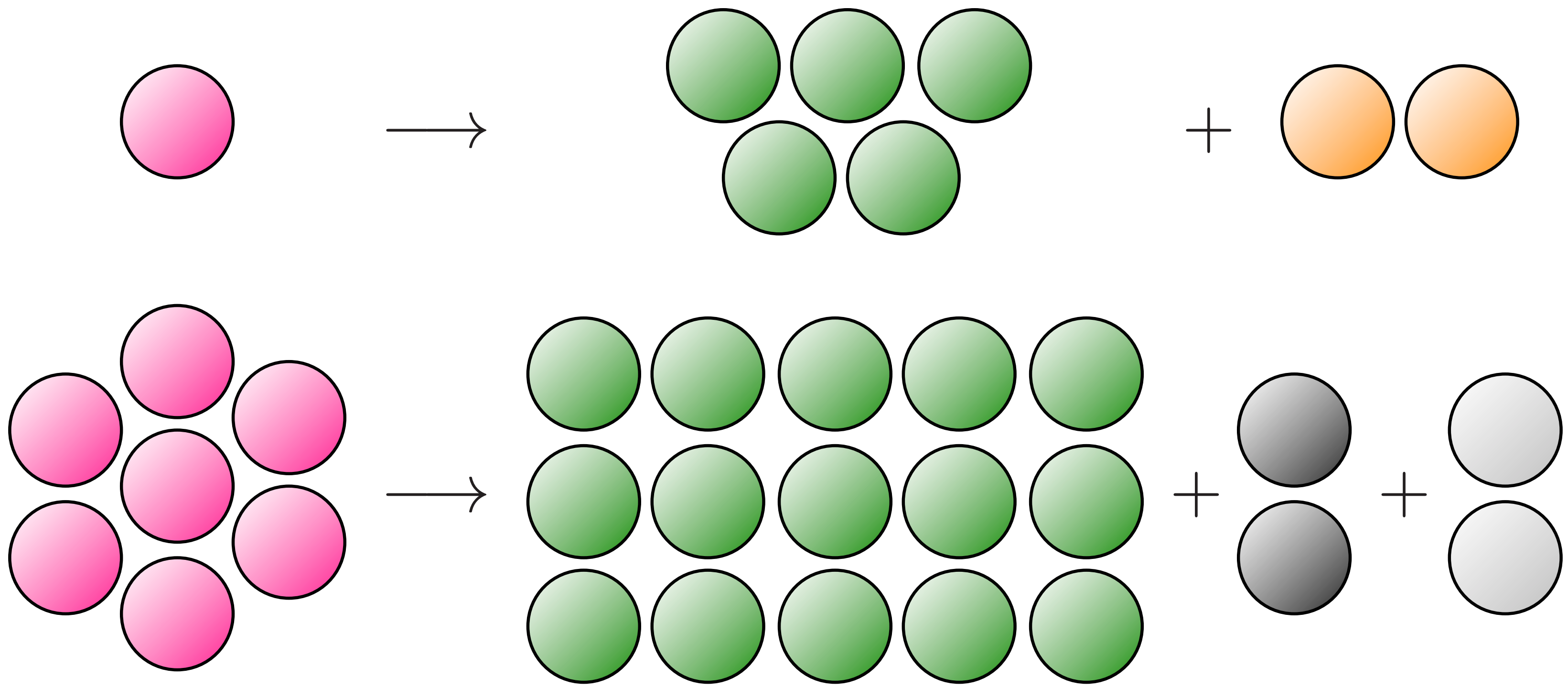
Fault-tolerant quantum computation with few qubits



Rui Chao   Ben W. Reichardt

University of Southern California

arXiv:1705.02329
arXiv:1705.05365

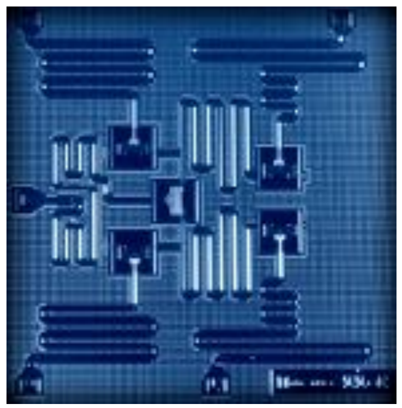
21th Annual Conference on Quantum Information Processing

January 15 – 19, 2018 at the TU Delft

Fault tolerance is expensive

- Factor 1024-bit integer using ~10^{11} gates.

  $\Rightarrow$ Need error $\leq 10^{-11}$ per gate



superconducting qubits 10^{-3}

[IBM Quantum Experience]

ion traps 10^{-2}

[Bermudez et al., arXiv:1705.02771]

(for FT QEC, we need $\leq 10^{-4}$)

- Even if error $\leq 10^{-5}$, **overhead =** $\dfrac{\textbf{physical}}{\textbf{logical}}$ = **3×10^{3}** (surface code)

  [Suchara et al., arXiv:1312.2316]

How to reduce the overhead

syndrome measurement

Effective fault-tolerant quantum computation with slow measurement

David P. DiVincenzo, Panos Aliferis

(Submitted on 6 Jul 2006 (v1), last revised 3 Aug 2006 (this version, v2))

How important is fast measurement for fault-tolerant quantum computation? Using a combination of existing and new id
that measurement times as long as even 1,000 gate times or more have a very minimal effect on the quantum accuracy t
shows that slow measurement, which appears to be unavoidable in many implementations of quantum computing, poses
obstacle to scalability.

Comments: 9 pages, 11 figures. v2: small changes and reference additions
Subjects: **Quantum Physics (quant-ph)**
Journal reference: Phys. Rev. Lett. 98 (2007) 220501
DOI: 10.1103/PhysRevLett.98.020501
Cite as: arXiv:quant-ph/0607047
(or arXiv:quant-ph/0607047v2 for this version)

decoding algorithm

Efficient Algorithms for Maximum Likelihood Decoding i

Sergey Bravyi, Martin Suchara, Alexander Vargo

(Submitted on 19 May 2014)

We describe two implementations of the optimal error correction algorithm known as the maxim
surface code with a noiseless syndrome extraction. First, we show how to implement MLD exact
of code qubits. Our implementation uses a reduction from MLD to simulation of matchgate qua
requires a special noise model with independent bit-flip and phase-flip errors. Secondly, we sh
for more general noise models using matrix product states (MPS). Our implementation has runn
that controls the approximation precision. The key step of our algorithm, borrowed from the DI
contracting a tensor network on the two-dimensional grid. The subroutine uses MPS with a bon
sequence of tensors arising in the course of contraction. We benchmark the MPS-based decoder
matching decoder observing a significant reduction of the logical error probability for $\chi \geq 4$.

Comments: 18 pages, 12 figures
Subjects: **Quantum Physics (quant-ph)**
Journal reference: Phys. Rev. A 90, 032326 (2014)
DOI: 10.1103/PhysRevA.90.032326
Cite as: arXiv:1405.4883 [quant-ph]
(or arXiv:1405.4883v1 [quant-ph] for this version)

complementary transversality

Fault-tolerant conversion between the Steane and Reed-Muller q

Jonas T. Anderson, Guillaume Duclos-Cianci, David Poulin

(Submitted on 11 Mar 2014)

Steane's 7-qubit quantum error-correcting code admits a set of fault-tolerant gates that generate the Clifford g
not universal for quantum computation. The 15-qubit Reed-Muller code also does not admit a universal fault-t
possesses fault-tolerant T and control-control-Z gates. Combined with the Clifford group, either of these two g
set. Here, we combine these two features by demonstrating how to fault-tolerantly convert between these two c
method to realize universal fault-tolerant quantum computation. One interpretation of our result is that both cc
same subsystem code in different gauges. Our scheme extends to the entire family of quantum Reed-Muller coc

Comments: 6 pages
Subjects: **Quantum Physics (quant-ph)**
Journal reference: Phys. Rev. Lett. 113, 080501 (2014)
DOI: 10.1103/PhysRevLett.113.080501
Cite as: arXiv:1403.2734 [quant-ph]
(or arXiv:1403.2734v1 [quant-ph] for this version)

logic synthesis

Logic Synthesis for Fault-Tolerant Quantum Computers

N. Cody Jones

(Submitted on 28 Oct 2013)

Efficient constructions for quantum logic are essential since quantum computation is experimentally challenging. T
quantum logic synthesis as a paradigm for reducing the resource overhead in fault-tolerant quantum computing. T
correction considered here is the surface code. After developing the theory behind general logic synthesis, the resc
state distillation for the $T = \exp(i\pi(I - Z)/8)$ gate are quantitatively analyzed. The resource costs for a relatively n
multi-qubit Fourier states are calculated for the first time. Four different constructions of the fault-tolerant Toffoli
which incorporate error detection, are analyzed and compared. The techniques of logic synthesis reduce the cost o
quantum computation by one to two orders of magnitude, depending on which benchmark is used.
Using resource analysis for T gates and Toffoli gates, several proposals for constructing arbitrary quantum gates a
including "Clifford+T" sequences, V-basis sequences, phase kickback, and programmable ancilla rotations. The ap
gates to quantum algorithms for simulating chemistry is discussed as well. Finally, the thesis examines the techniq
efficient constructions of quantum logic, and these observations point to even broader applications of logic synthe

Comments: PhD Thesis. 201 pages, 10 chapters, 62 figures. Original version on Stanford archives at [this http URL]. Incorporates arXiv:1010.5022, arXiv:1204.0567, arXiv:1205.2402, arXiv:1210.3388, arXiv:1212.5069, and arXiv:1303.3066
Subjects: **Quantum Physics (quant-ph)**
Cite as: arXiv:1310.7290 [quant-ph]
(or arXiv:1310.7290v1 [quant-ph] for this version)

state distillation

Magic state distillation with low overhead

Sergey Bravyi, Jeongwan Haah

(Submitted on 11 Sep 2012)

We propose a new family of error detecting stabilizer codes with an encoding rate 1/3 that perı
pi/8-rotation T on all logical qubits. The new codes are used to construct protocols for distillir
Clifford group gates and Pauli measurements. The distillation overhead has a poly-logarithmic
accuracy, where the degree of the polynomial is $\log_2 3 \approx 1.6$. To construct the desired family c
triorthogonal matrix --- a binary matrix in which any pair and any triple of rows have even ove
to a stabilizer code with a transversal T-gate on all logical qubits, possibly augmented by Cliff
for generating triorthogonal matrices is proposed. Our techniques lead to a two-fold overhead
output accuracy 10^{-12} compared with the best previously known protocol.

Comments: 11 pages, 3 figures
Subjects: **Quantum Physics (quant-ph)**
Journal reference: Phys. Rev. A 86, 052329 (2012)
DOI: 10.1103/PhysRevA.86.052329
Cite as: arXiv:1209.2426 [quant-ph]
(or arXiv:1209.2426v1 [quant-ph] for this version)

circuit architecture

Noise Threshold for a Fault-Tolerant Two-Dimensional Lattice Archite

Krysta M. Svore, David P. DiVincenzo, Barbara M. Terhal

(Submitted on 12 Apr 2006 (v1), last revised 3 Nov 2006 (this version, v3))

We consider a model of quantum computation in which the set of operations is limited to nearest-neighbor interactions o
We model movement of qubits with noisy SWAP operations. For this architecture we design a fault-tolerant coding scheme
concatenated [[7,1,3]] Steane code. Our scheme is potentially applicable to ion-trap and solid-state quantum technologie
a lower bound on the noise threshold for our local model using a detailed failure probability analysis. We obtain a thresho
10^-5 for the local setting, where memory error rates are one-tenth of the failure rates of gates, measurement, and prepa
For the analogous nonlocal setting, we obtain a noise threshold of 3.61 x 10^-5. Our results thus show that the additiona
operations required to move qubits in the local model affect the noise threshold only moderately.

Comments: 20 pages, 11 figures. v2 has some small changes and a link to a website with supplementary material. v3: Corrects earl
tolerant T gate construction. Describes different strategy for non-Clifford fault tolerance
Subjects: **Quantum Physics (quant-ph)**
Journal reference: Quant. Inf. Comp. Vol. 7, No. 4, pp. 297-318 (2007)
Cite as: arXiv:quant-ph/0604090
(or arXiv:quant-ph/0604090v3 for this version)

ancilla preparation

Noise Threshold for a Fault-Tolerant Two-Dimensional Lattice Architectu

Krysta M. Svore, David P. DiVincenzo, Barbara M. Terhal

(Submitted on 12 Apr 2006 (v1), last revised 3 Nov 2006 (this version, v3))

We consider a model of quantum computation in which the set of operations is limited to nearest-neighbor interactions on a 2D
We model movement of qubits with noisy SWAP operations. For this architecture we design a fault-tolerant coding scheme using
concatenated [[7,1,3]] Steane code. Our scheme is potentially applicable to ion-trap and solid-state quantum technologies. We c
a lower bound on the noise threshold for our local model using a detailed failure probability analysis. We obtain a threshold of 1
10^-5 for the local setting, where memory error rates are one-tenth of the failure rates of gates, measurement, and preparation
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operations required to move qubits in the local model affect the noise threshold only moderately.

Comments: 20 pages, 11 figures. v2 has some small changes and a link to a website with supplementary material. v3: Corrects earlier erro
tolerant T gate construction. Describes different strategy for non-Clifford fault tolerance
Subjects: **Quantum Physics (quant-ph)**
Journal reference: Quant. Inf. Comp. Vol. 7, No. 4, pp. 297-318 (2007)
Cite as: arXiv:quant-ph/0604090
(or arXiv:quant-ph/0604090v3 for this version)

measurement

Fault-Tolerant Measurement-Based Quantum Computing with Variable Cluster States

Nicolas C. Menicucci

(Submitted on 28 Oct 2013 (v1), last revised 3 Apr 2014 (this version, v3))

A long-standing open question about Gaussian continuous-variable cluster states is whether they enable fa
based quantum computation. The answer is yes. Initial squeezing in the cluster above a threshold value of 2
from finite squeezing acting on encoded qubits are below the fault-tolerance threshold of known qubit-bas
concatenating with one of these codes and using ancilla-based error correction, fault-tolerant measuremen
computation of theoretically indefinite length is possible with finitely squeezed cluster states.

