Capacity Approaching Coding for Low-noise Interactive Quantum Communication

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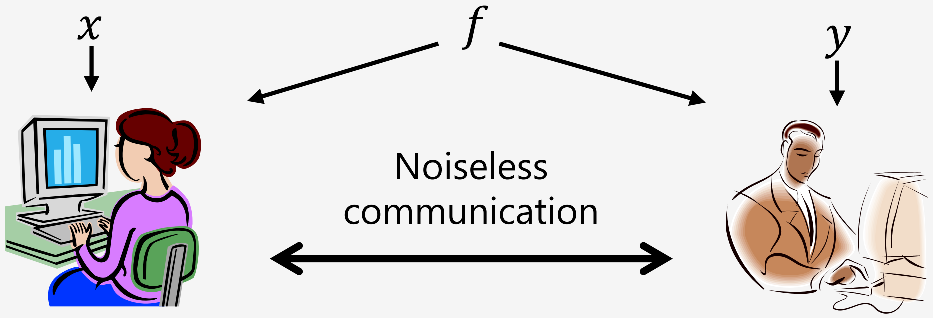
Motivation

- **Communication Complexity**

  Two parties (Alice & Bob) with classical inputs x and y, resp.
  Function f known to both
  Goal: Compute $f(x, y)$ by communicating over a noiseless channel



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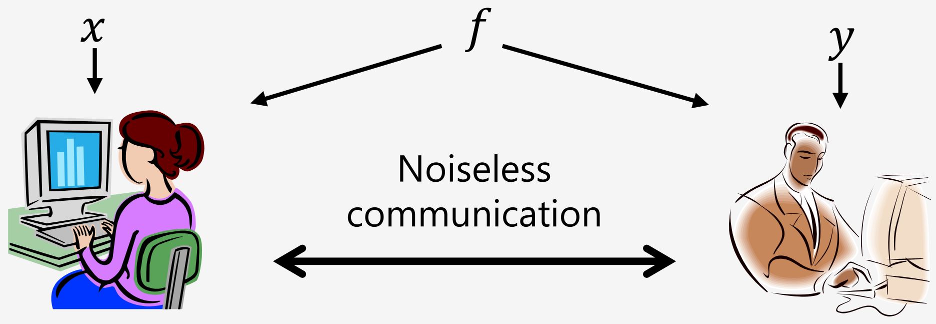

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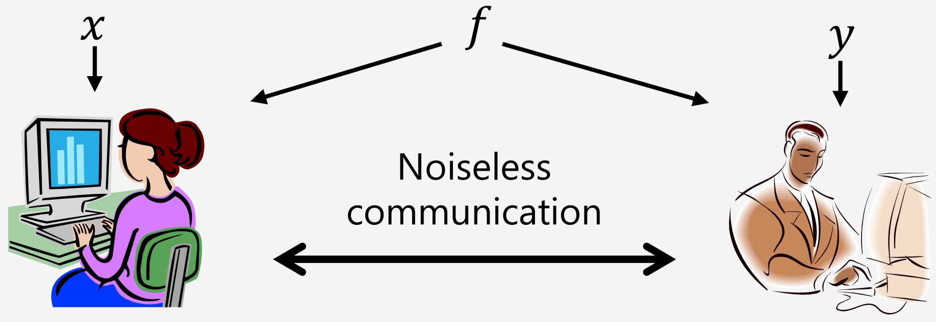

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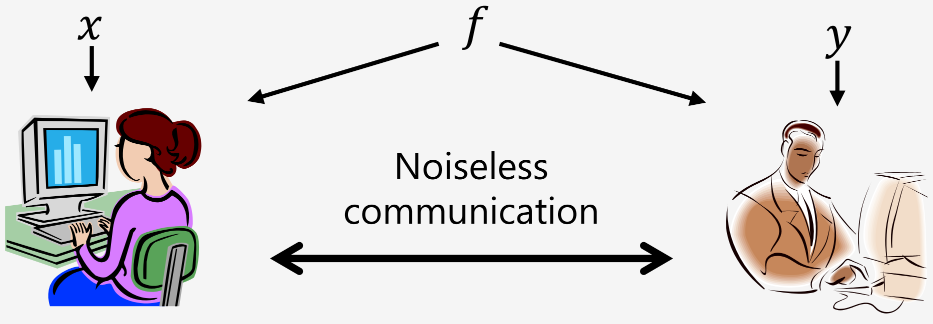

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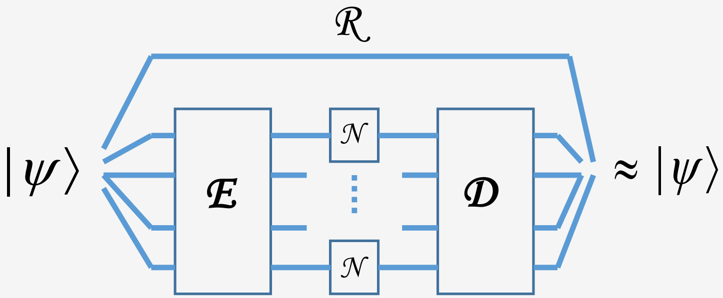
  Achieve noiseless one-way communication using a noisy one-way channel

  Channel capacity : Optimal asymptotic achievable rate of such a procedure



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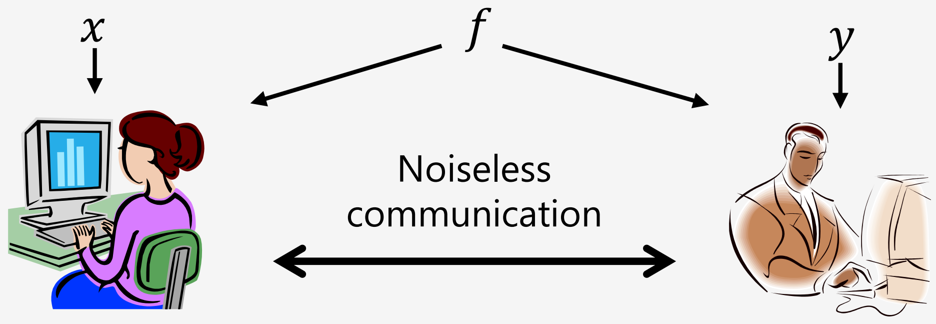

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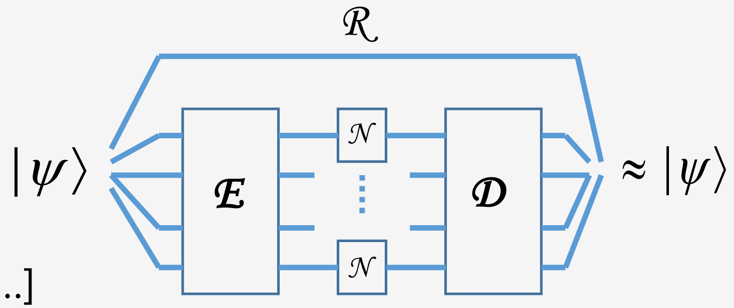

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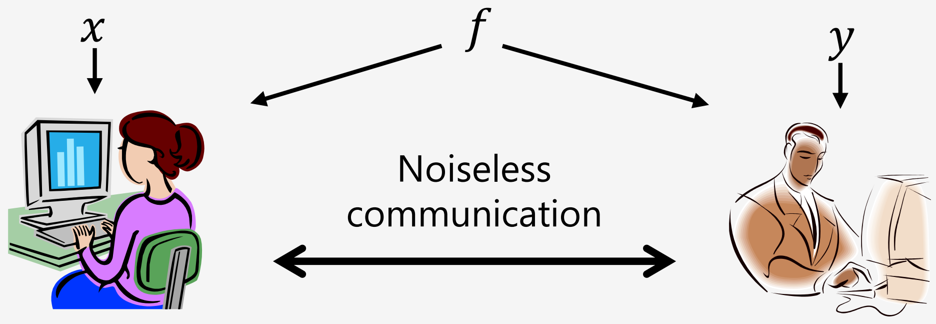

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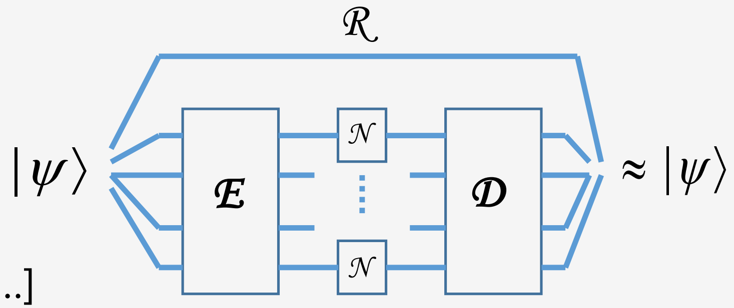

