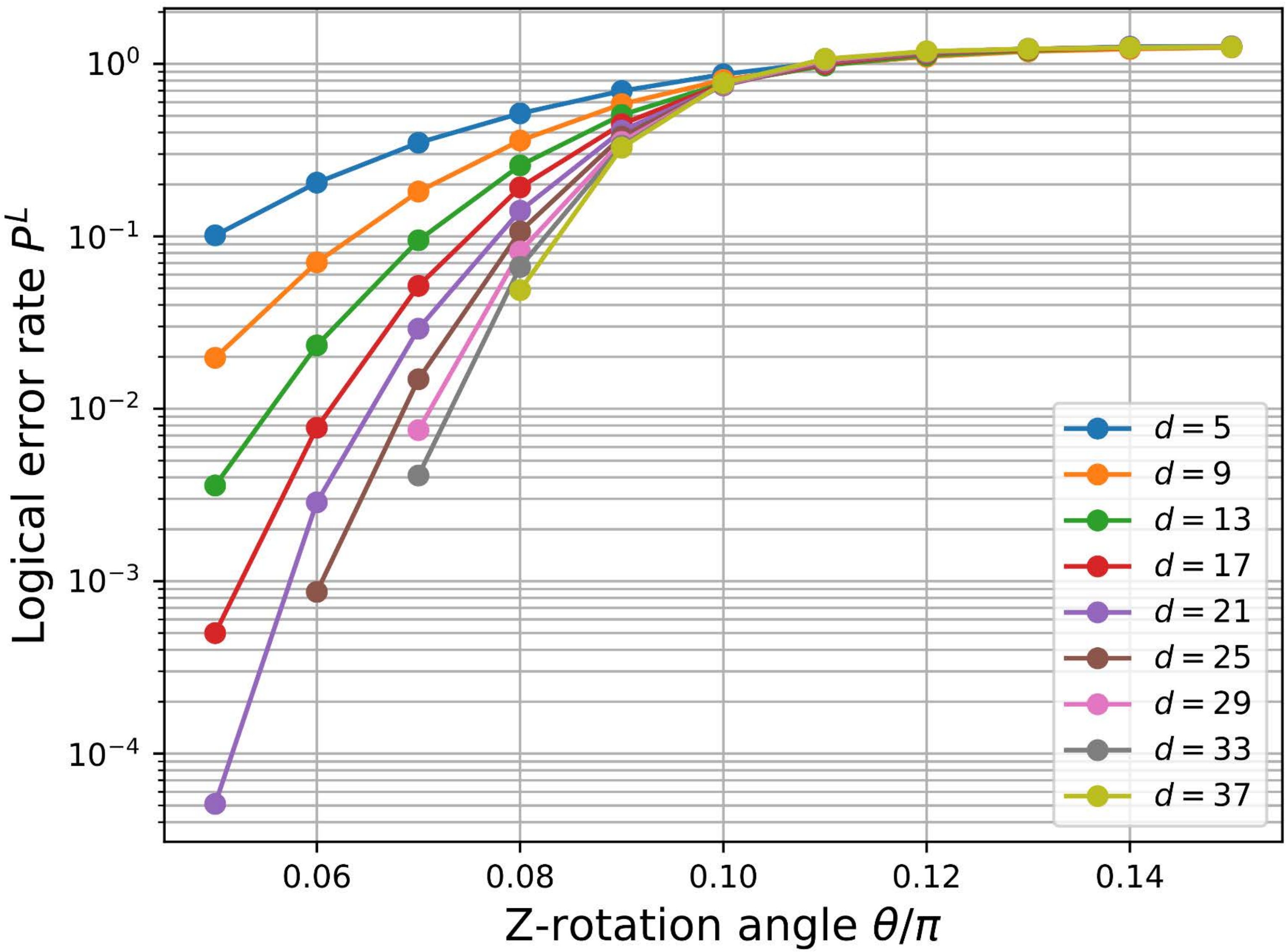
average diamond-norm distance between the logical channel Φ_s^L and the identity