Comments: (v3) consistent with published version, more accessible for general audience; (v2) condensed presentati
generation and a comparison of currently achievable squeezing to the threshold; (v1) 13 pages, a few fi
Subjects: **Quantum Physics (quant-ph)**
Journal reference: Phys. Rev. Lett. 112, 120504 (2014)
DOI: 10.1103/PhysRevLett.112.120504
Cite as: arXiv:1310.7596 [quant-ph]
(or arXiv:1310.7596v3 [quant-ph] for this version)

logical gate operation

Universal fault-tolerant gates on concatenated stabilizer codes

Theodore J. Yoder, Ryuji Takagi, Isaac L. Chuang

(Submitted on 12 Mar 2016 (v1), last revised 18 Sep 2016 (this version, v2))

It is an oft-cited fact that no quantum code can support a set of fault-tolerant logical gates that is both universal and transversal. This no-go theorem is generally responsible for the interest in alternative universality constructions including magic state distillation. Widely overlooked, however, is the possibility of non-transversal, yet still fault-tolerant, gates that work directly on small quantum codes. Here we demonstrate precisely the existence of such gates. In particular, we show how the limits of non-transversality can be overcome by performing rounds of intermediate error-correction to create logical gates on stabilizer codes that use no ancillas other than those required for syndrome measurement. Moreover, the logical gates we construct, the most prominent examples being Toffoli and controlled-controlled-Z, often complete universal gate sets on their codes. We detail such universal constructions for the smallest quantum codes, the 5-qubit and 7-qubit codes, and then proceed to generalize the approach. One remarkable result of this generalization is that any nondegenerate stabilizer code with a complete set of fault-tolerant single-qubit Clifford gates has a universal set of fault-tolerant gates. Another is the interaction of logical qubits across different stabilizer codes, which, for instance, implies a broadly applicable method of code switching.

Comments: 18 pages + 5 pages appendix, 12 figures
Subjects: **Quantum Physics (quant-ph)**
Journal reference: Phys. Rev. X 6, 031039 (2016)
DOI: 10.1103/PhysRevX.6.031039
Cite as: arXiv:1603.03948 [quant-ph]
(or arXiv:1603.03948v2 [quant-ph] for this version)

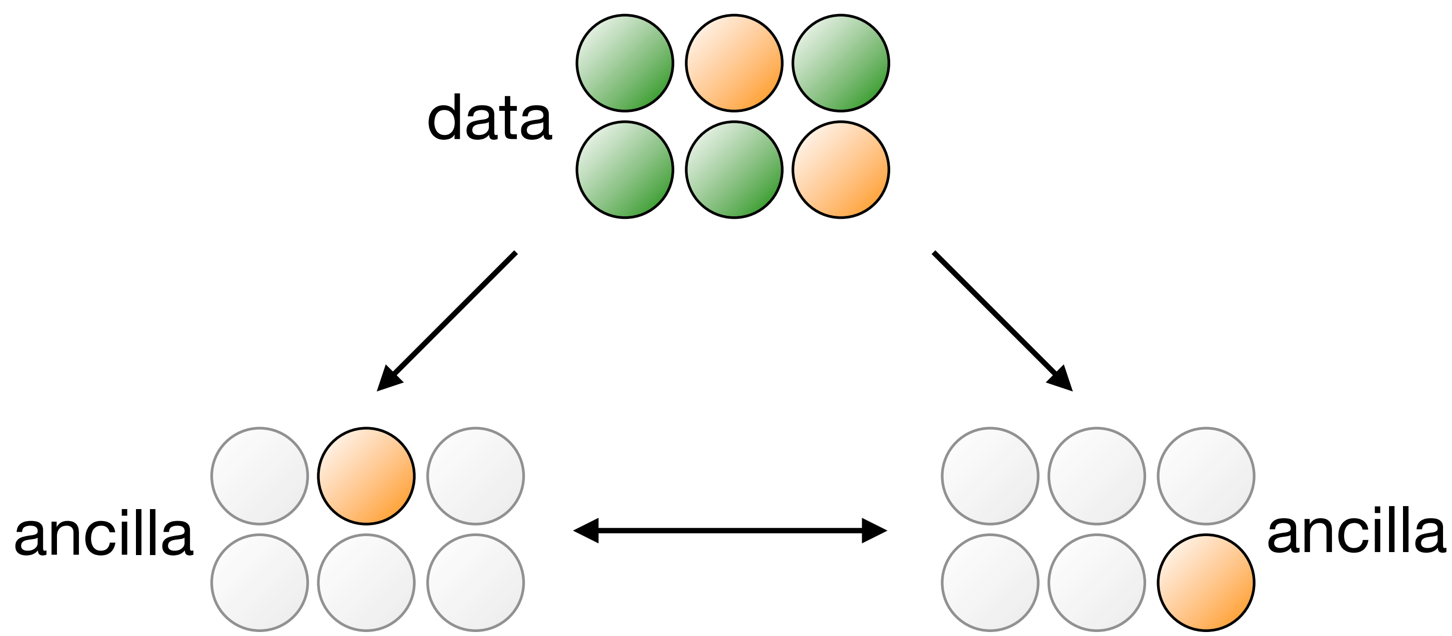
Choose efficient codes

- Encode multiple qubits in a block

[Gottesman, arXiv:9702029]
[Steane et al., arXiv:0311014]
[Steane, arXiv:0412165]
[Gottesman, arXiv:1310.2984]

⇒ Higher rate, but difficult to compute fault tolerantly

⇒ Constant overhead is possible, but mostly a theoretical result, not practical



Our focus: Reduce FT overhead on small quantum devices, e.g. with ~10s qubits

Fault-tolerance theory has not yet been tested

Non-fault tolerant error correction experiments

- 3-qubit repetition code using liquid NMR [Cory et al., arXiv:9802018]
- 2 by 2 surface codes using superconducting qubits [Corcoles et al., arXiv:1410.6419]
- 7-qubit color code using ion traps [Nigg et al., arXiv:1403.5426]

…………

Recent fault-tolerance experiments

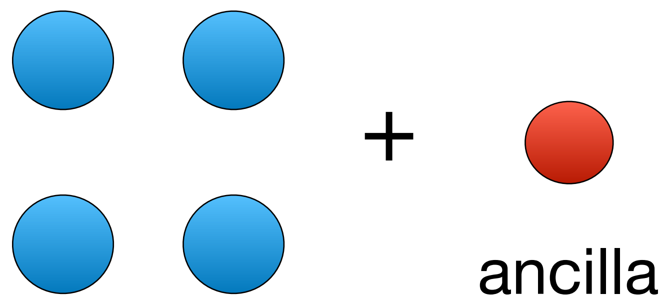
- FT error detection on ion traps

  [Linke et al., arXiv:1611.06946]

- FT preparation and computation on superconducting qubits

  [Vuillot, arXiv:1705.08957]

  [Takita et al., arXiv:1705.09259]



[[4, 2, 2]] error detecting code

Test fault tolerance on small devices

Demonstrating FT is a daunting task see [Gottesman, arXiv:1610.03507]

- Choose a circuit family of interest
- Test full circuit, including error correction & computation
- Start from very few qubits, but scalable and representative
- For every circuit in the family, the error rate for the encoded circuit should be lower than that for the unencoded
- Efficient and reasonable verification

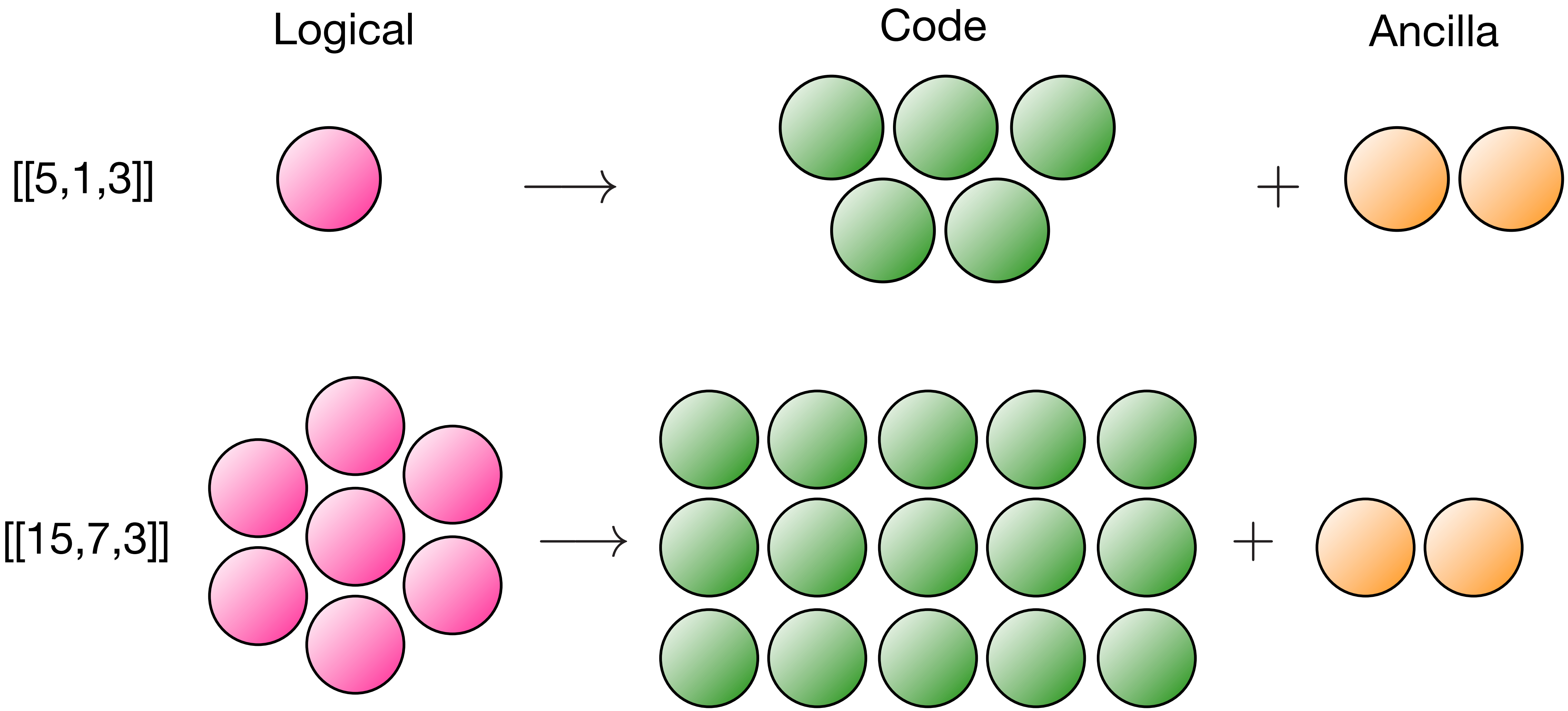
Experimentalists need good tests

- To test/demonstrate the theory
- To assess FT schemes' performance with real error models
- To adapt FT schemes to real noise

Theorists need to come up with them!

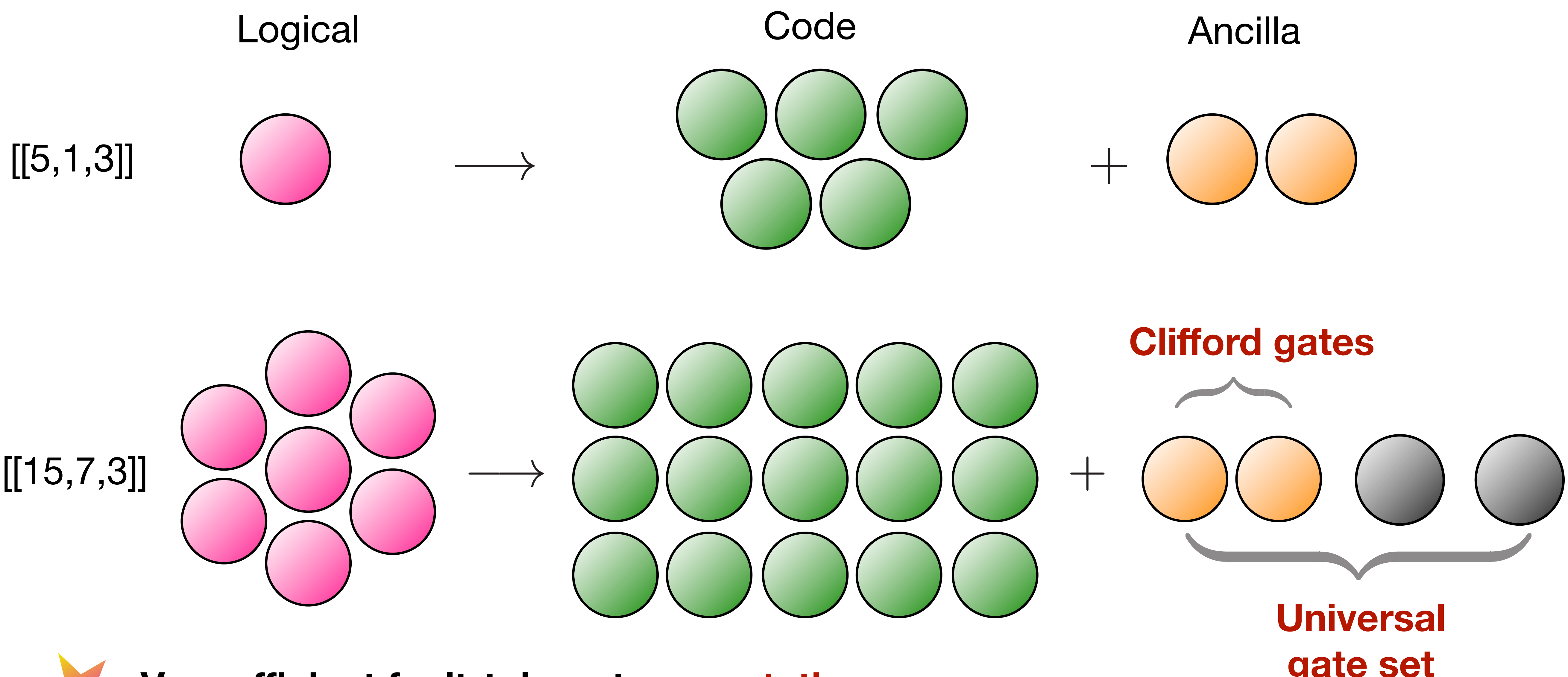
Our results

Very efficient fault-tolerant error correction



Our results

Very efficient fault-tolerant error correction



Very efficient fault-tolerant computation

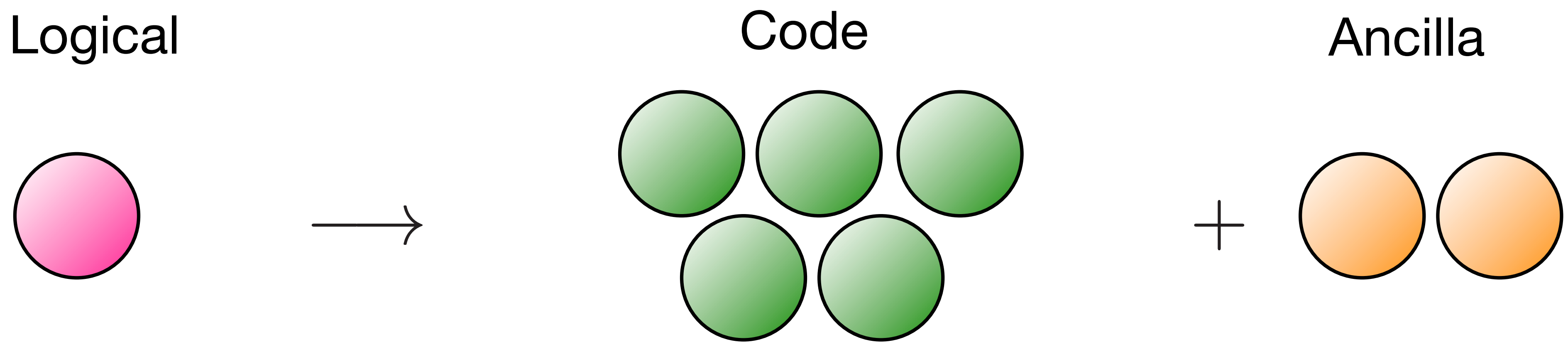
Previously:

With [[7,1,3]] code, need 53=49+4 qubits to test FT computation on 7 logical qubits

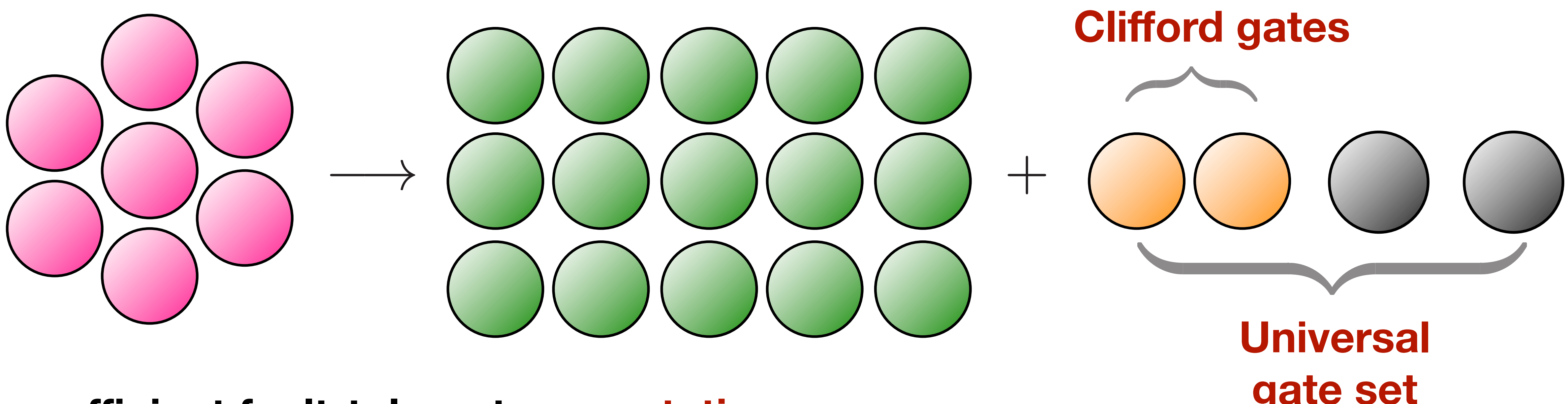
Our results

Very efficient fault-tolerant error correction

[[5,1,3]]



[[15,7,3]]



Very efficient fault-tolerant computation

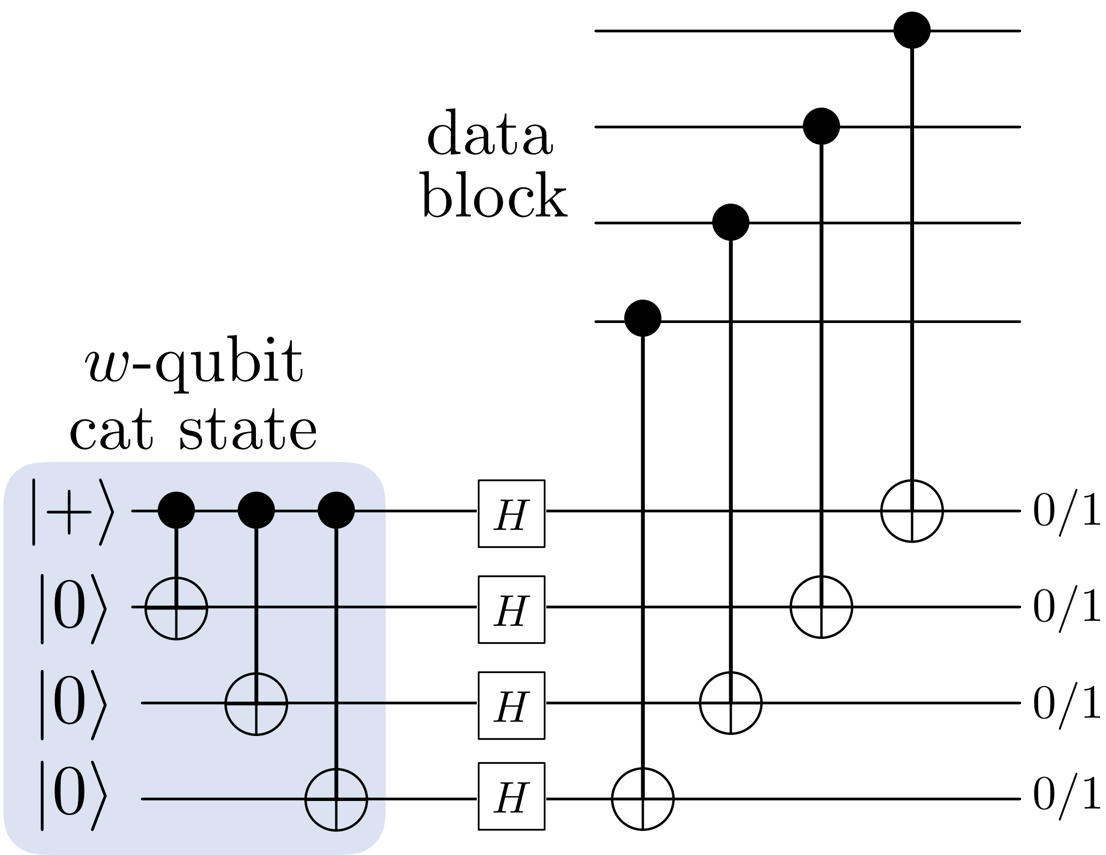
Previously:

With [[7,1,3]] code, need 53=49+4 qubits to test FT computation on 7 logical qubits

Main idea: Flags to catch "bad" errors

Typical error-correction paradigms

Shor's measurement of $Z^{\otimes w}$



Typical error-correction paradigms

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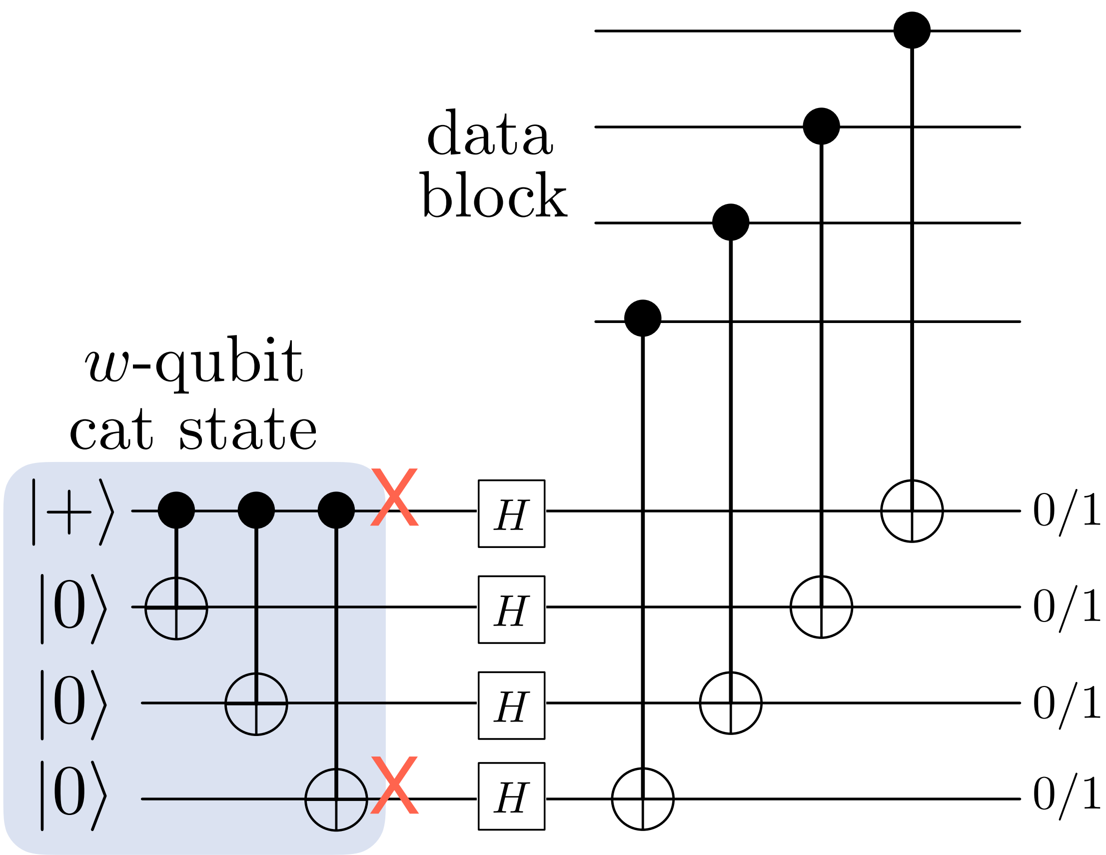

Typical error-correction paradigms

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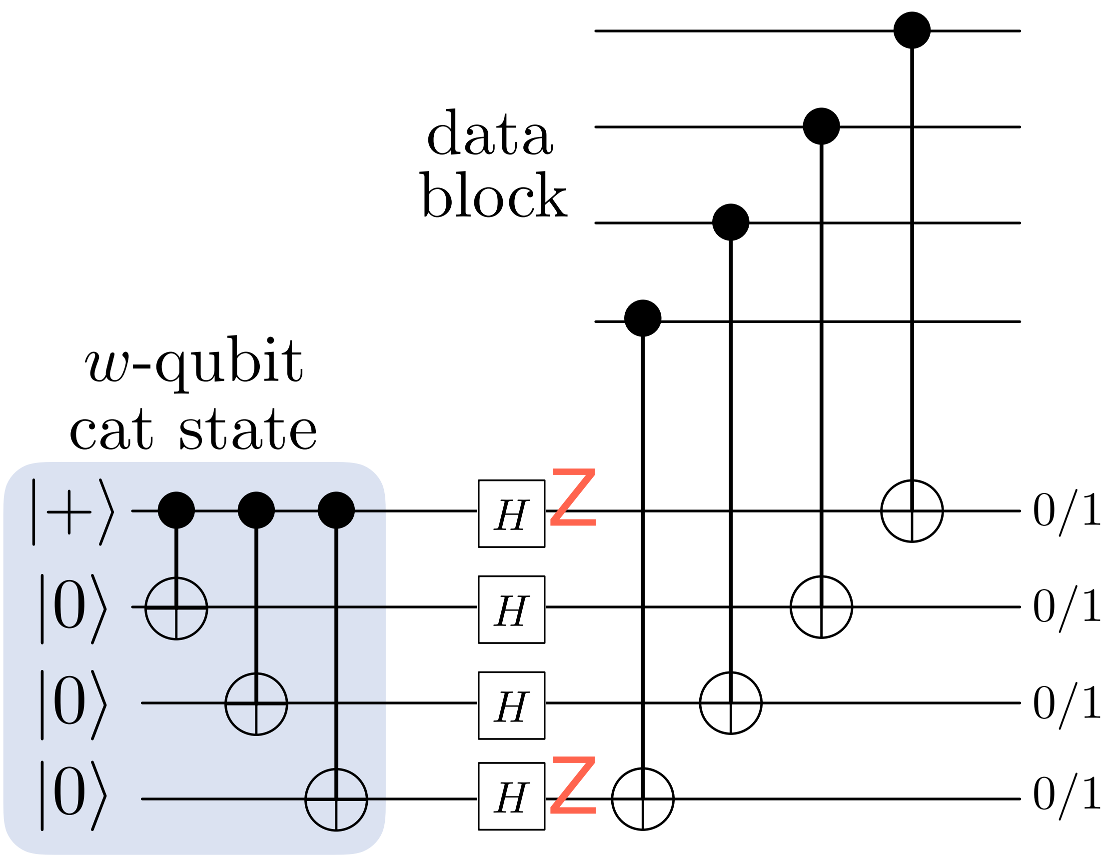

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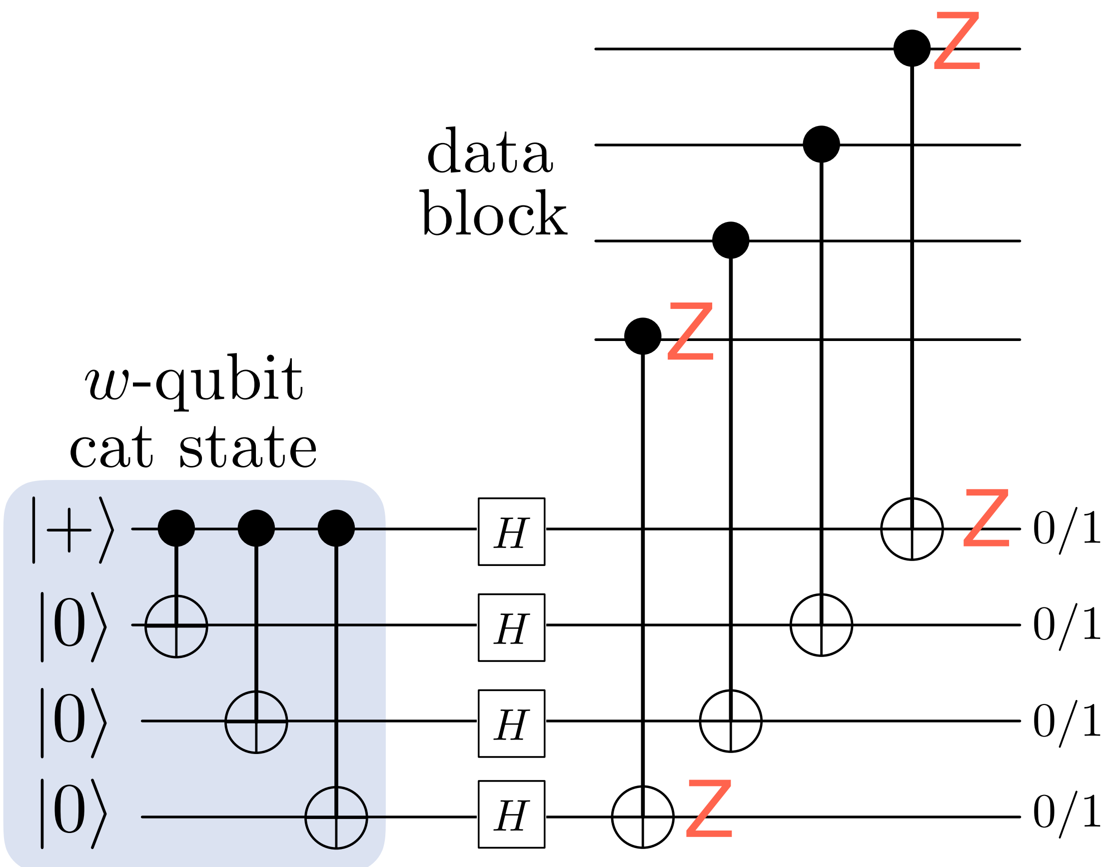