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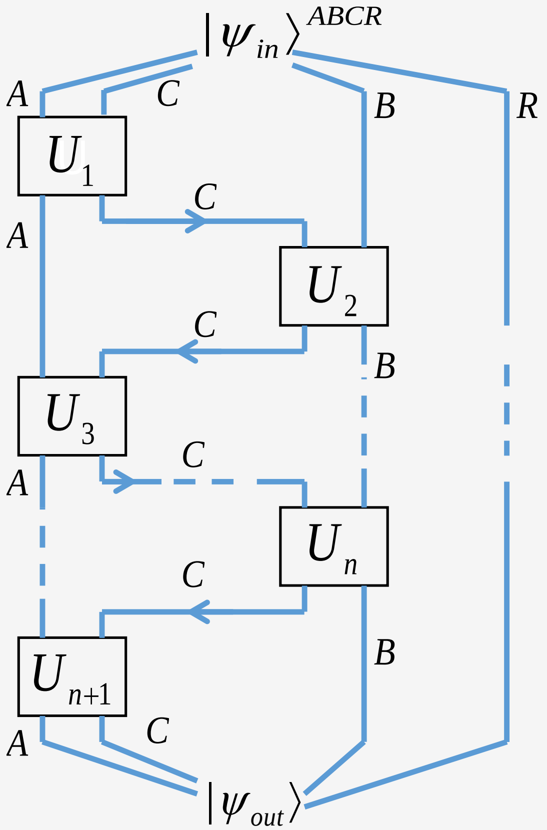
-What about two-way/interactive capacity of a channel?

Noisy Interactive Quantum Communication

Noiseless protocol Π



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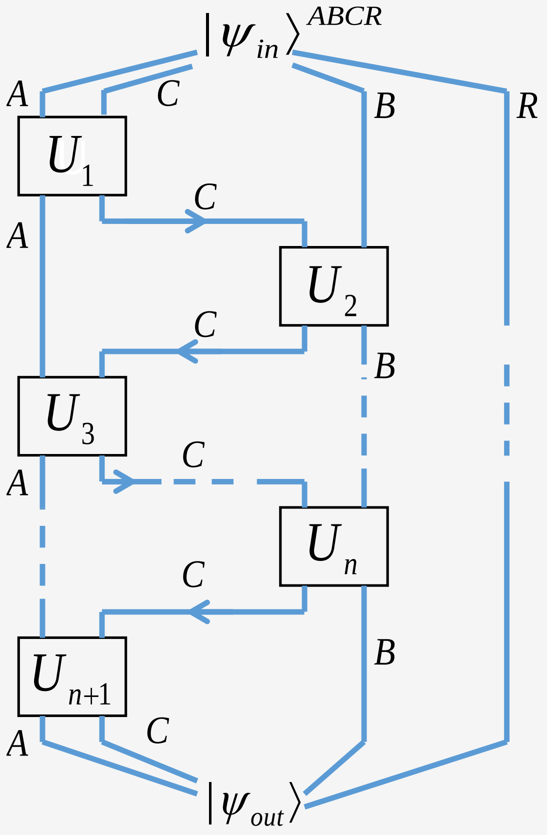

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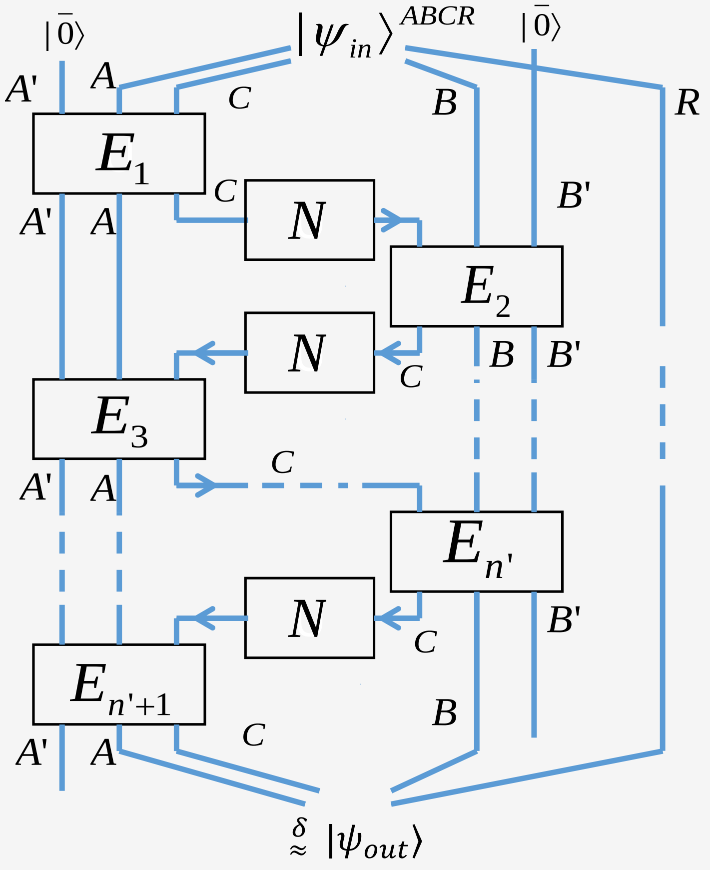

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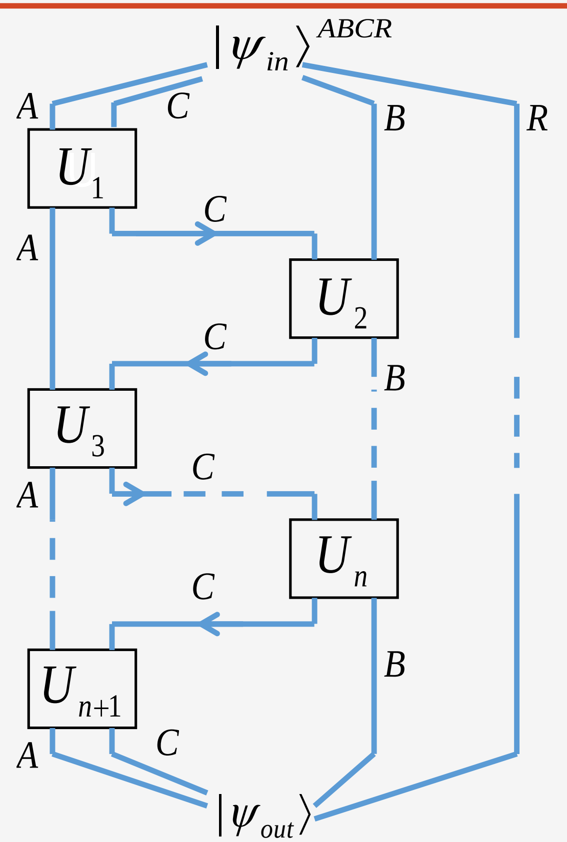

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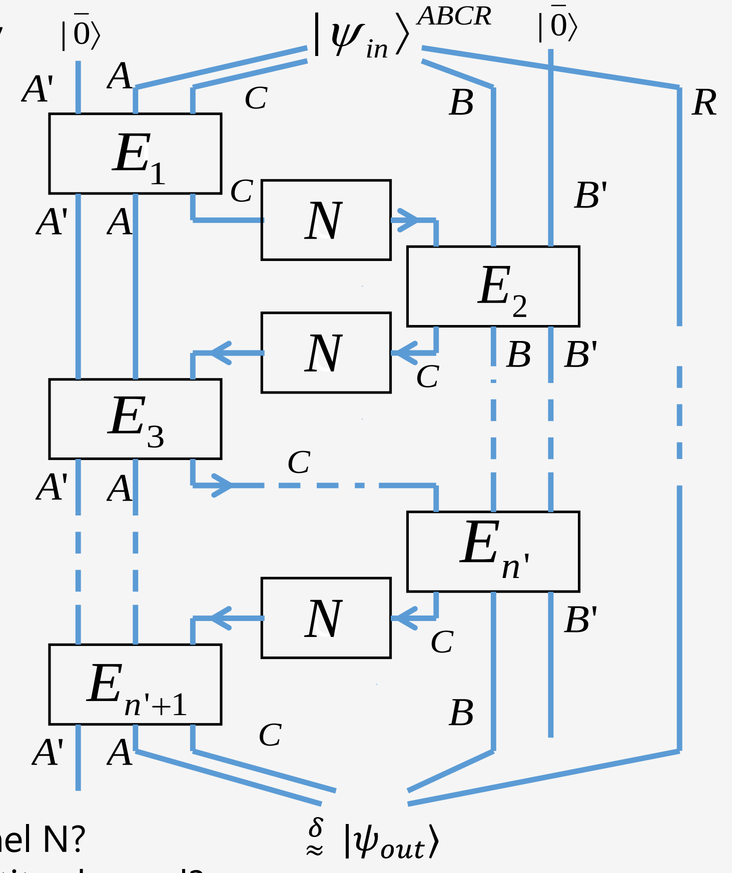