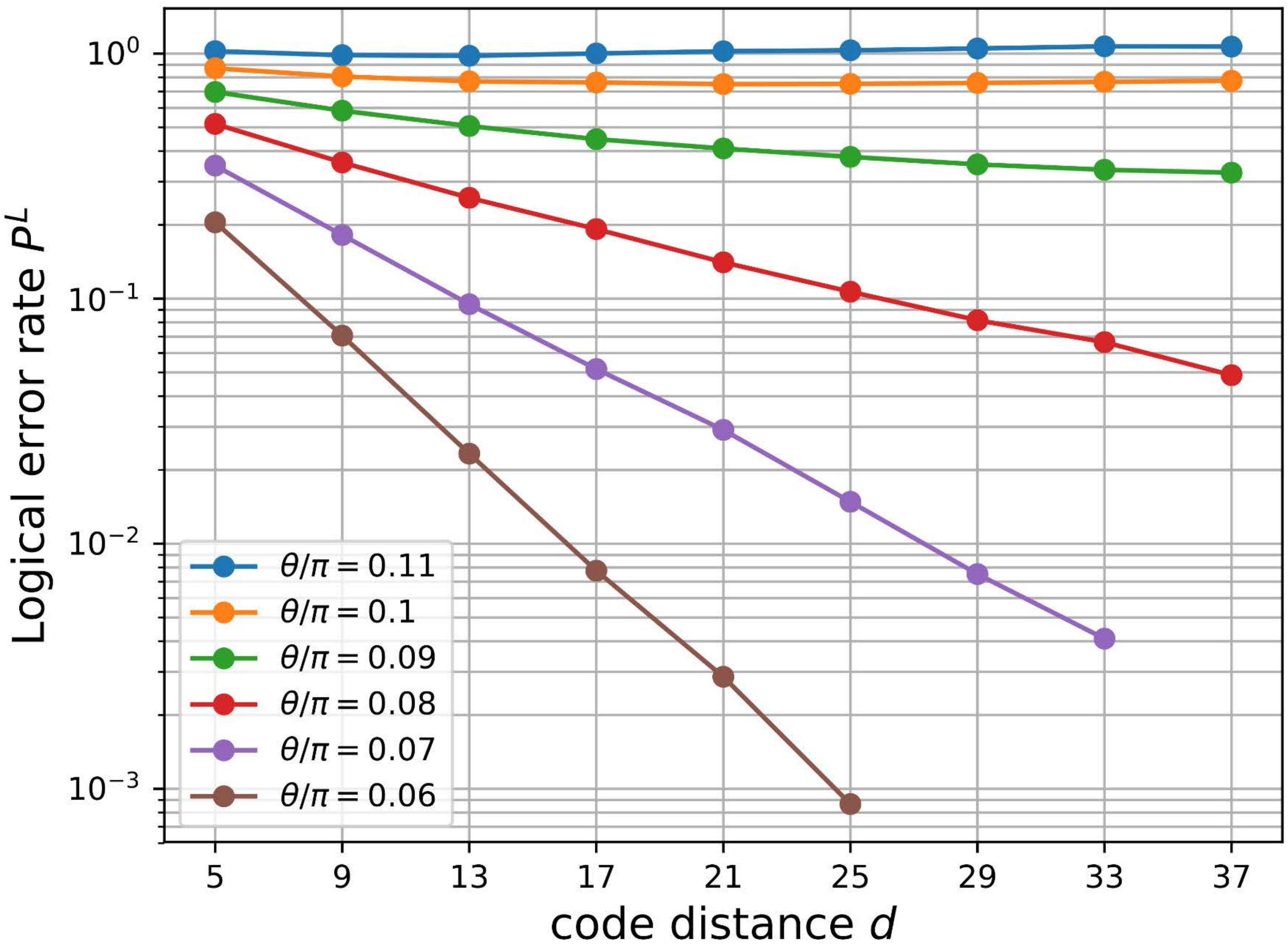
Pauli correction was computed using the min-weight matching decoder.

The decoder does not depend on θ (all edge weights are set to one)



50,000 syndrome samples per data point