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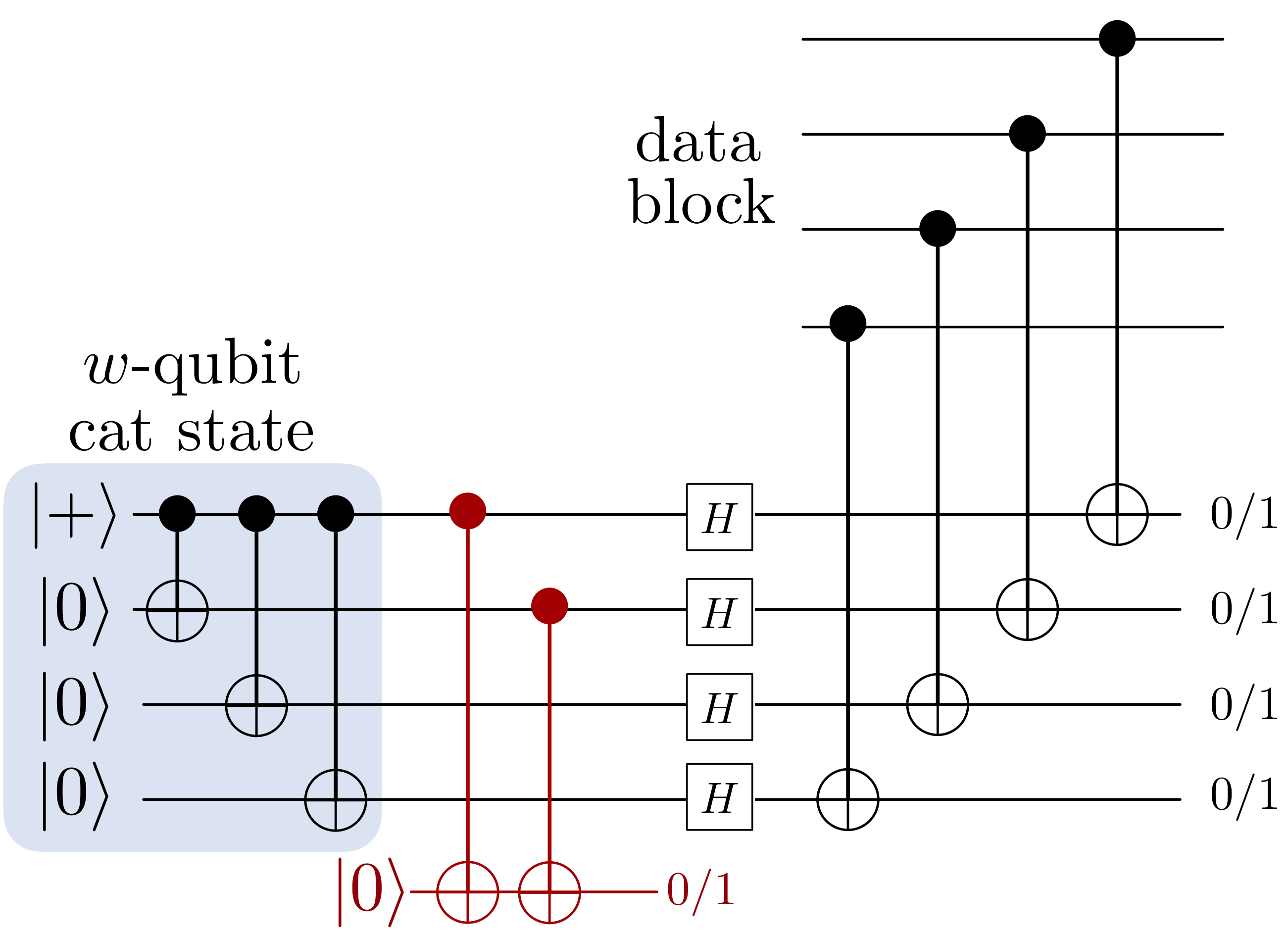

Typical error-correction paradigms

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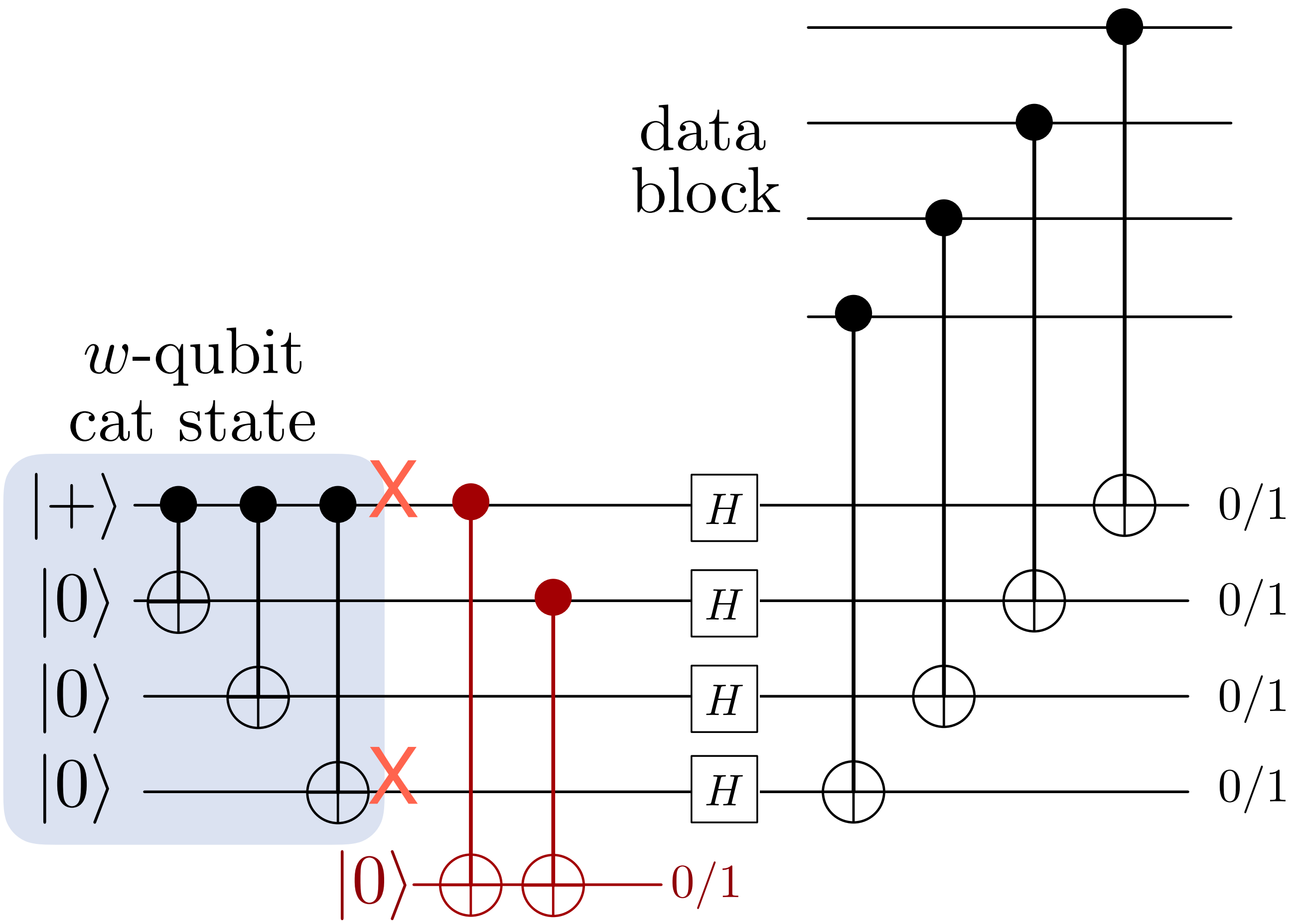

Typical error-correction paradigms

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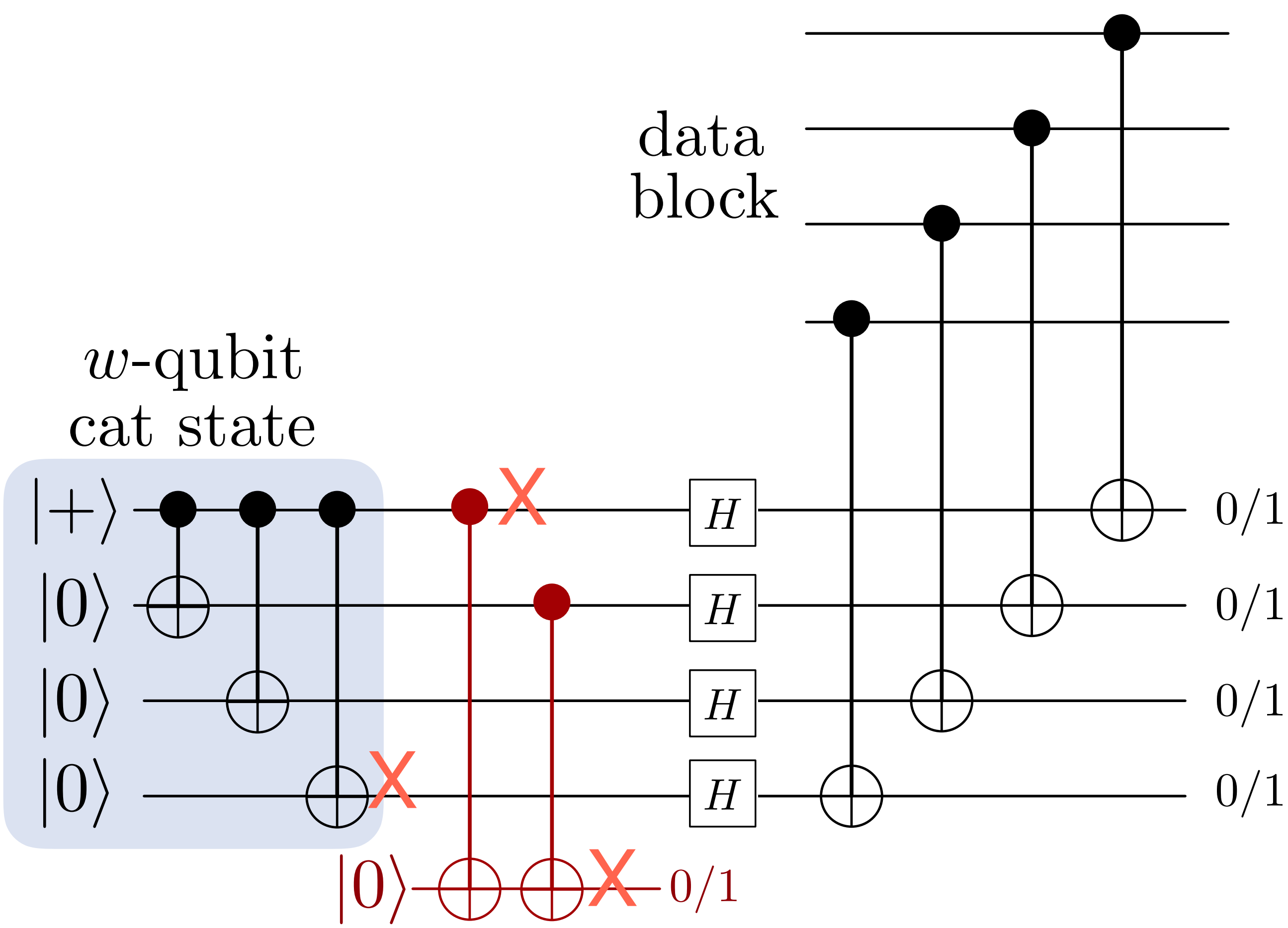
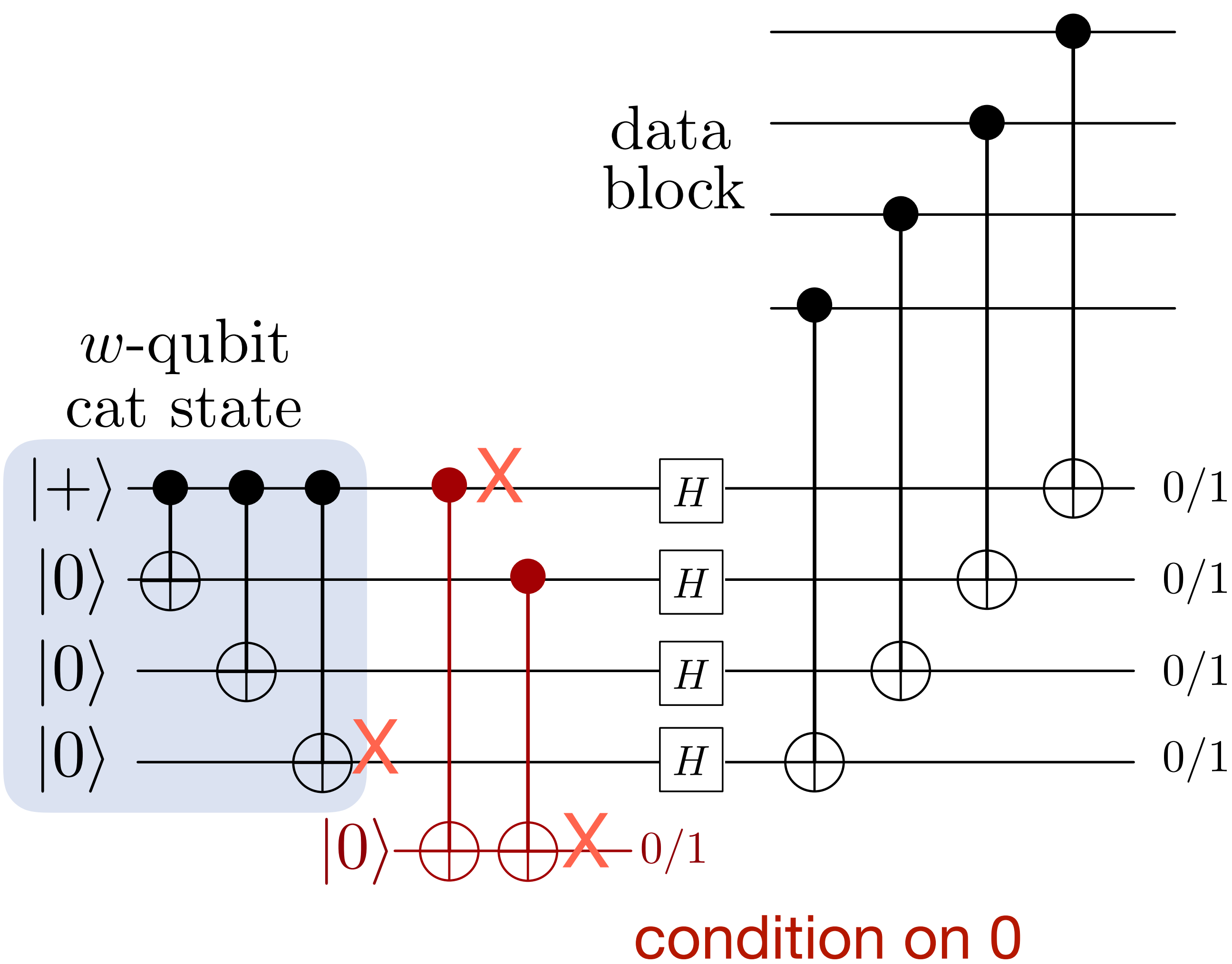