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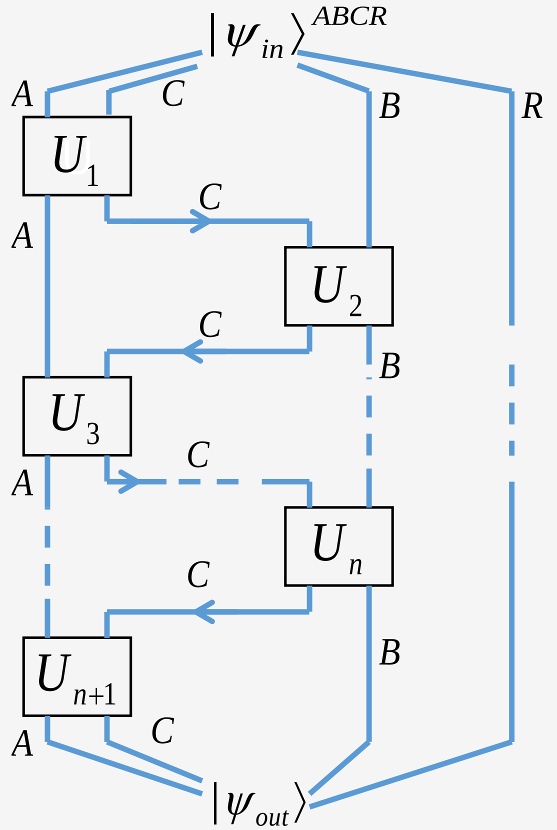

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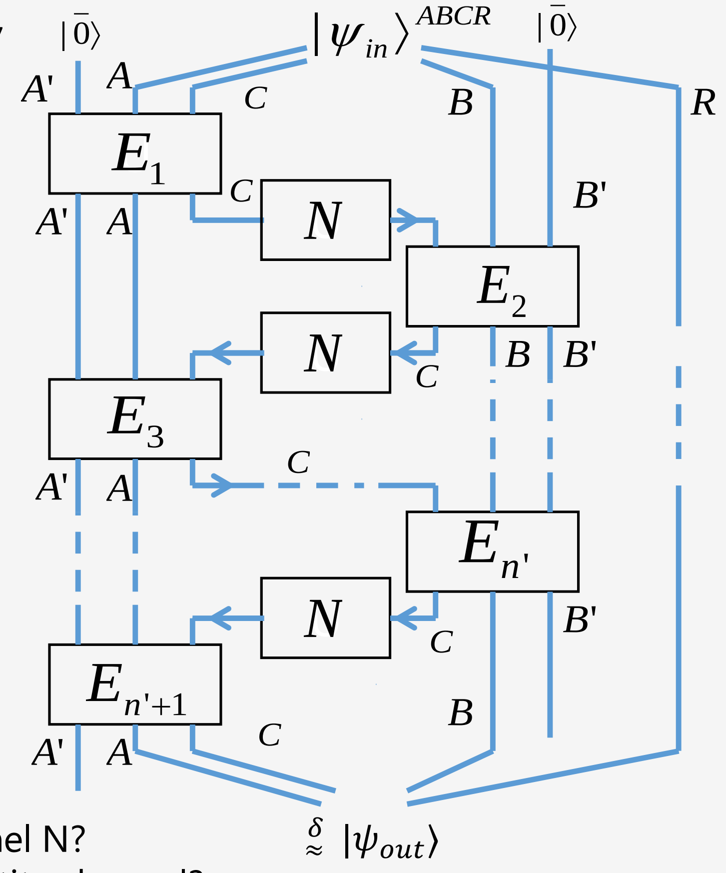

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Communication rate: R := n/n′

Interactvie/two-way capacity of N: Optimal communication rate in the limit of large n and vanishing distance δ

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Constant dilation of each message not sufficient to get constant overall fidelity!

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  - Need to **actively reverse** the simulation
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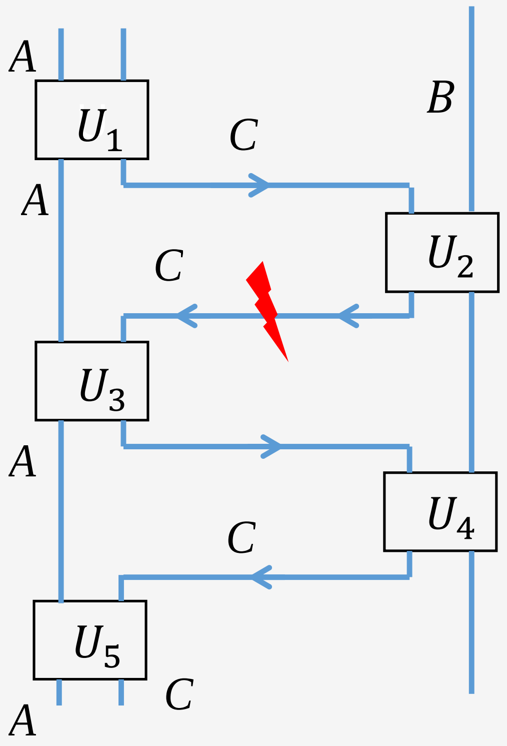

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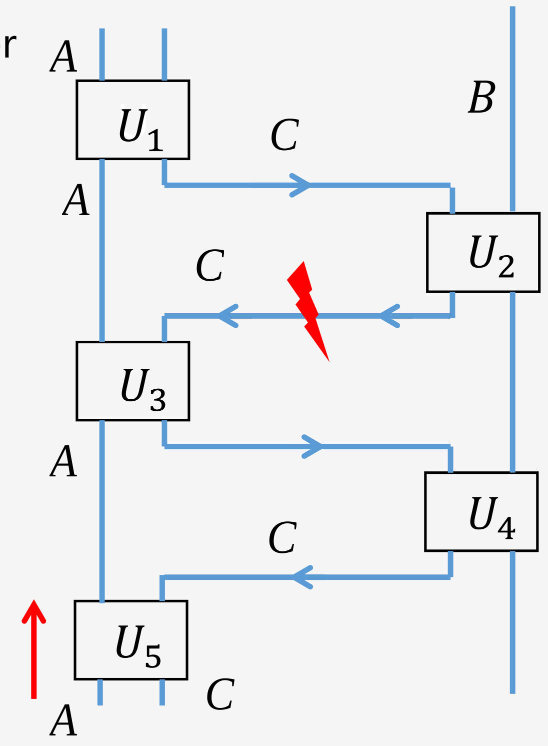

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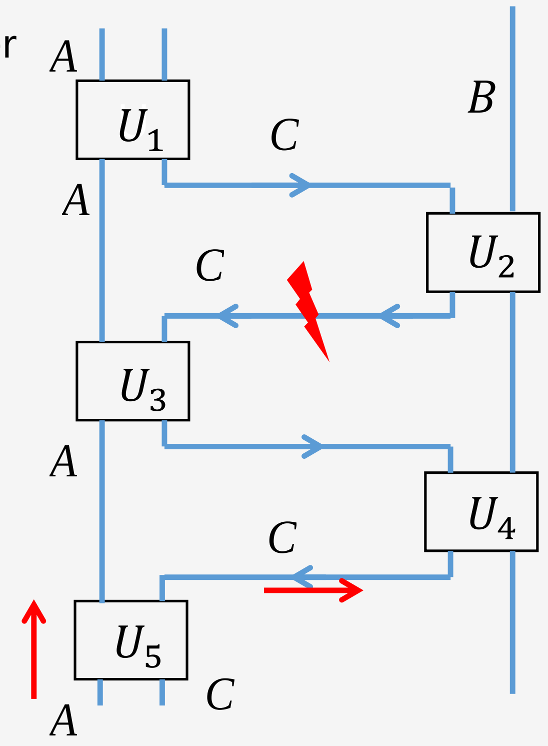