Estimated error threshold:
$$0.09\pi \leq \theta_0 \leq 0.11\pi$$



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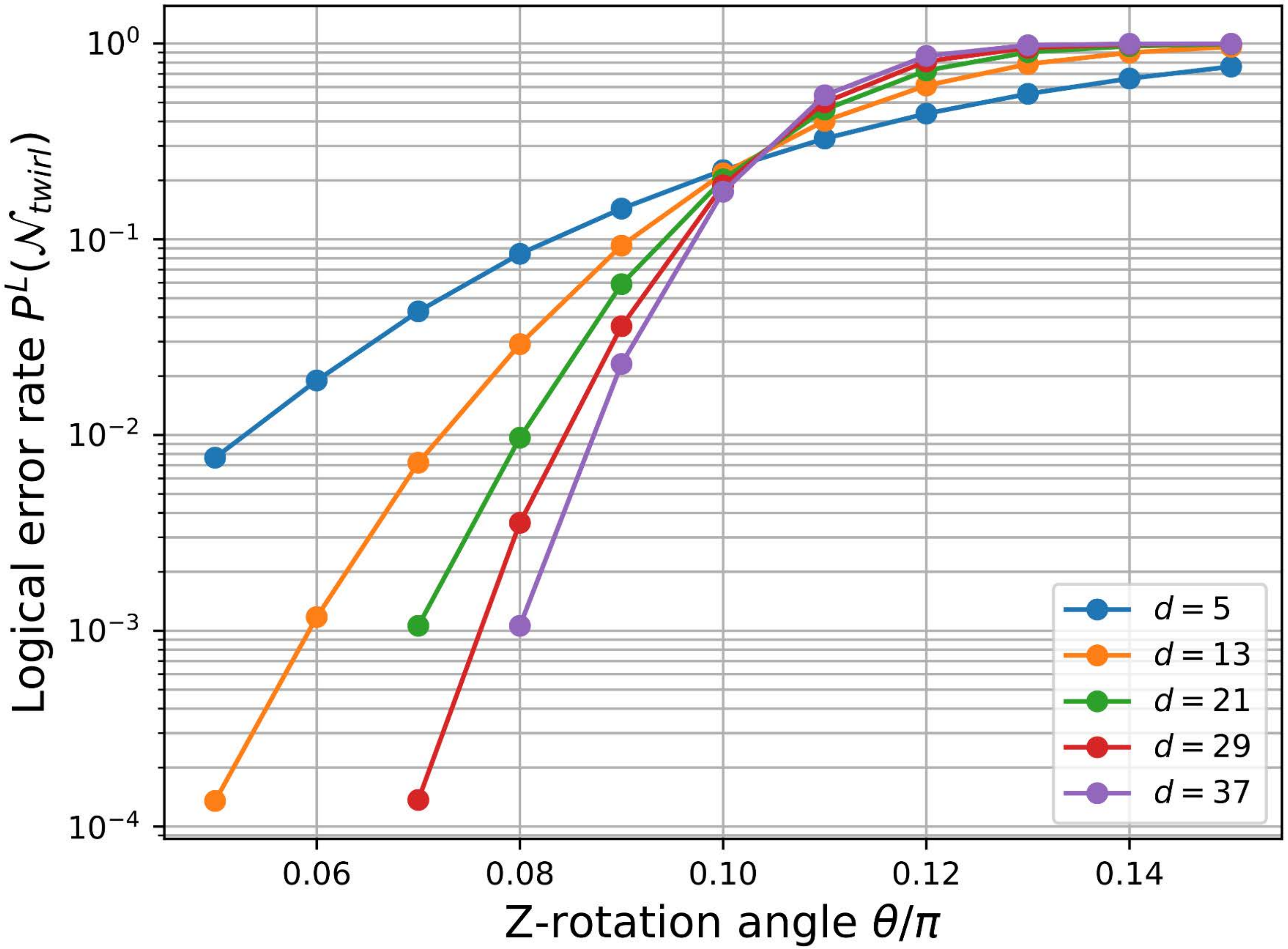
Coherent vs twirled physical noise