condition on 0

w+1 ancilla qubits

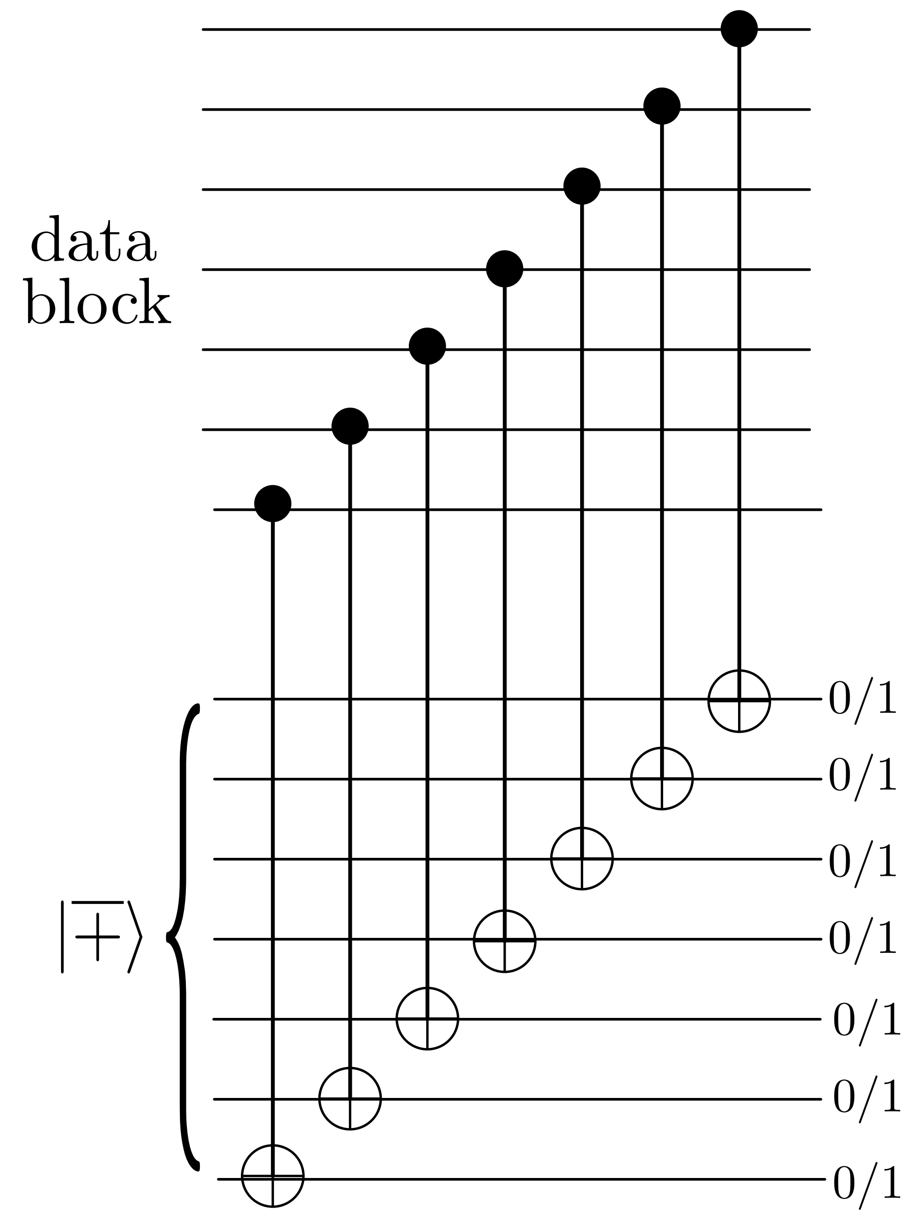
Typical error-correction paradigms

Shor's measurement of $Z^{\otimes w}$



w+1 ancilla qubits

Steane's measurement of Z stabilizers of CSS code

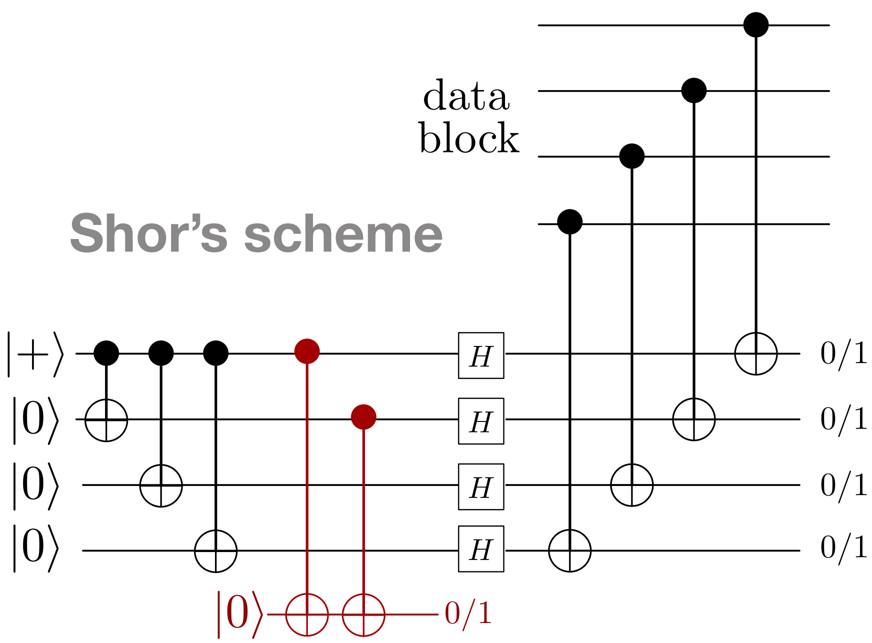


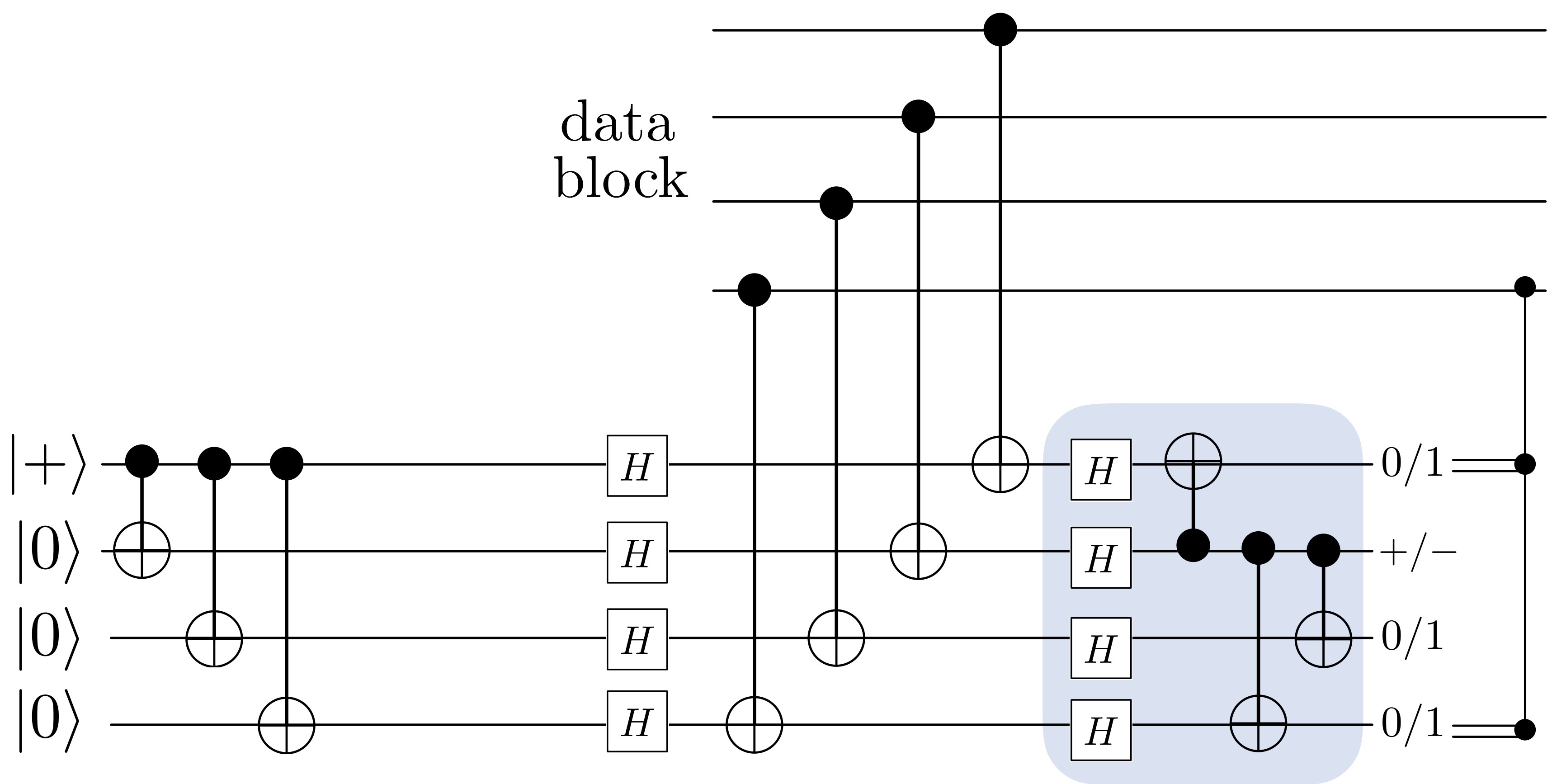
preparation of $|\overline{+}\rangle$ requires

> n ancilla qubits

Previous improvement: DiVincenzo-Aliferis decoding trick

Shor's scheme





no verification needed

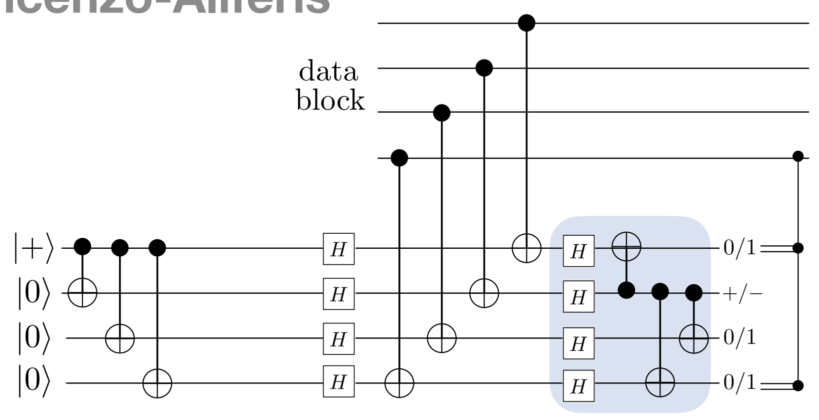
w ancilla qubits

[arXiv:0607047]

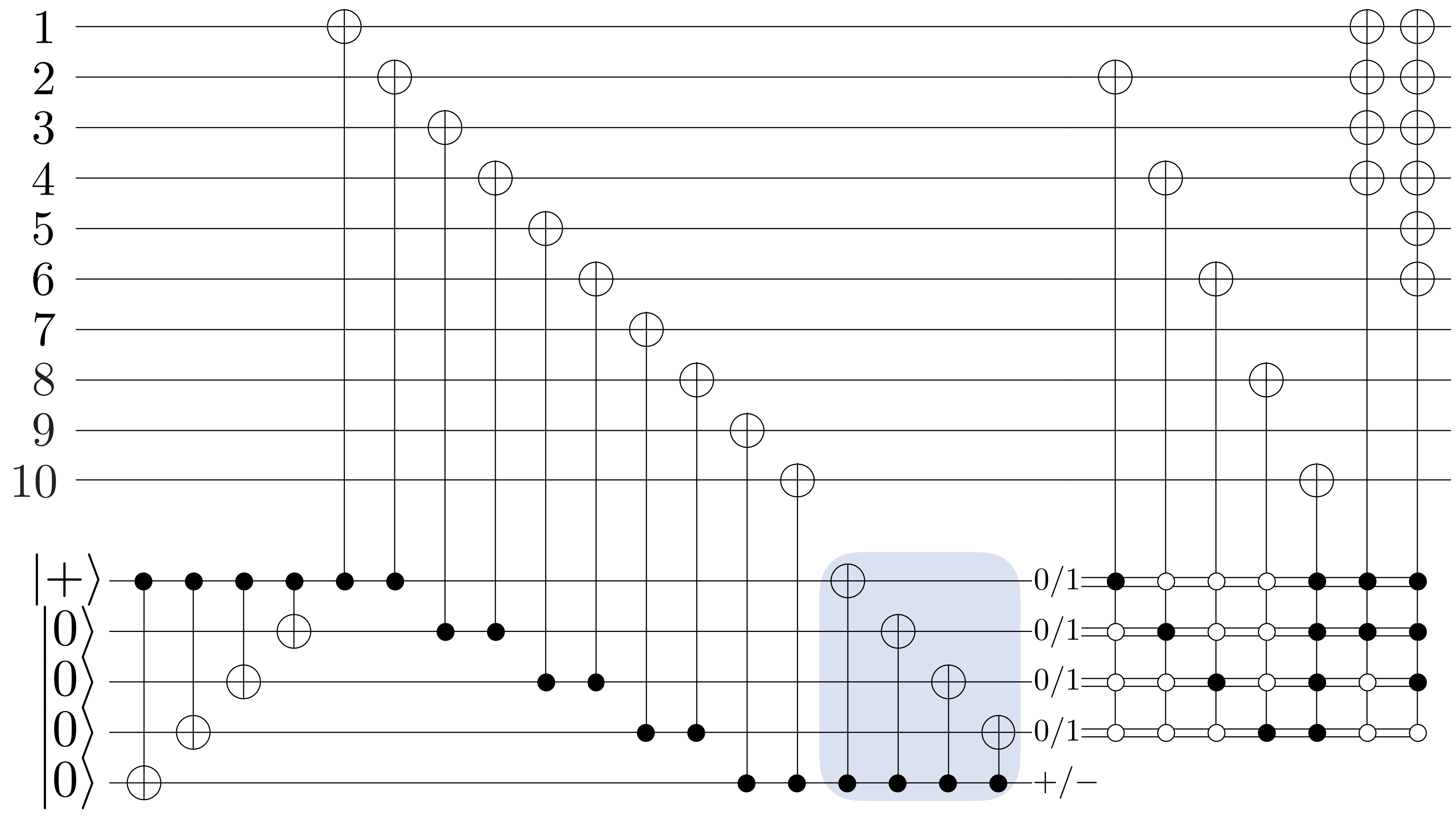
Previous improvement: Stephens-Yoder-Kim half cat state trick