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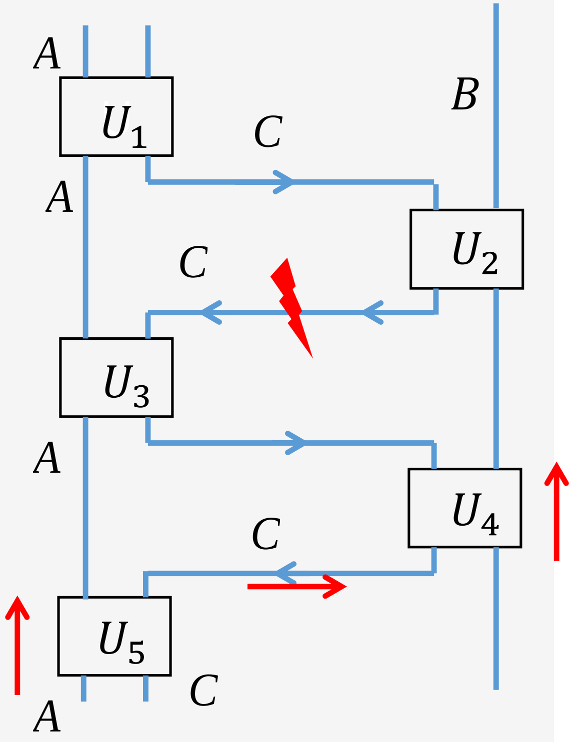

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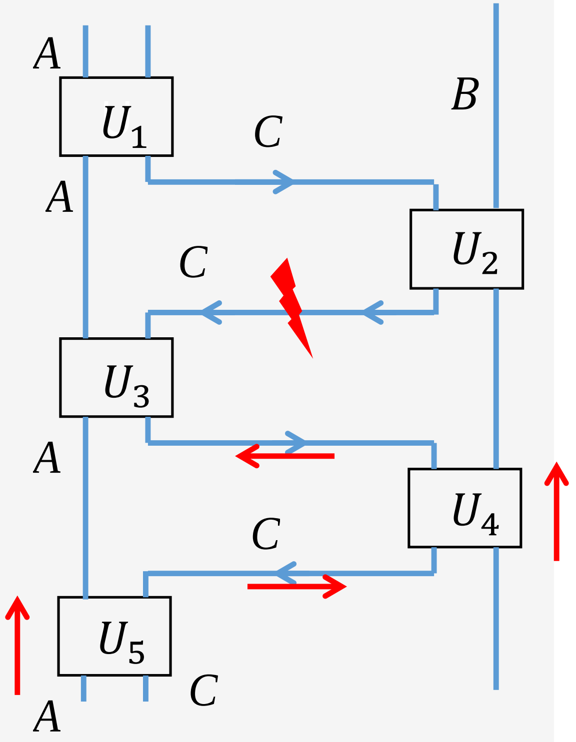

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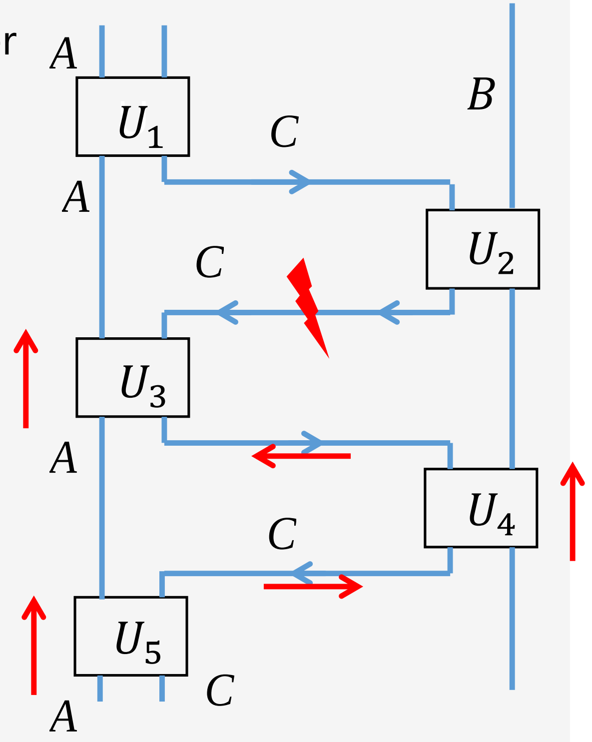

Previous Work

Classical :

- Noisy interactive communication problem introduced by Schulman [Sch92,Sch93]

  Possible to simulating noiseless interactive communication over a two-way noisy channel with constant overhead ($C > 0$)

- Active field of research:
  - Results focused on improving tolerable error-rate and computational efficiency :

    [BR11, GMS11, BK12, FGOS13, BN13, BE14, GH14, GHS14, BKN14, EGH15, …]

    Mostly based on tree codes, Huge communication overhead even for vanishing error rate
  - [KR13], [Hae14] introduced capacity approaching codes :

    Characterized interactive capacity up to leading order : $C \longrightarrow 1$ with error-rate $\epsilon \longrightarrow 0$

    Random noise: $\boldsymbol{C > 1 - O(\sqrt{\epsilon})}$ **,** **Adversarial noise:** $\boldsymbol{C > 1 - O\left(\sqrt{\epsilon \log\log \frac{1}{\epsilon}}\right)}$
  - More recent results: [BEGH16, GH17, HV17, BE17 , …]

Quantum :

- Recently, [BNTTU14] proved constant factor communication overhead is possible ($C > 0$)

  Computationally inefficient, Huge communication overhead even for vanishing error rate ($C \ll 1$)

Main Result

Theorem: Rate $1 - O(\sqrt{\epsilon})$ is achievable, with success prob. $1 - 2^{-\Omega(n\epsilon)}$, over fully adversarial qubit channel of error rate at most ϵ.

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  Characterizing interactive/two-way capacity to leading order: : $C \longrightarrow 1$ as error-rate $\epsilon \longrightarrow 0$

- First **computationally efficient** coding scheme

  Computational complexity of coding operations: $O(n^2)$

- **Plain quantum model**: No pre-shared resources

  Outperforms conjectured optimal bound in plain classical model!

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Note: This work is not an extension of [BNTTU14]:

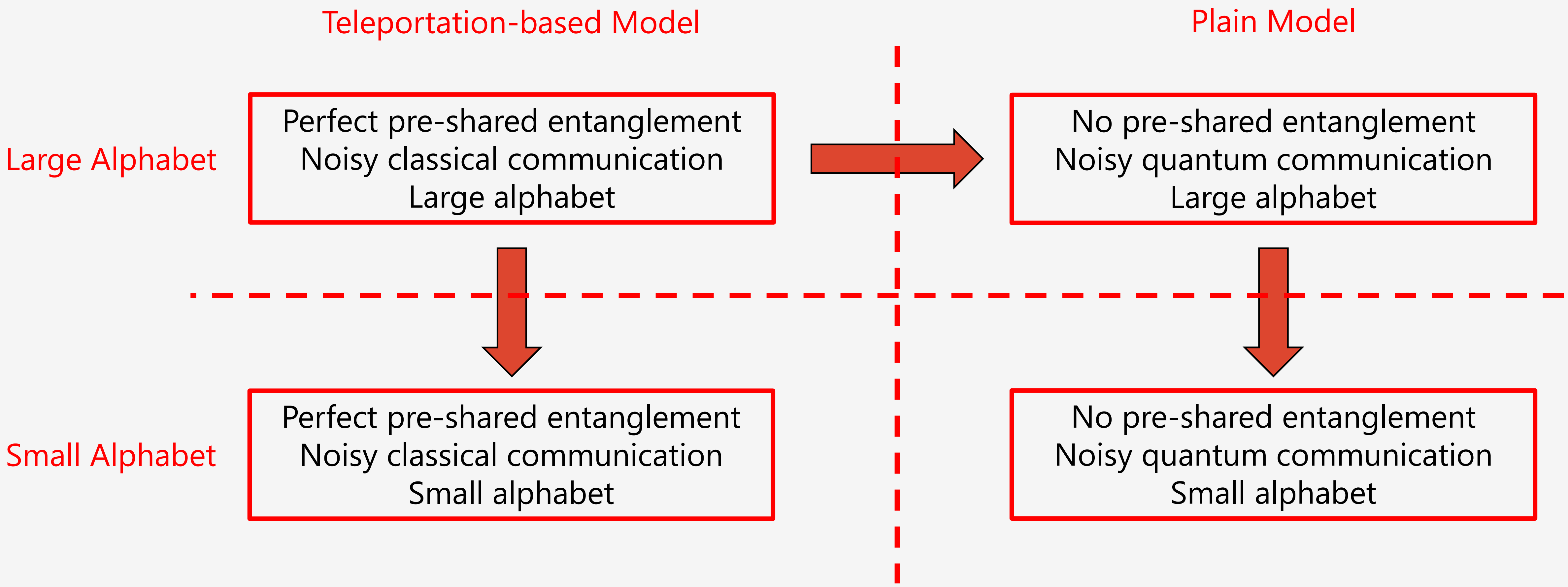
[BNTTU14] : Based on tree codes (computationally inefficient)