Pauli Twirl Approximation: replace coherent physical noise

$$N(\rho) = e^{i\theta Z}\rho e^{-i\theta Z}$$

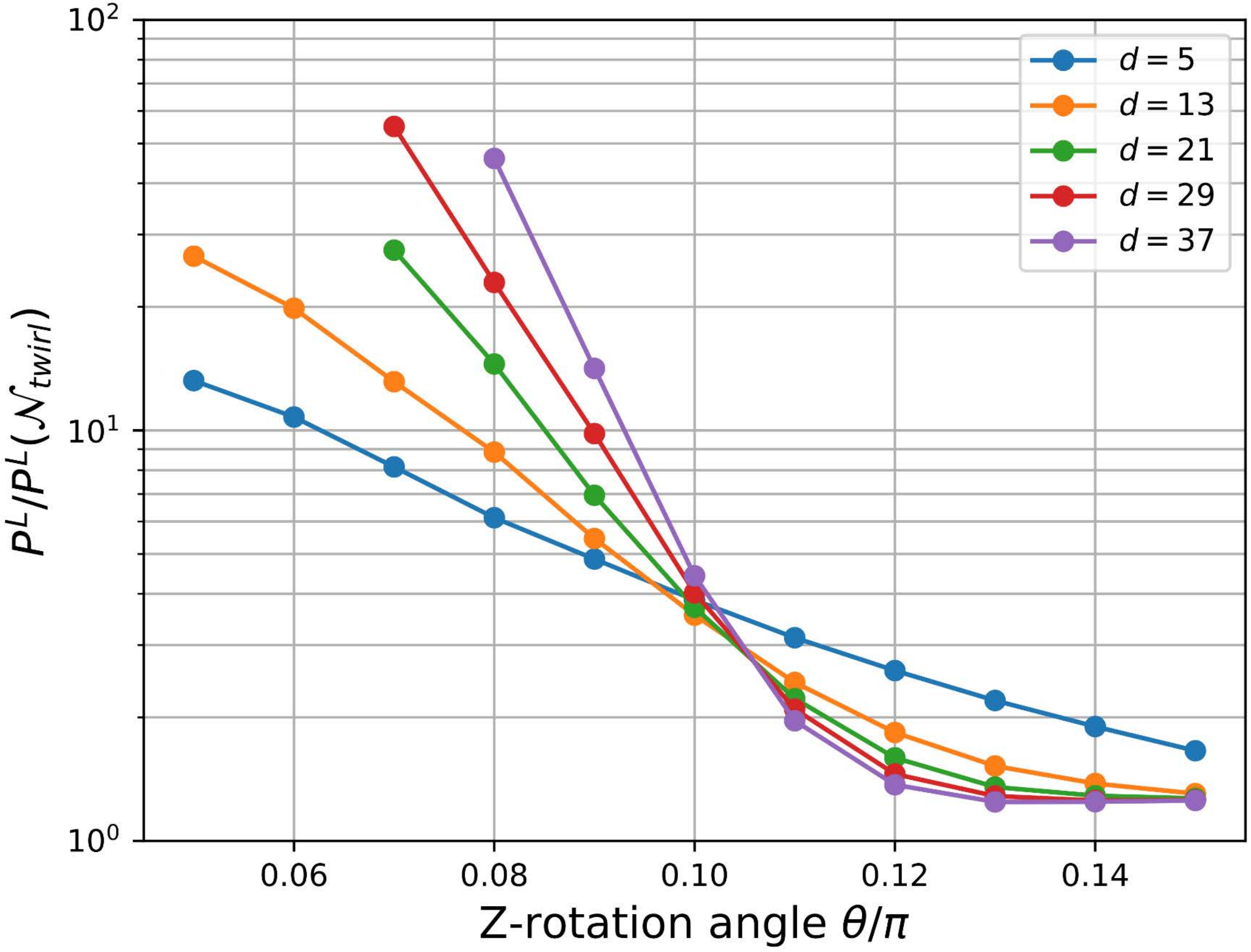
by its Pauli twirled version

$$N_{twirl}(\rho) = (1-\epsilon)\rho + \epsilon Z\rho Z$$

$$\epsilon = \sin^2(\theta)$$

The same error threshold !