3 ancilla qubits for w =4
4 ancilla qubits for w ≤8

DiVincenzo-Aliferis

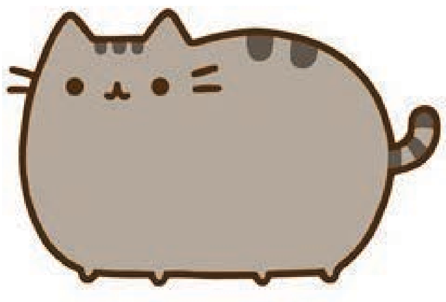
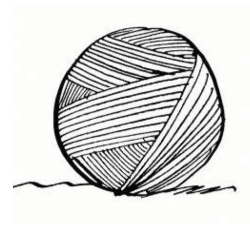


For general w, only max{ 3, ⌈w/2⌉ } ancilla qubits



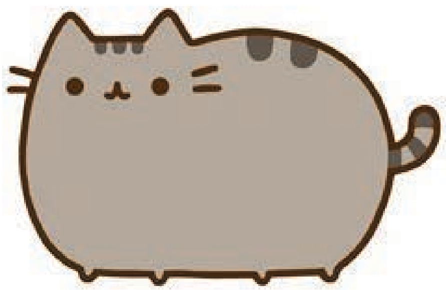
[arXiv:1612.04795] [arXiv: 1402.3037]

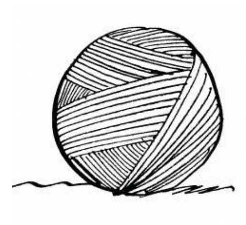
Shor w+1

Shor

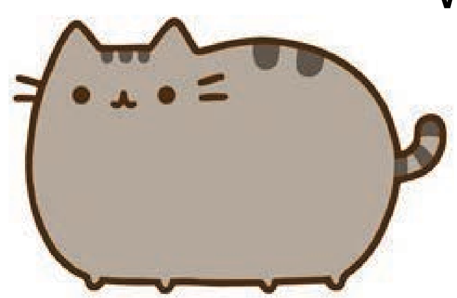
w+1

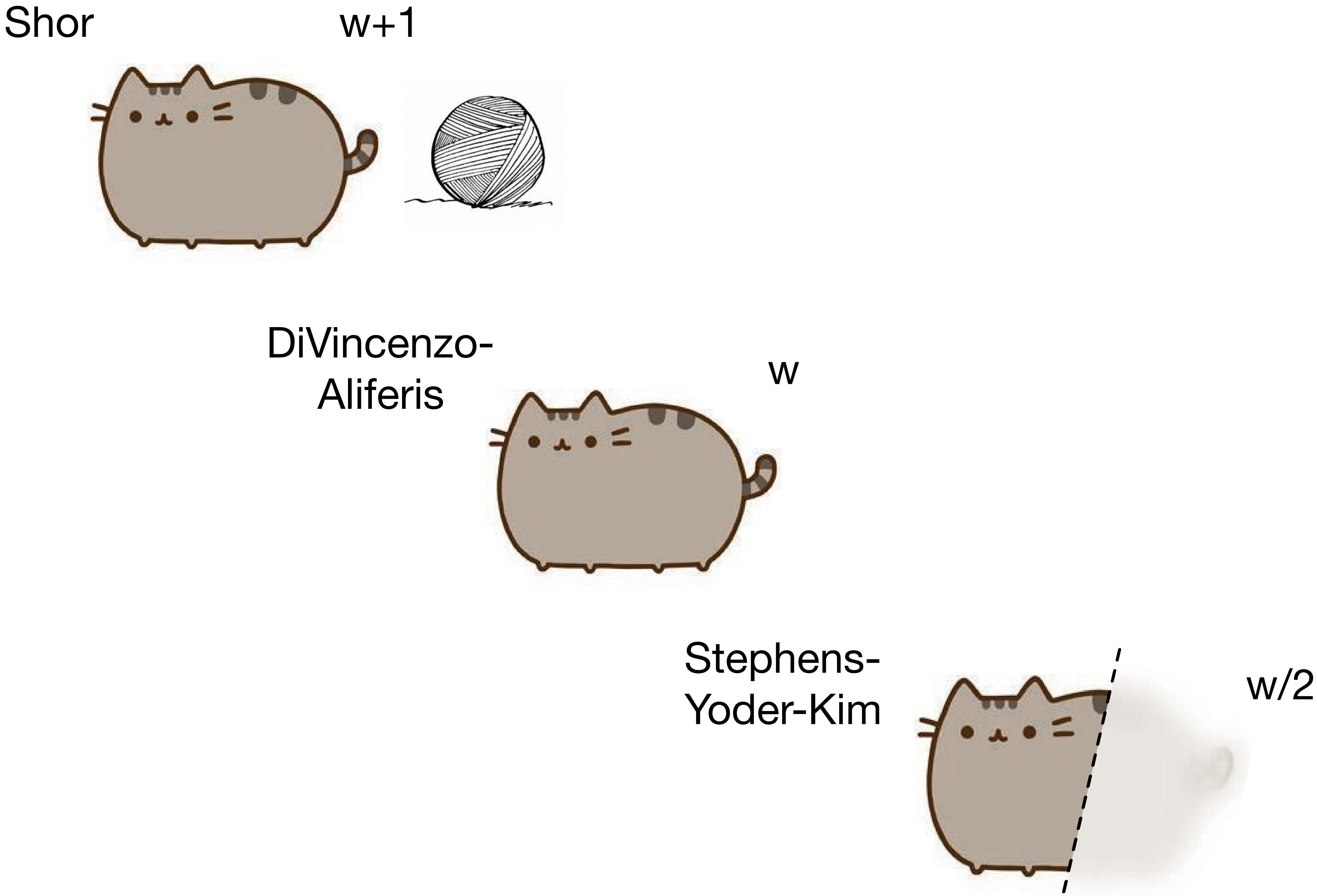




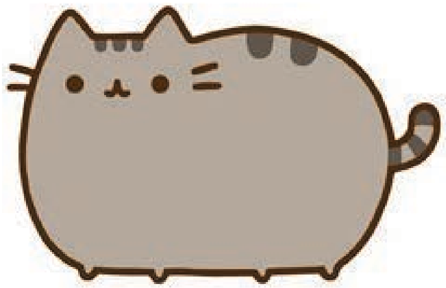
DiVincenzo-
Aliferis

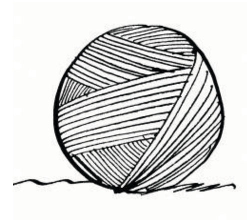
w



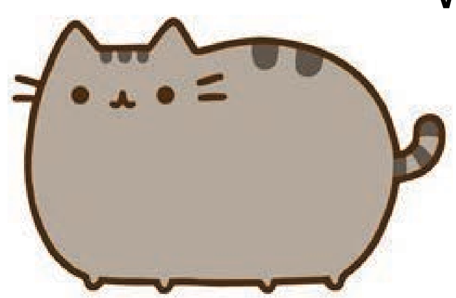

Shor
w+1
DiVincenzo-
Aliferis
w
Stephens-
Yoder-Kim
w/2

Shor w+1

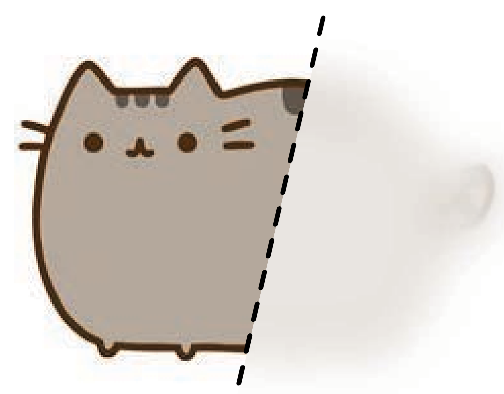




DiVincenzo-Aliferis w

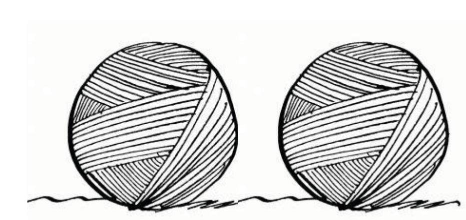


Stephens-Yoder-Kim w/2



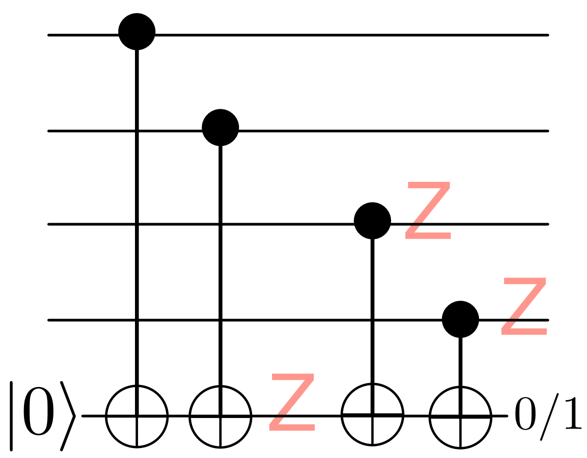
2

us!



Flag paradigm

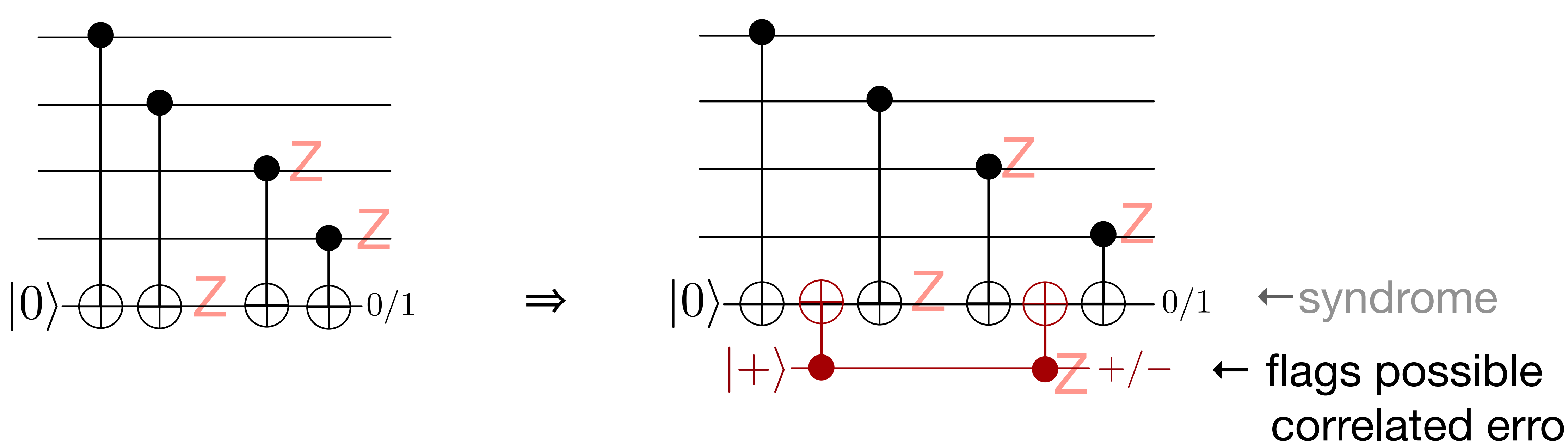
Circuit to measure $Z^{\otimes 4}$



Problem: Errors can spread
Previous approaches: Try to avoid this
Our main idea: Catch the errors that can spread

Flag paradigm

Circuit to measure $Z^{\otimes 4}$



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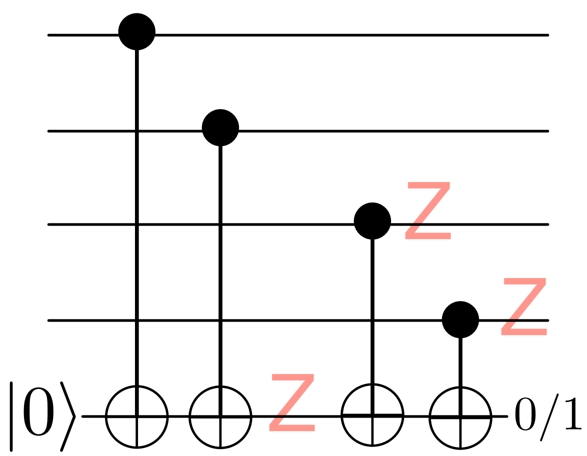
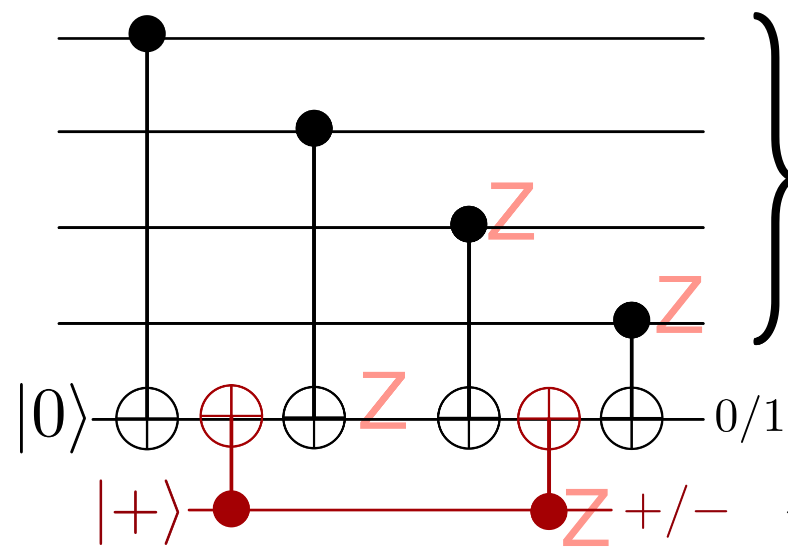
Flag paradigm