$C \ll 1$ even for vanishing error $\epsilon \longrightarrow 0$

Tolerates adversarial error rates up to 1/2

Development of Framework

Focus on adversarial noise (includes random noise)



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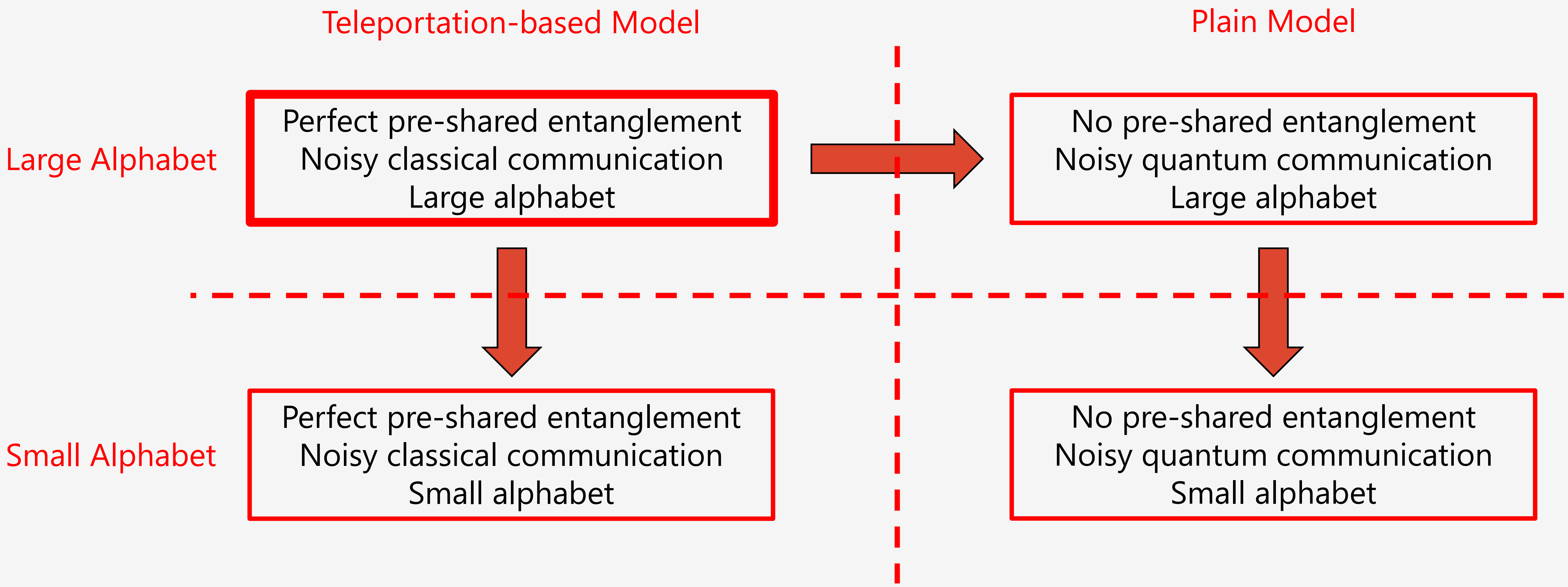

Noisy Interactive Communication: Natural Approach

Haeupler's Template (Classical)

- Both parties conduct their original conversation as if there were no noise
- At regular intervals exchange concise summaries of the conversation so far
- If summaries consistent, continue
- Otherwise, error detected, backtrack to earlier stage and resume

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  - Trivial encoding of each message
  - Summaries measure the error syndrome

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- Efficient: involves evaluating hash functions
- As simulation proceeds, gain more trust in earlier conversation $\longrightarrow$ any detected error is recent with high prob.

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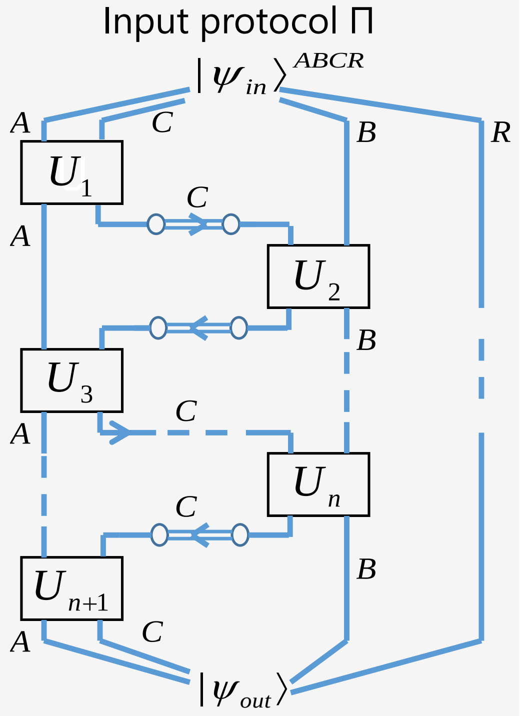
- How to backtrack?

  Requirement: communication wasted by a single error should be constant!

Our Framework

Follow natural approach!

Make sure both parties know joint quantum state before deciding their next action!



- Introduce sufficient but concise data structure to track :
  - Stage in protocol
  - Type of action in each iteration
  - Teleportation measurement outcomes
  - Received instructions for teleportation decoding
  - Recovery operations
  - Which MESs to use next for teleportation
  - …
- Each party maintains their own data and an estimate of other party's data
- At the beginning of each iteration, check if the estimates match the actual data (by hashing)

No ⟶ resolve the inconsistency in classical data

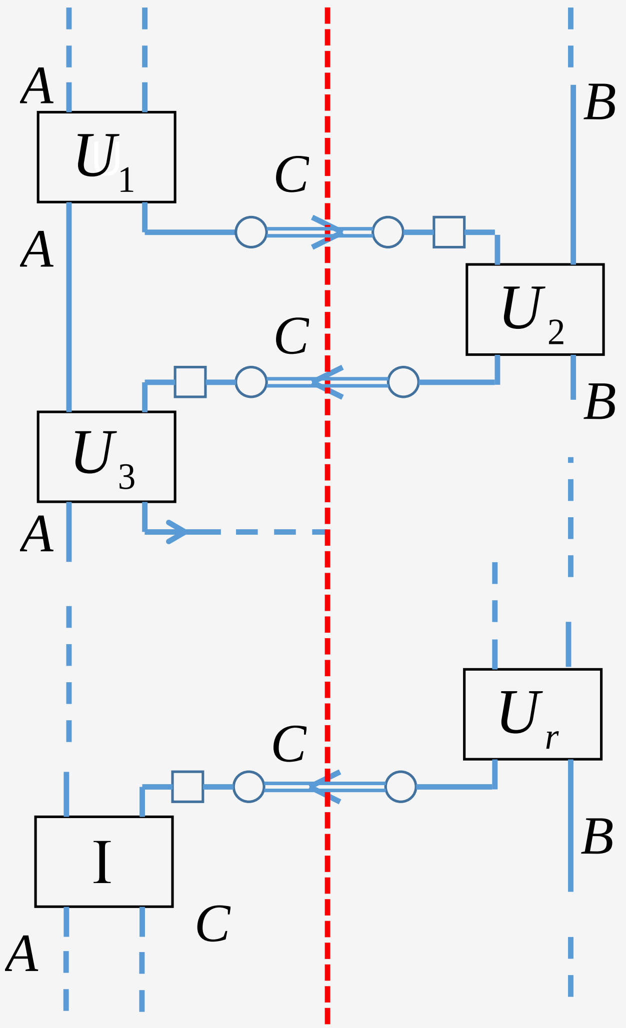
Adapt synchronization mechanism developed by [Hae14] in classical setting