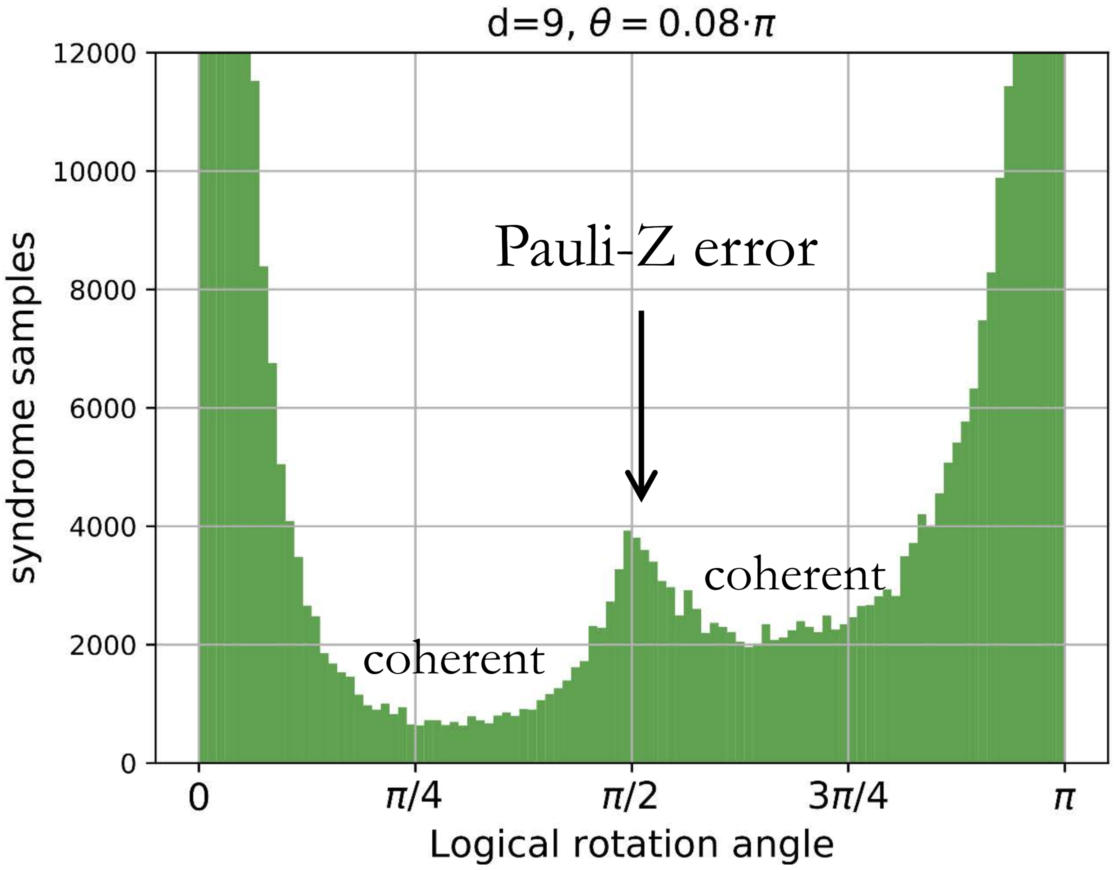
Coherent vs twirled physical noise

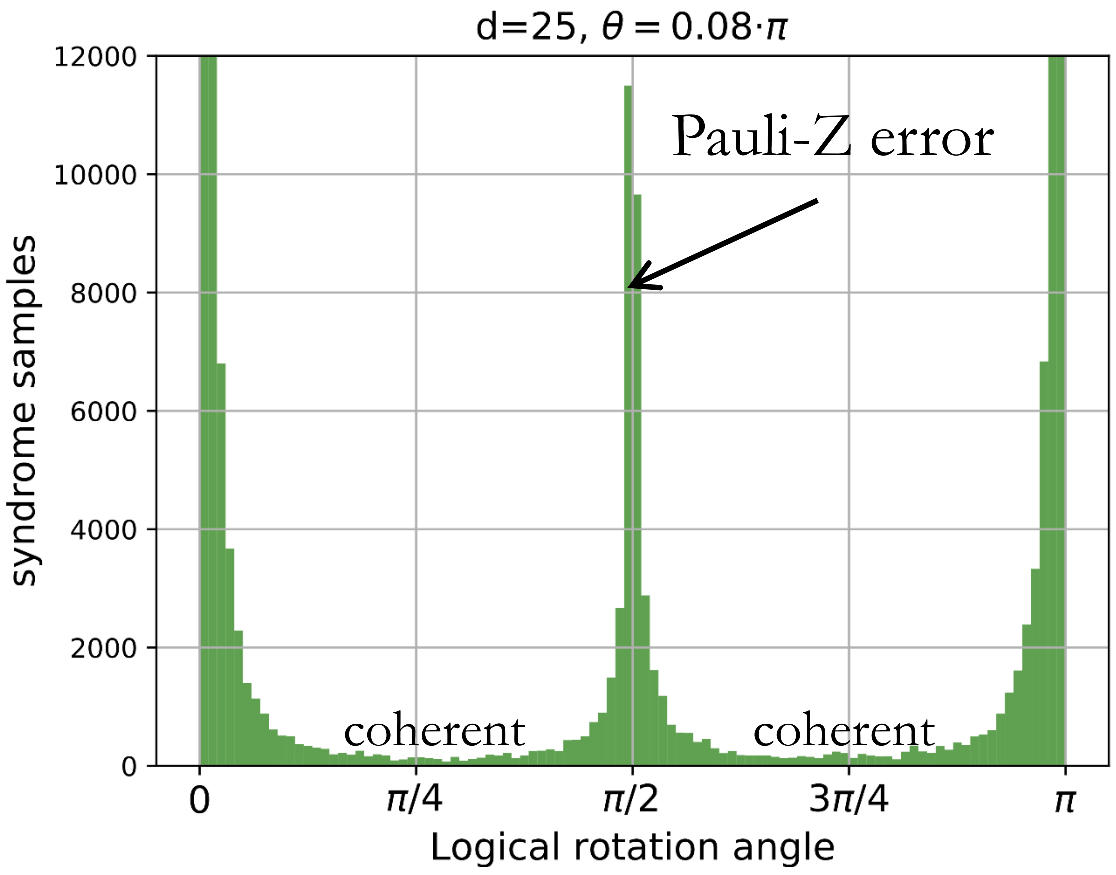


PTA **underestimates** the logical error rate in the sub-threshold regime

PTA gives a good estimate of the error threshold

Logical Z-rotation angle: probability distribution





$d = 9$ $\qquad$ $d = 25$

How to quantify coherence of the logical-level noise ?

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Now we have two different logical error rates:

$$P^L = \sum_s p(s) \|\Phi_s - \mathrm{Id}\|_\diamond \qquad P^L_{twirl} = \sum_s p(s) \|\Phi_s^{twirl} - \mathrm{Id}\|_\diamond \leq P^L$$