Circuit to measure $Z^{\otimes 4}$



⇒



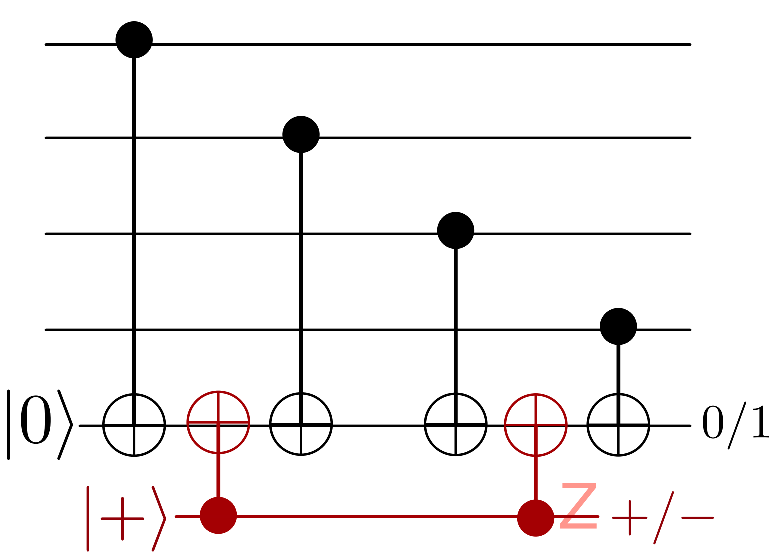
Flagged errors must be distinguishable!

← syndrome

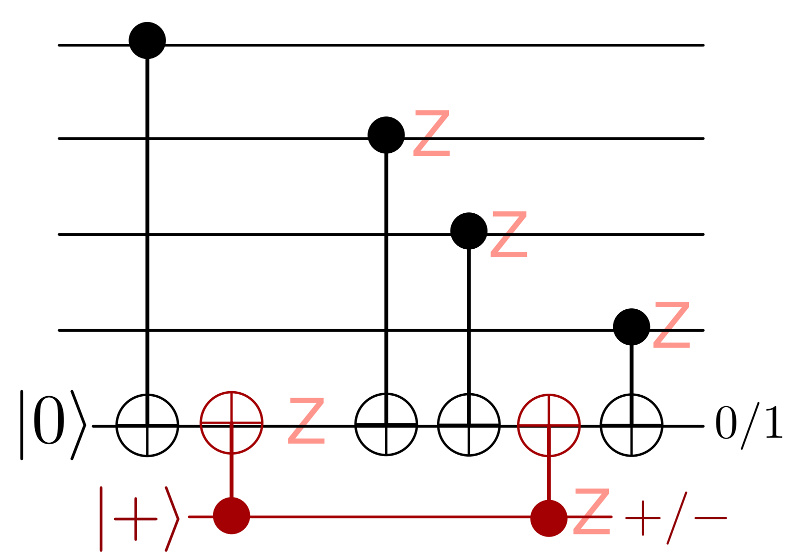
← flags possible correlated error

$IIZZ$

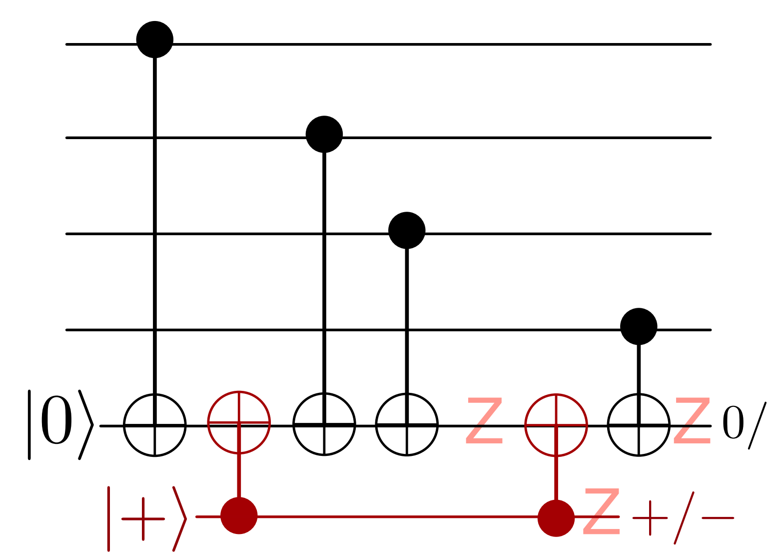
Problem: Errors can spread
Previous approaches: Try to avoid this
Our main idea: Catch the errors that can spread



$IIII$



$IZZZ$

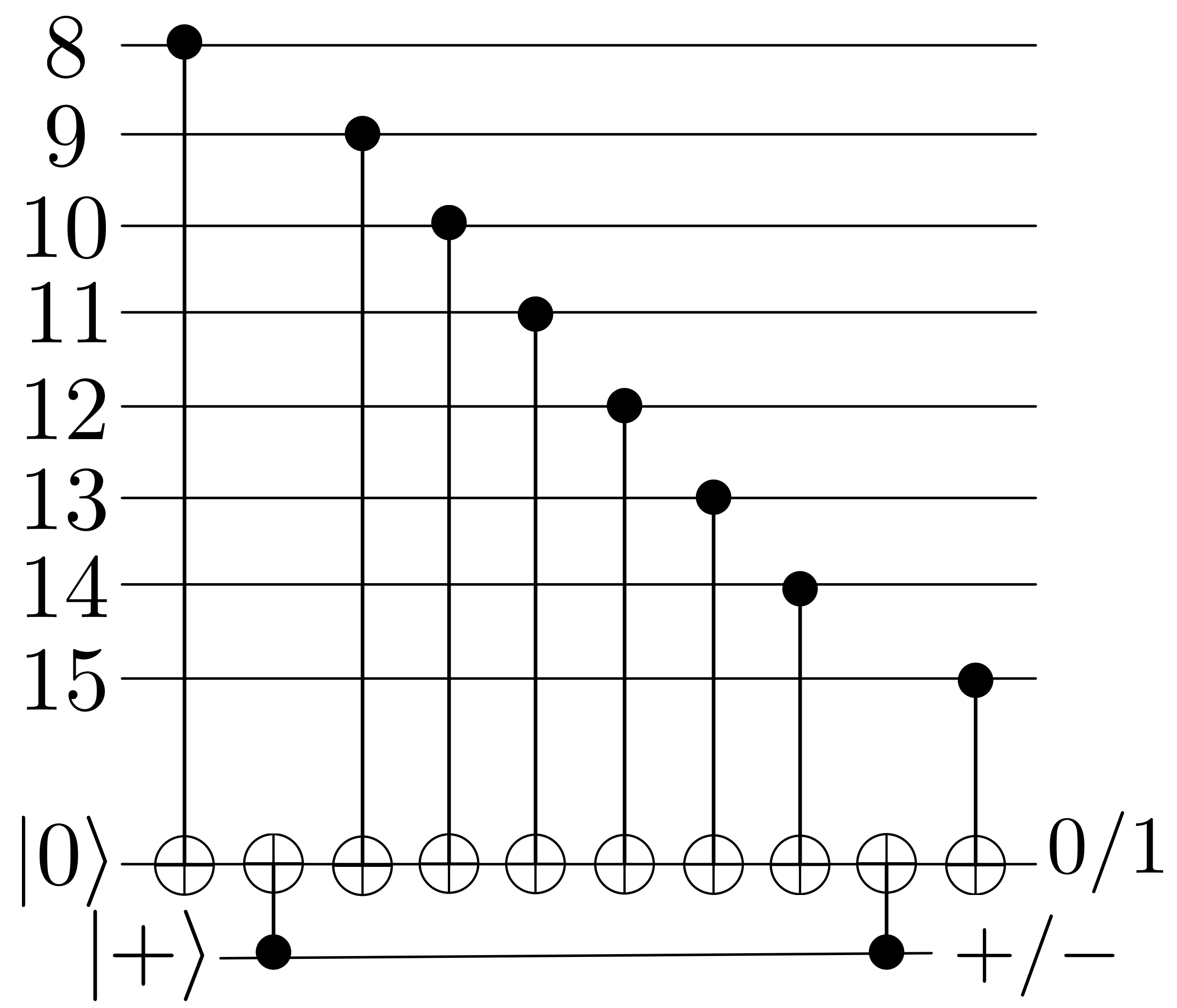


$IIIZ$

Order of CNOTs matters

[[15,7,3]] code

0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
1	0	1	0	1	0	1	0	1	0	1	0	1	0	1



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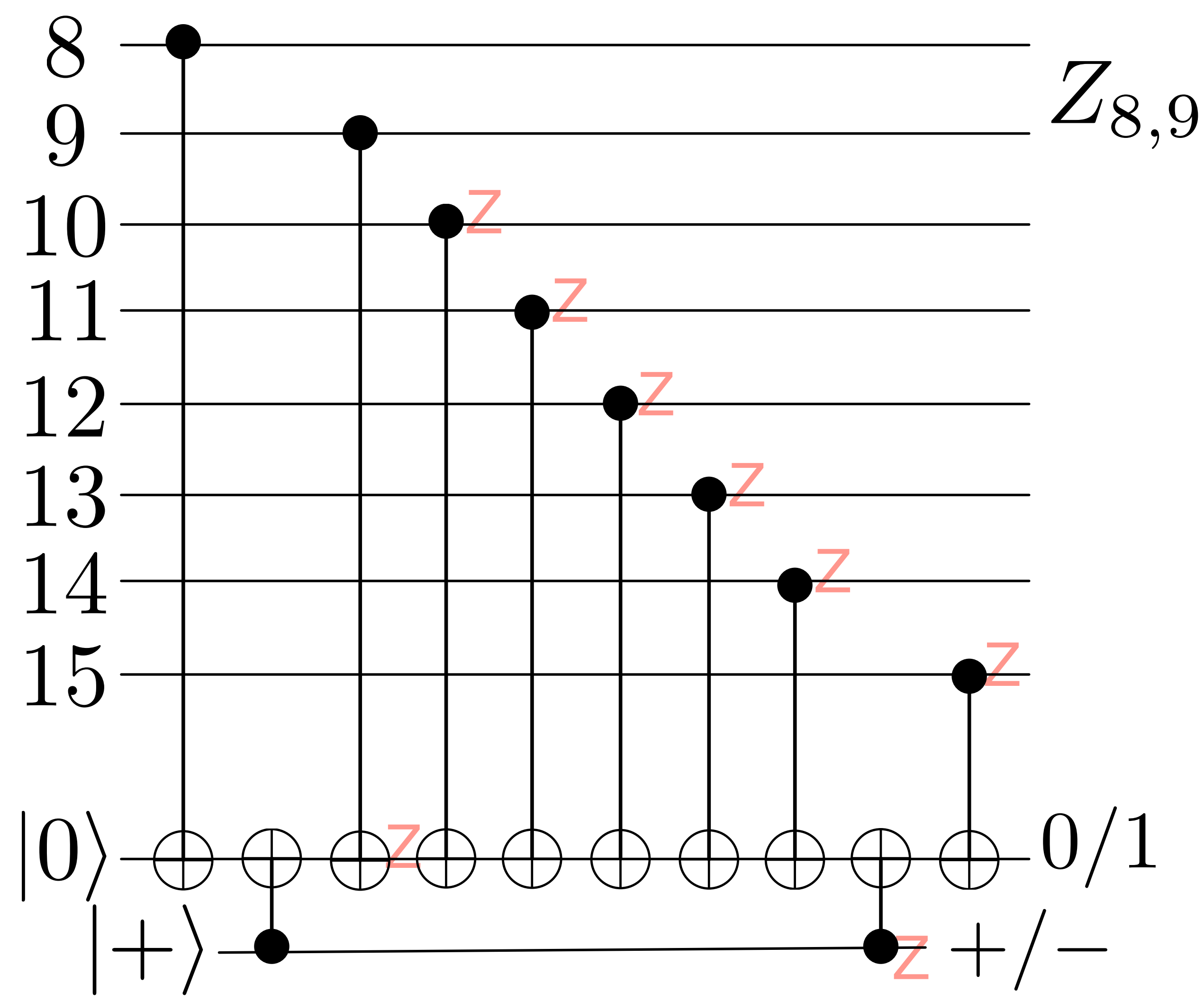
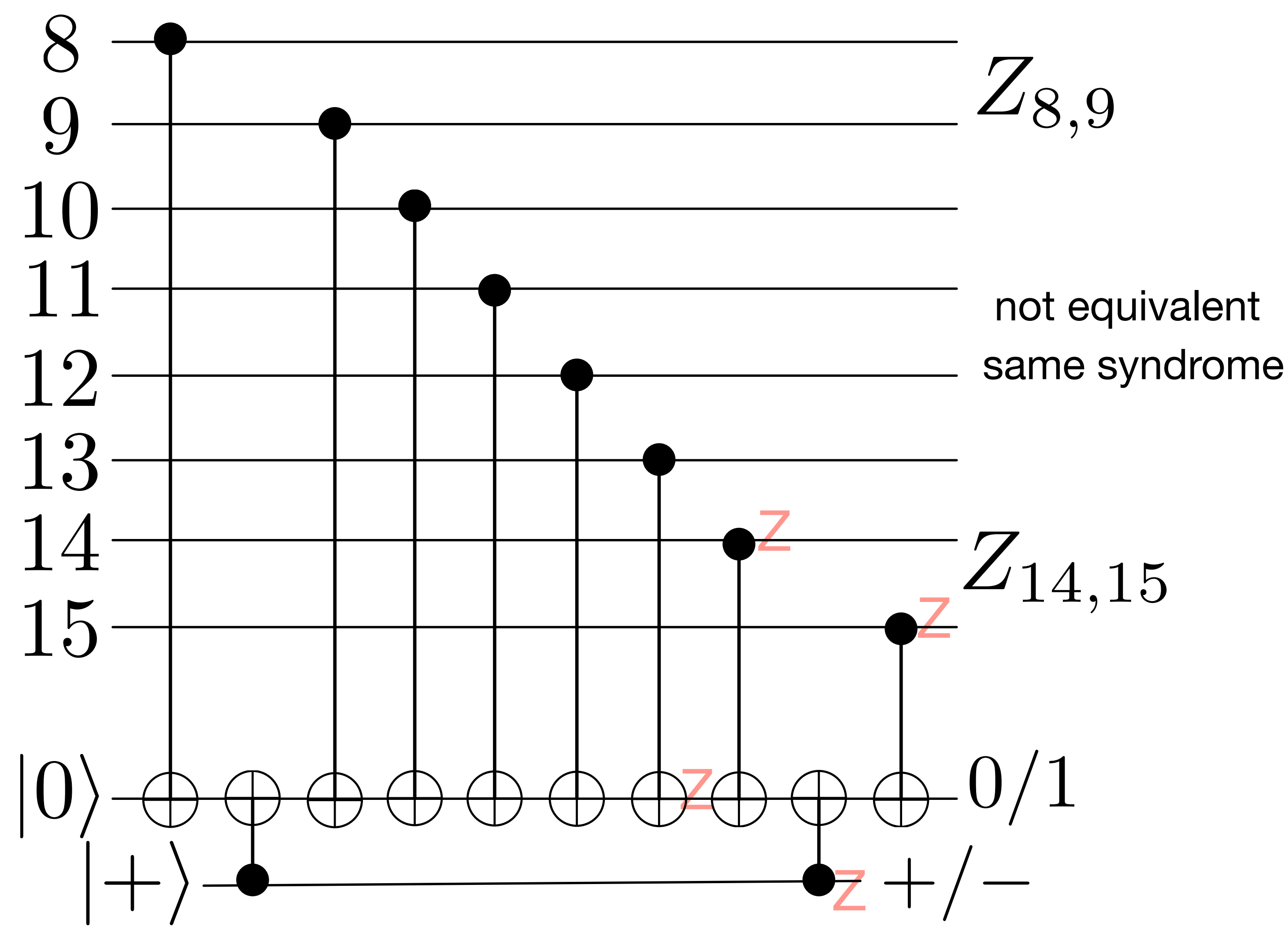
0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
1	0	1	0	1	0	1	0	1	0	1	0	1	0	1