Yes ⟶ Compute the joint state ⟶ Decide next action

Our Framework

In each iteration, Alice & Bob engage in one of three actions :

1. Simulate next block in Π
2. Reveres the last block of simulation
3. Exchange classical data



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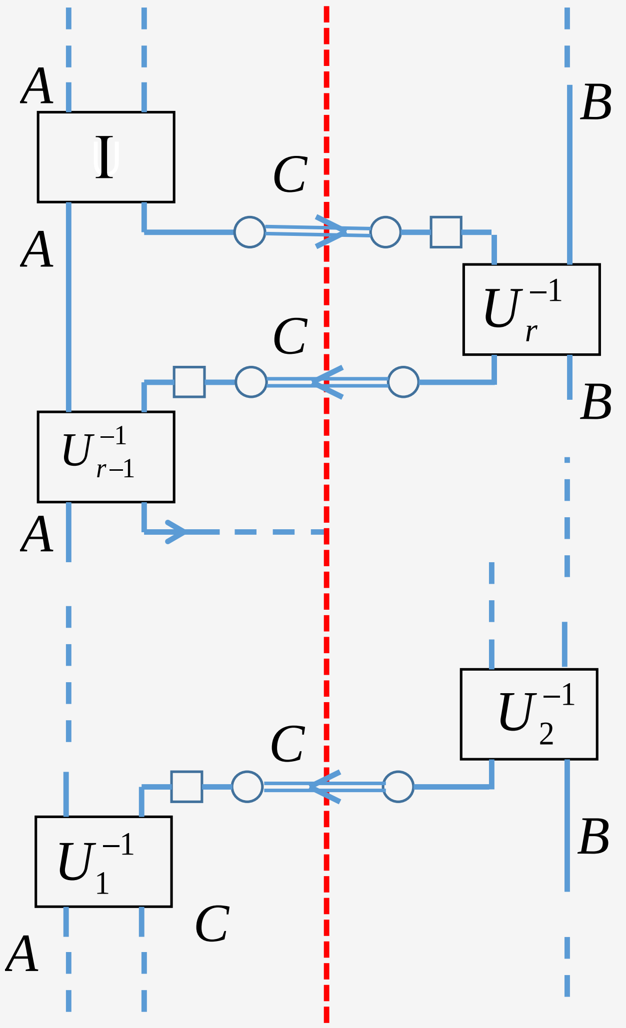

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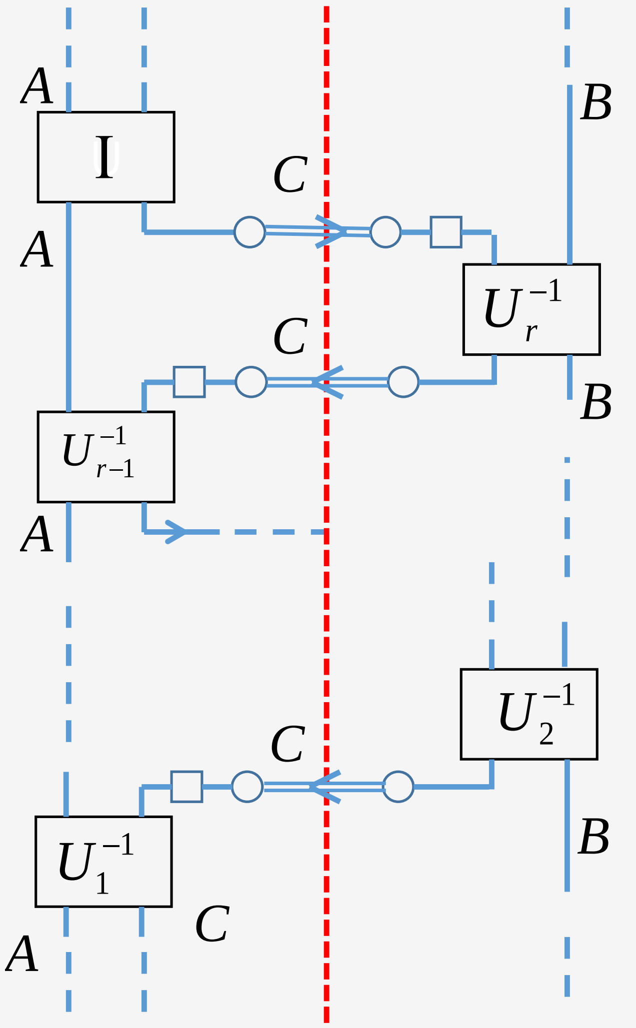
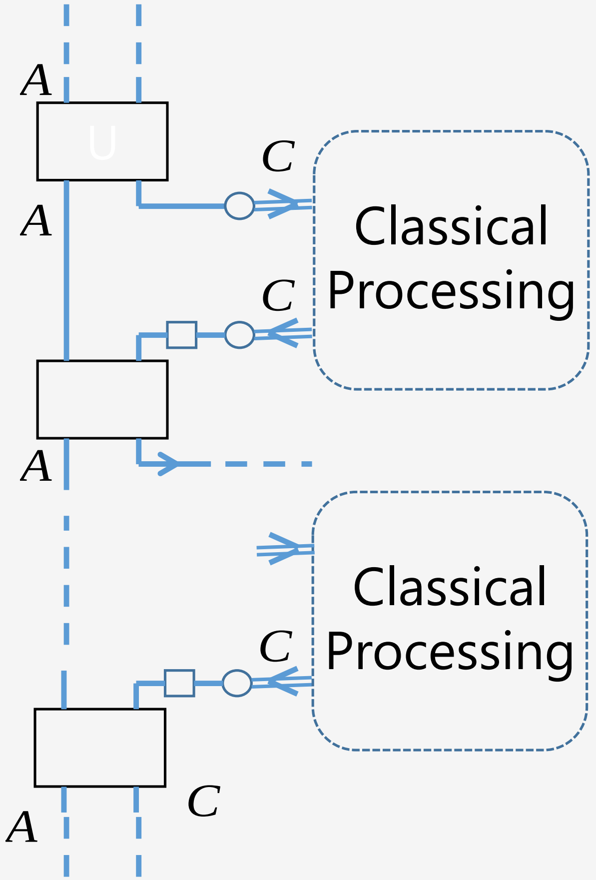


Error or hash collision ⟶ different actions!

Out-of-Sync Teleportation

What if Alice proceeds with simulation of Π (forward or reverse) while Bob exchanges classical data?!

- Alice : teleports quantum data, interprets Bob's classical data as teleportation measurement outcomes
- Bob : sends classical data, interprets Alice's instructions for teleportation decoding as classical data
- They become out-of-sync on which MESs to use to teleport next



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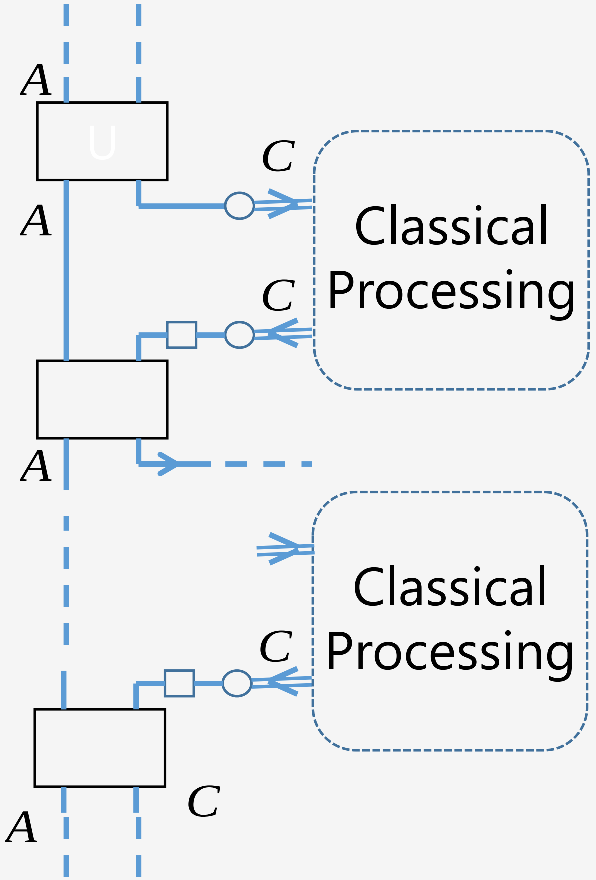
Can Alice and Bob recover from this?!