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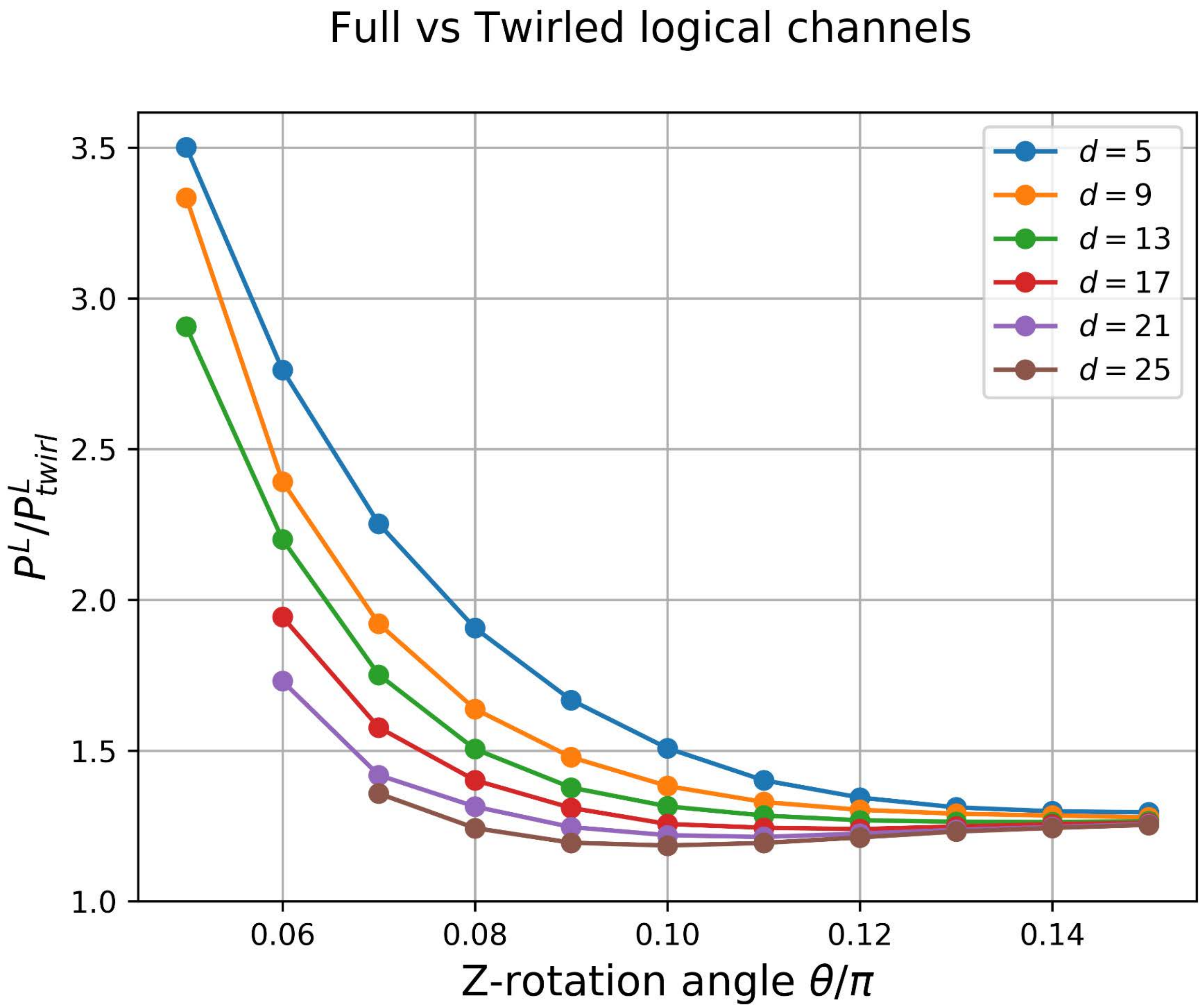
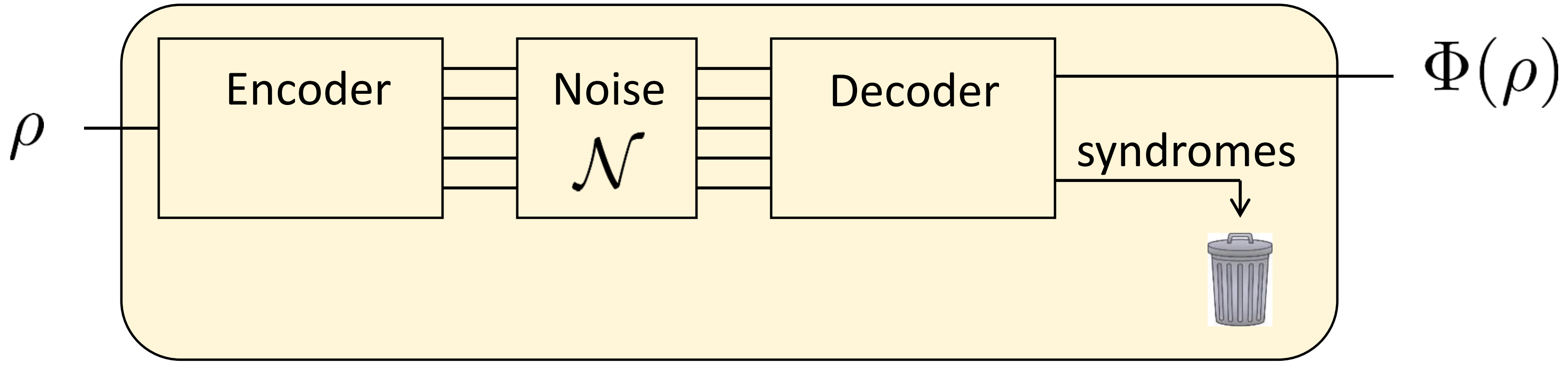
We shall use the ratio P^L/P^L_{twirl} to quantify coherence of the logical noise.



The degree of coherence of the logical channel.

Increasing the code distance makes the logical noise less coherent

Average logical channel



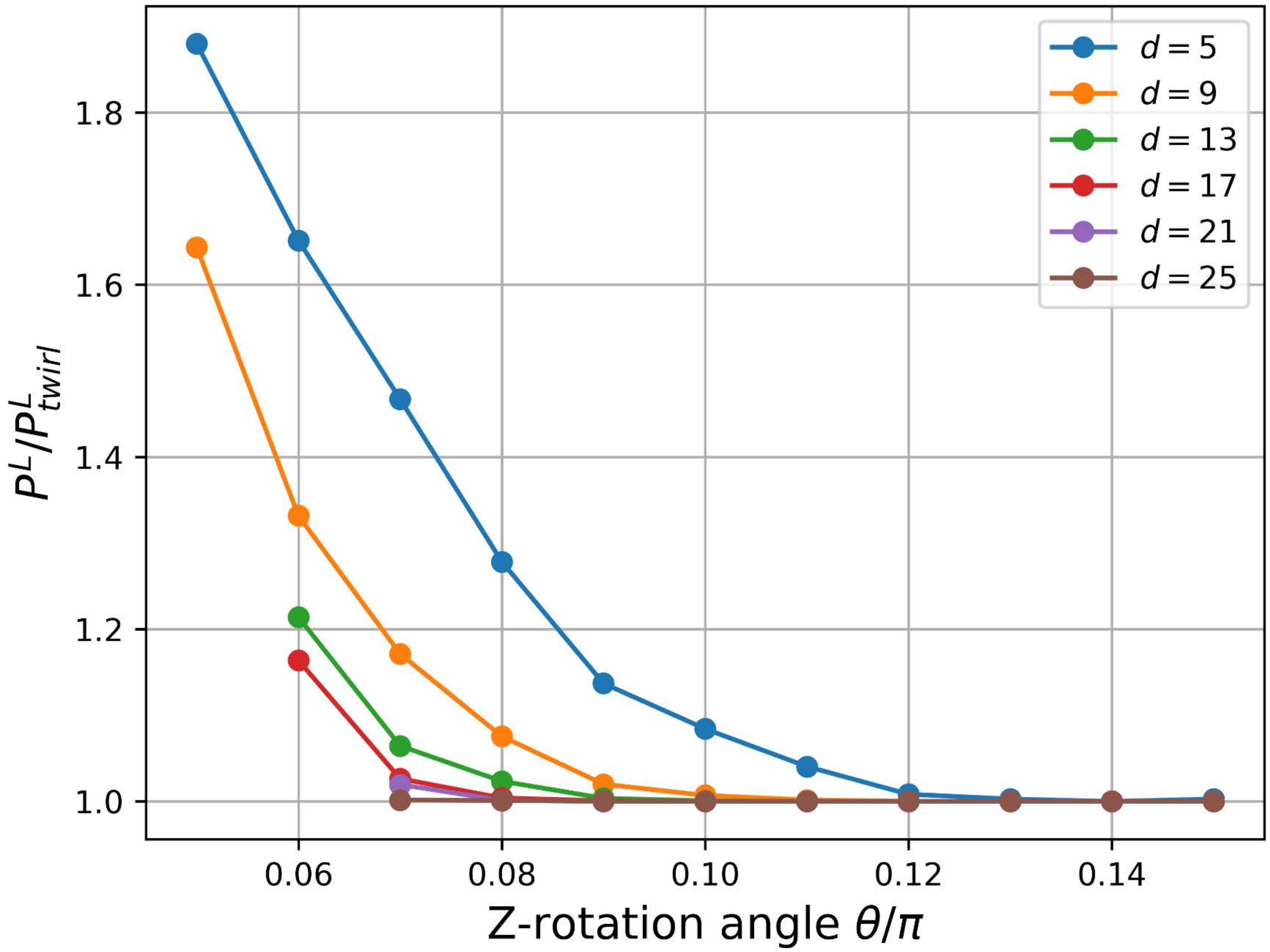
$$\Phi(\rho) = \sum_s p(s) e^{i\eta(s)Z} \rho e^{-i\eta(s)Z}$$

Appropriate model if the environment has no access to the measured syndromes.

Quantify coherence of the average logical channel using the ratio of two error rates

$$P^L = \|\Phi - \mathrm{Id}\|_\diamond \qquad P^L_{twirl} = \|\Phi^{twirl} - \mathrm{Id}\|_\diamond \leq P^L$$