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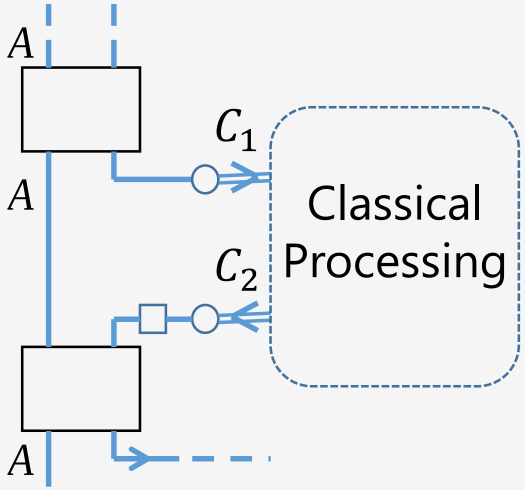
- Information does not leak to environment (adversary)

  Quantum data reside somewhere in the **closed system**



Out-of-Sync Teleportation

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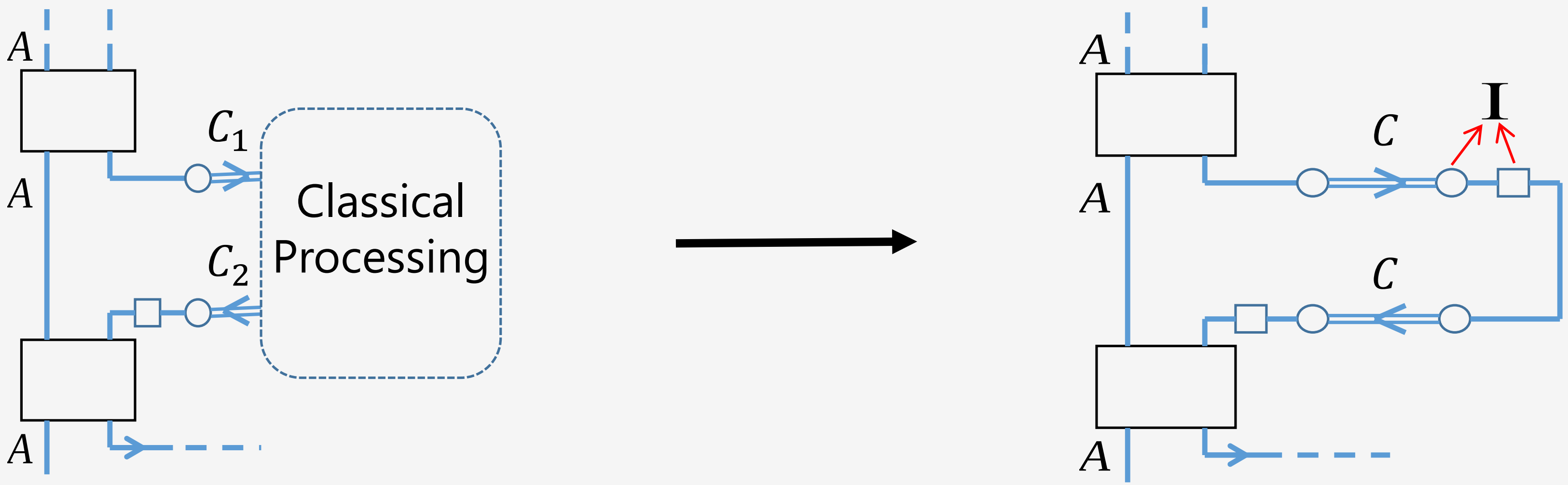
  Quantum data reside somewhere in the **closed system**

- Need to **redirect** quantum data back to A, B, C registers
  - Resolve inconsistencies in classical data
  - Determine which MES to use next
  - “Complete the teleportations”



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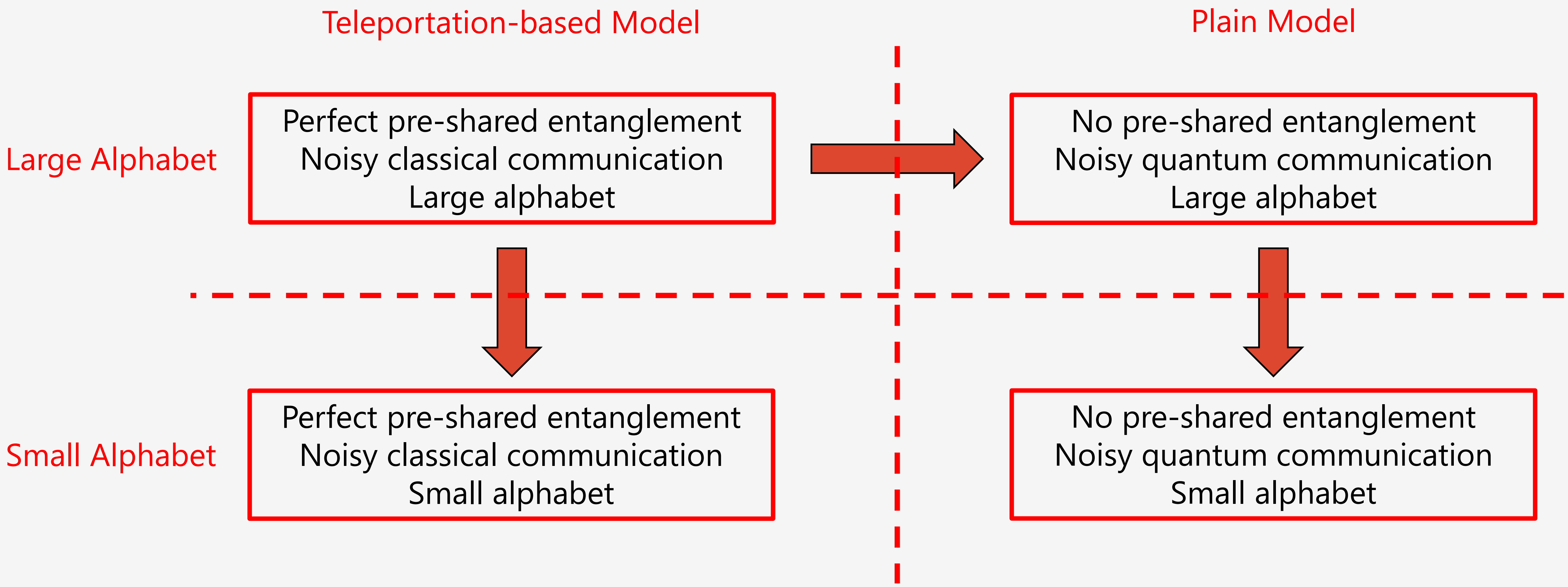

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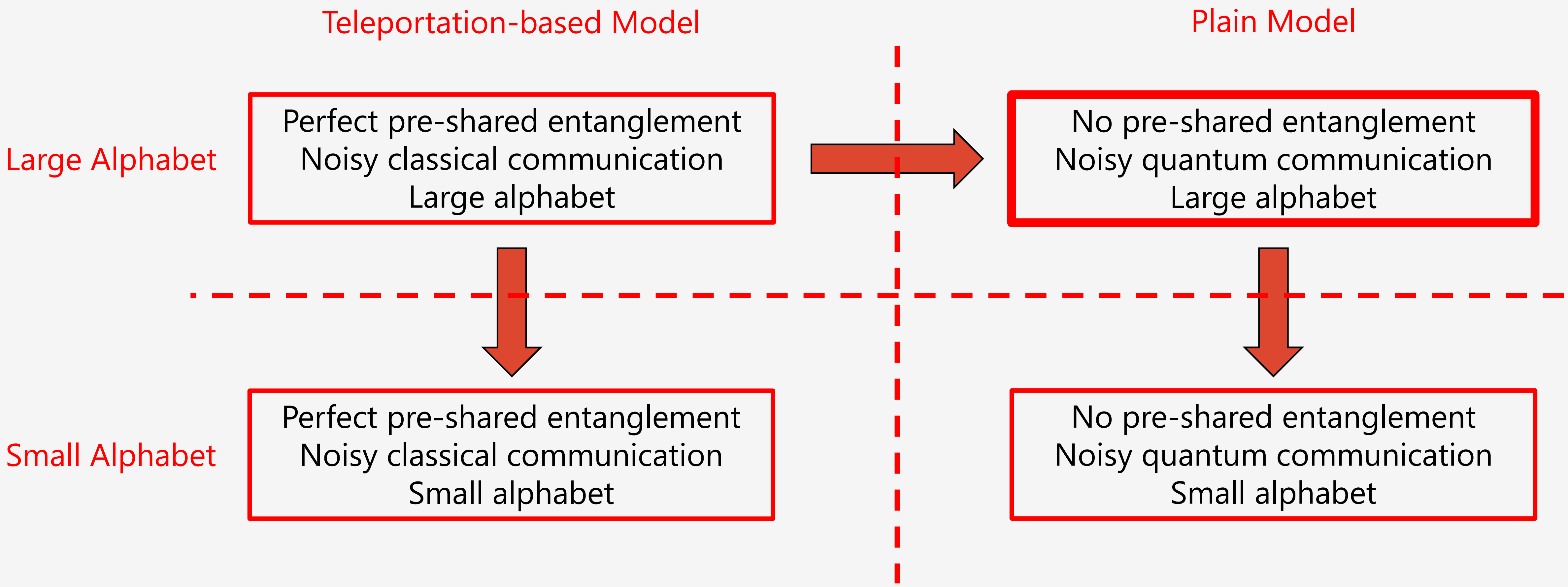

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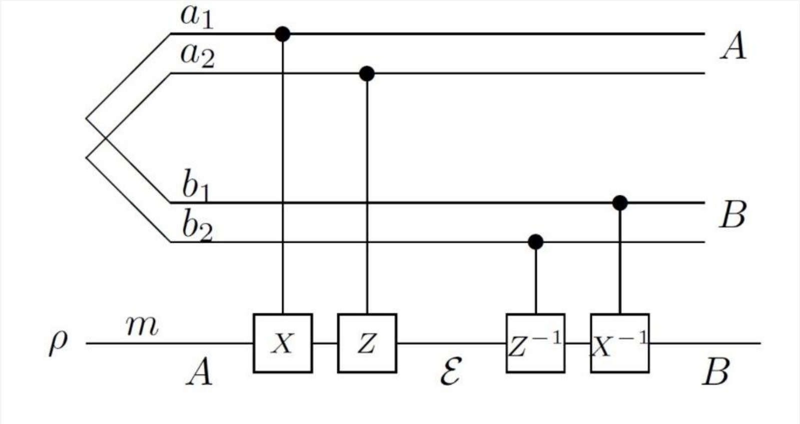

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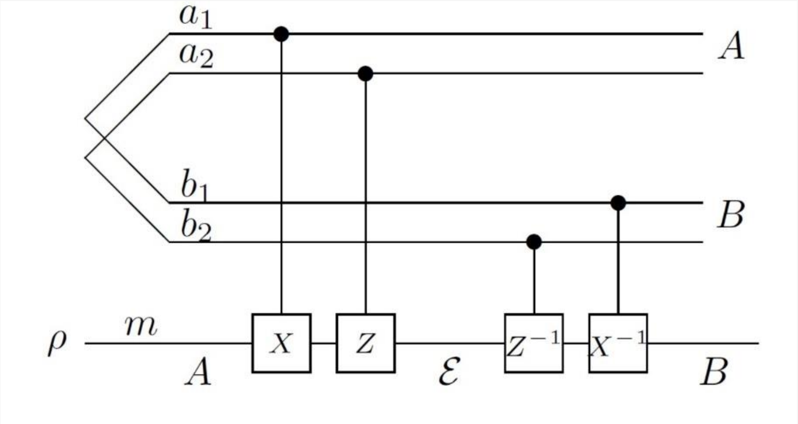
Key features: Allows for recycling MESs when no errors
Detection of errors with distributed syndrome



Framework for Plain Model

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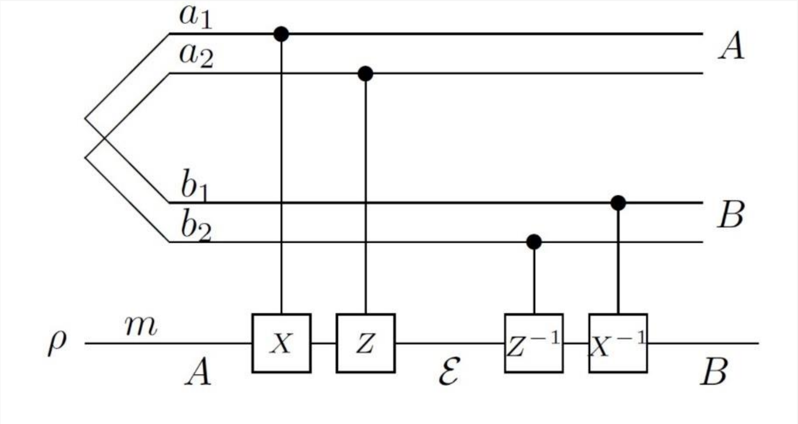

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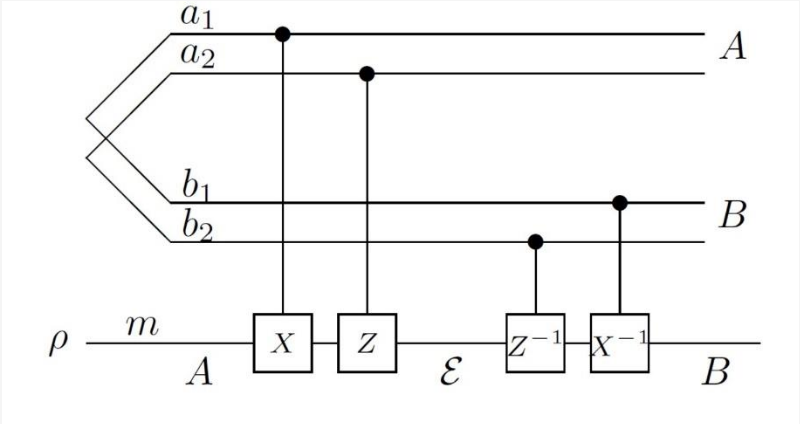

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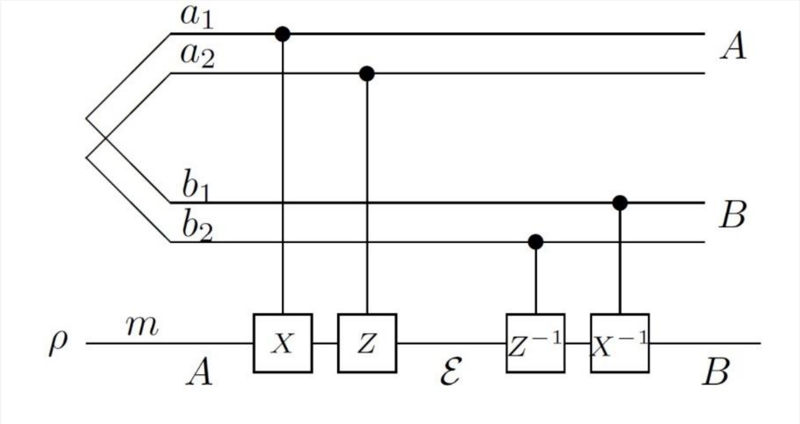

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- New Obstacles : out-of-sync QVC, out-of-sync hashing, out-of-sync recycling

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Thanks!

Crude Analysis for Rate

Noiseless protocol of length n , $\frac{n}{r}$ blocks of length r

Number of errors = $\epsilon \cdot \frac{n}{c} = O(\epsilon n)$

Number of iterations to recover from an error = $O(1)$

Total # of iterations = # of iteration of forward simulation + # of iterations of recovery = $\frac{n}{r} + O(\epsilon n)$

Communication in each iteration = $r + O(1)$ (for checks)

Total communication = $\left(\frac{n}{r} + O(\epsilon n)\right)(r + O(1)) = n\left(1 + O\left(\epsilon r + \frac{1}{r}\right)\right) = n\left(1 + O(\sqrt{\epsilon})\right)$ for $r = \Theta\left(\frac{1}{\sqrt{\epsilon}}\right)$

$$\longrightarrow \quad R = \frac{n}{n\left(1 + O(\sqrt{\epsilon})\right)} = 1 - O(\sqrt{\epsilon})$$