Full vs Twirled logical channels

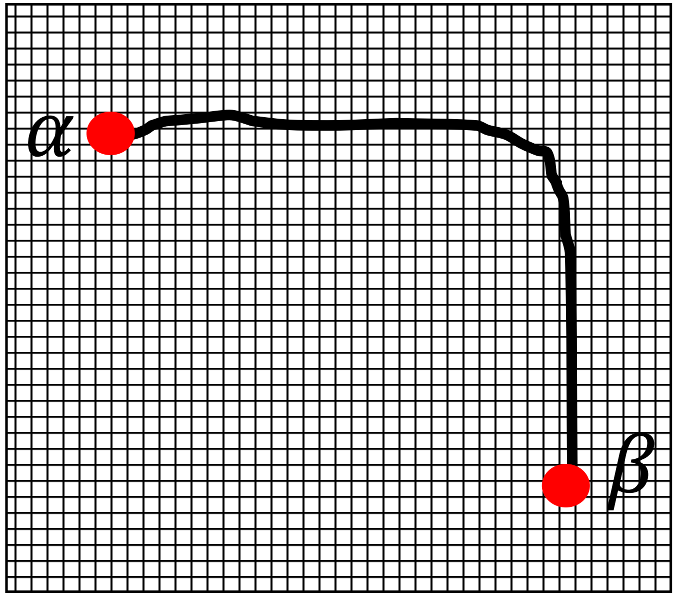


The degree of coherence for the **average logical channel**

Increasing the code distance makes the logical noise less coherent

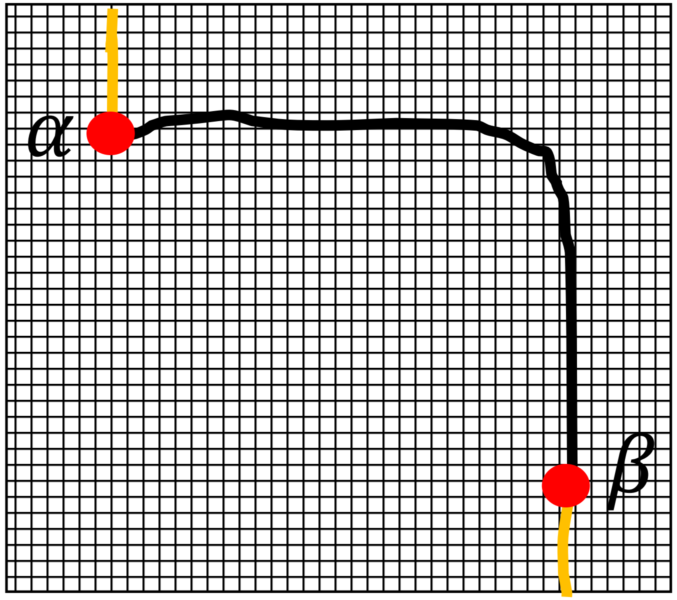
Conjecture:
the average logical channel has no coherent part in the limit $d \to \infty$

Coherent vs Pauli noise models



Example of a uncorrectable Z-error

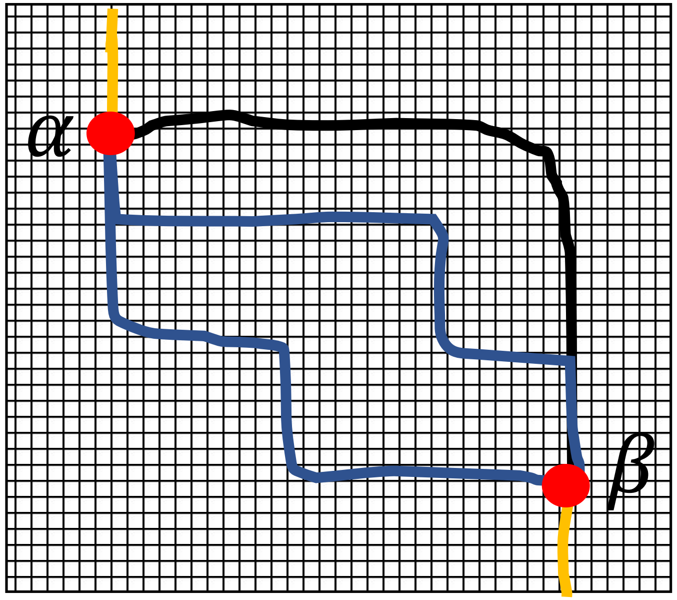
Coherent vs Pauli noise models



Example of a uncorrectable Z-error

Minimum weight correction

Coherent vs Pauli noise models



Example of a uncorrectable Z-error

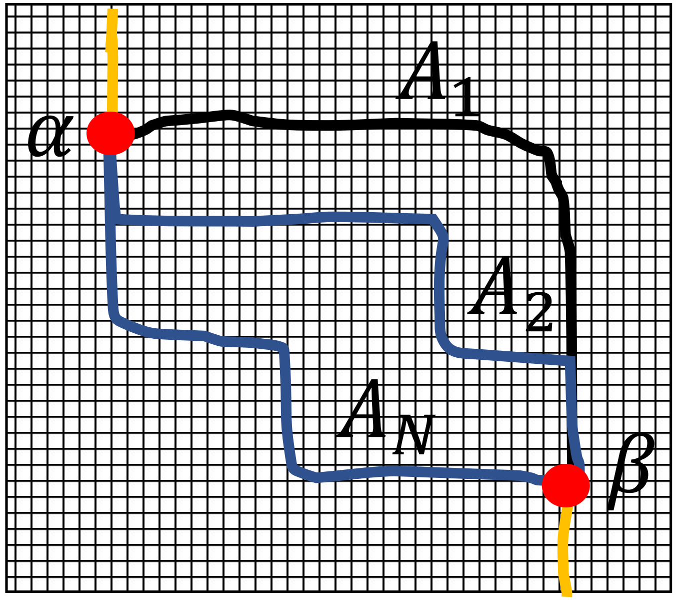
Minimum weight correction

N equivalent errors

$$N \sim 2^{R/2}$$

R = Manhattan distance between α and β

Coherent vs Pauli noise models



Example of a uncorrectable Z-error

Minimum weight correction

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R = Manhattan distance between α and β

Total probability of all uncorrectable errors connecting α and β can be amplified due to a constructive interference.

Coherent noise:

$$P_f \sim \left| \sum_{j=1}^{N} A_j \right|^2$$

?

$$\gg$$

Pauli noise:

$$P_f \sim \sum_{j=1}^{N} |A_j|^2$$

Conclusions

Efficient simulation algorithm for error correction with Z-rotation errors
Runtime $O(d^4)$. Simulated systems with up to 2400 qubits.

The observed error threshold is close to 0.1π which agrees very well with the threshold of the Pauli twirled noise model.

Pauli twirl approximation significantly underestimates the logical error rate in the sub-threshold regime.

Increasing the code distance makes the logical-level noise less coherent.