Order of CNOTs matters

[[15,7,3]] code

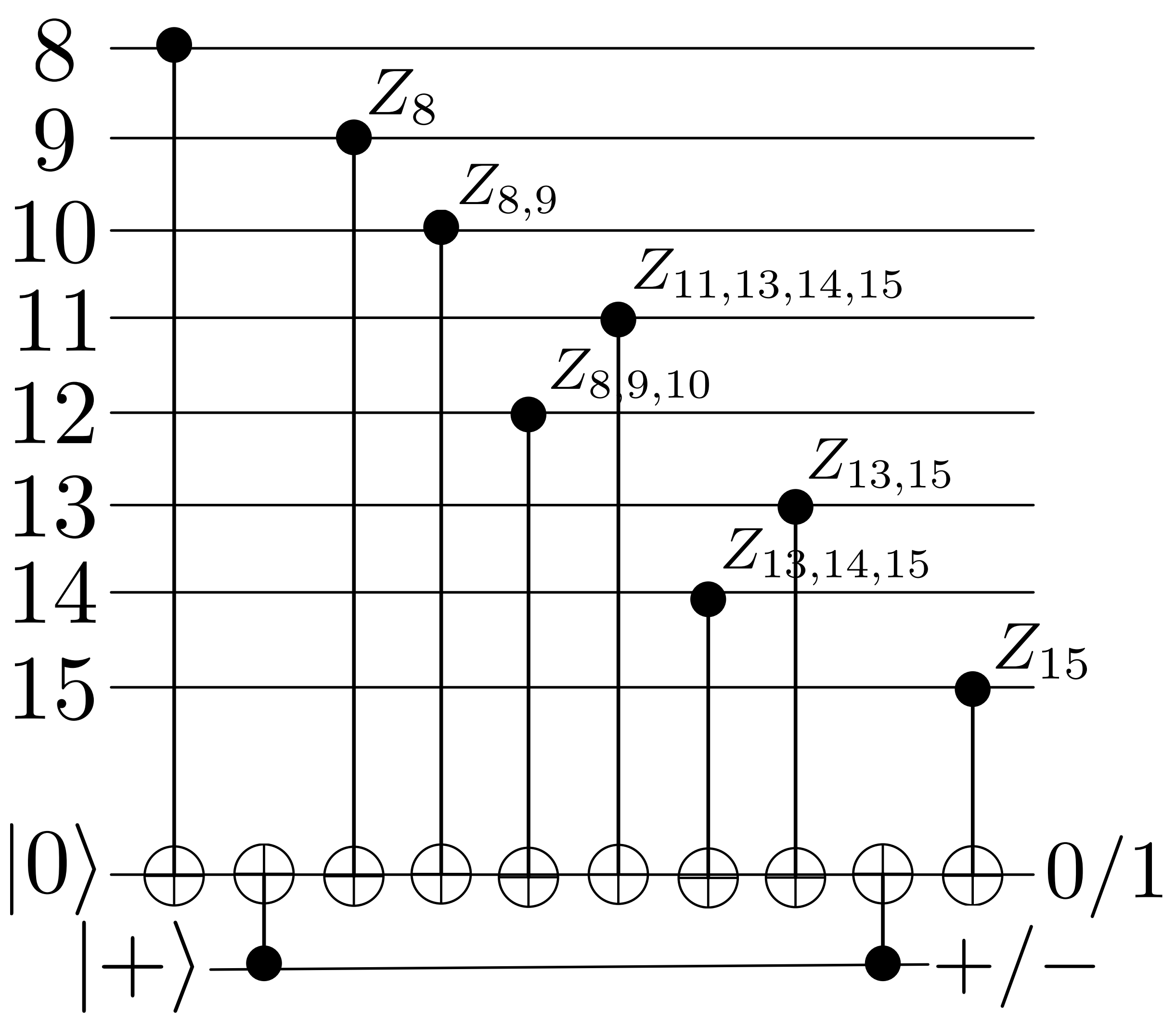
$$\begin{matrix}
0&0&0&0&0&0&0&1&1&1&1&1&1&1&1\\
0&0&0&1&1&1&1&0&0&0&0&1&1&1&1\\
0&1&1&0&0&1&1&0&0&1&1&0&0&1&1\\
1&0&1&0&1&0&1&0&1&0&1&0&1&0&1
\end{matrix}$$



Order of CNOTs matters

[[15,7,3]] code

0 0 0 0 0 0 0 1 1 1 1 1 1 1 1
0 0 0 1 1 1 1 0 0 0 0 1 1 1 1
0 1 1 0 0 1 1 0 0 1 1 0 0 1 1
1 0 1 0 1 0 1 0 1 0 1 0 1 0 1

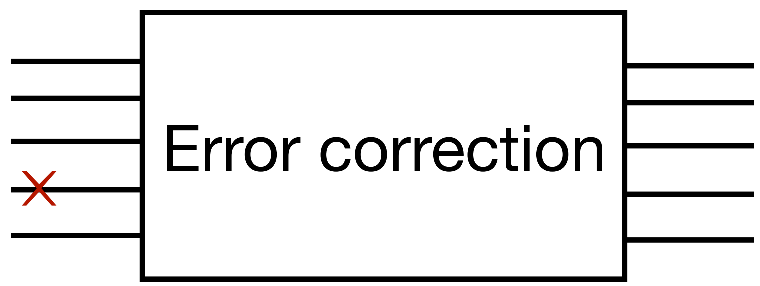


Flag trick works for many codes

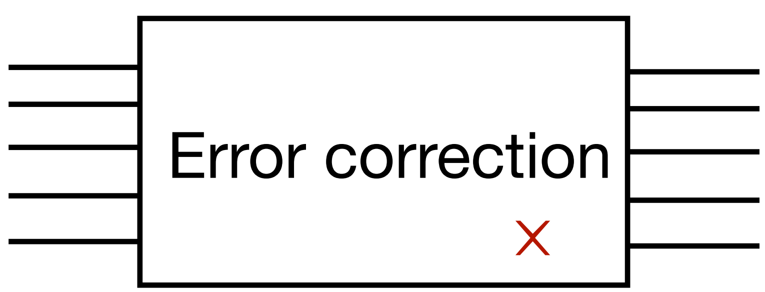
	Code	Ancilla qubits required for Shor	Half cat	Flag
	$[\![5,1,3]\!]$	5	3	2
Hamming	$[\![7,1,3]\!]$	5	3	2
	$[\![9,1,3]\!]$	1	–	–
	$[\![8,3,3]\!]$	7	3	2
	$[\![10,4,3]\!]$	9	4	2
	$[\![11,5,3]\!]$	9	4	2
	$[\![15,7,3]\!]$	9	4	2
	$[\![31,21,3]\!]$	17	8	2
	$[\![2^r-1,2^r-1-2r,3]\!]$	$2^{r-1}+1$	2^{r-2}	2

Criteria for FT QEC on distance-3 codes

- Only consider at most 1 fault

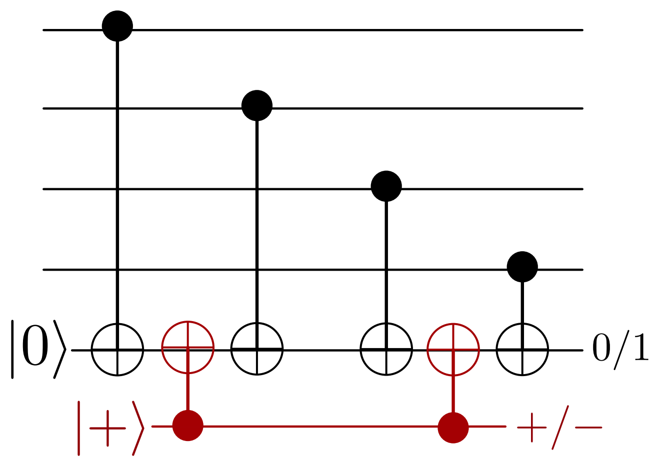


no error



at most weight-1 error

Error correction procedure



Measure stabilizer
using flag gadget

no flag

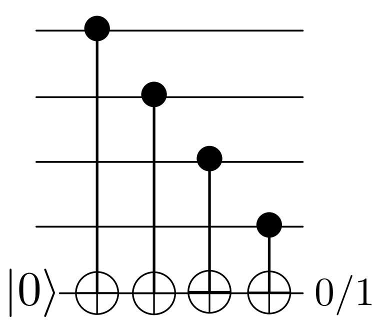
trivial syndrome

Measure next stabilizer
using flag gadget

no flag

nontrivial syndrome

Get all syndromes
without flags
Correct weight-1 error

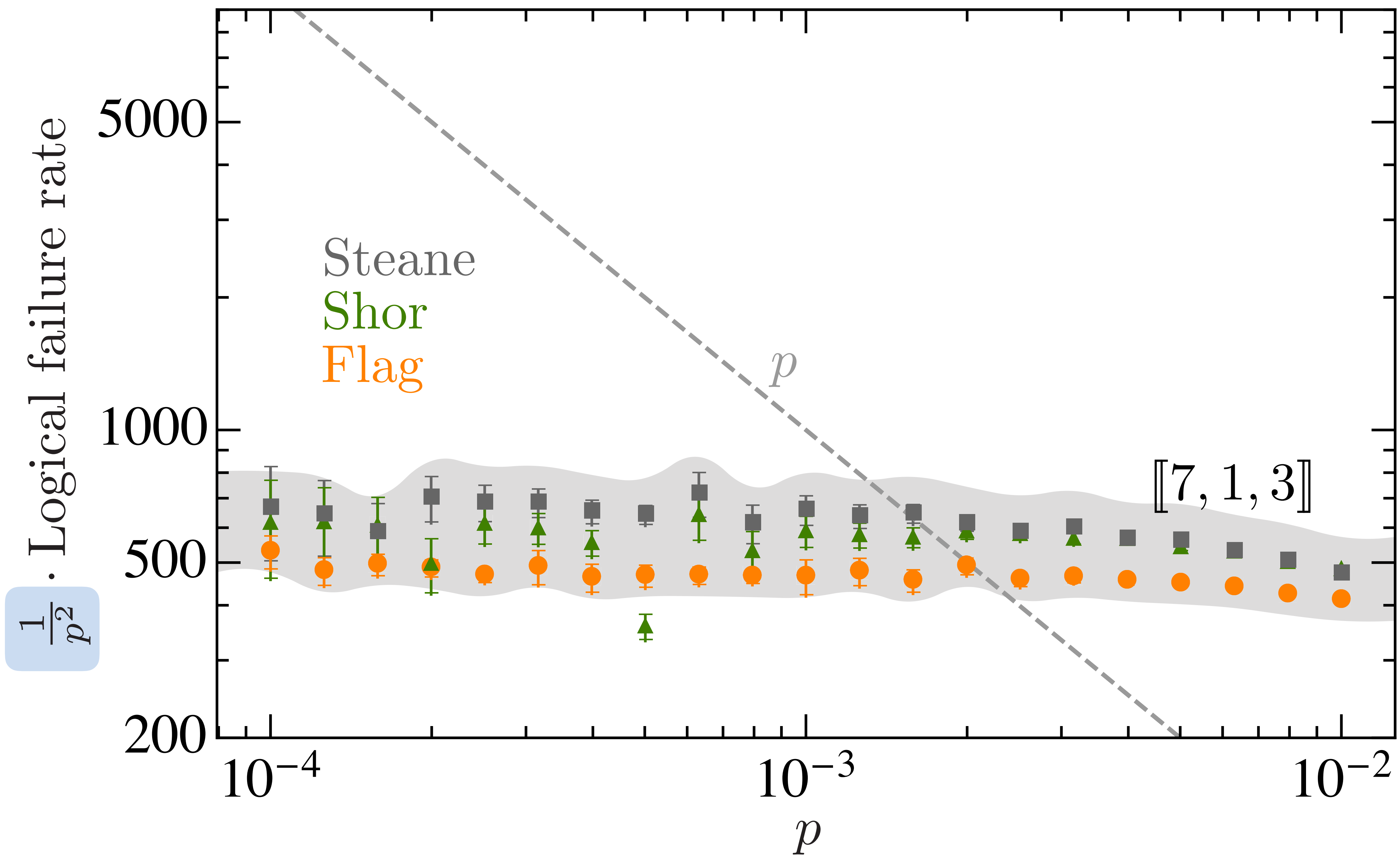


flag raised

Get all syndromes
without flags
Correct correlated error

Simulations

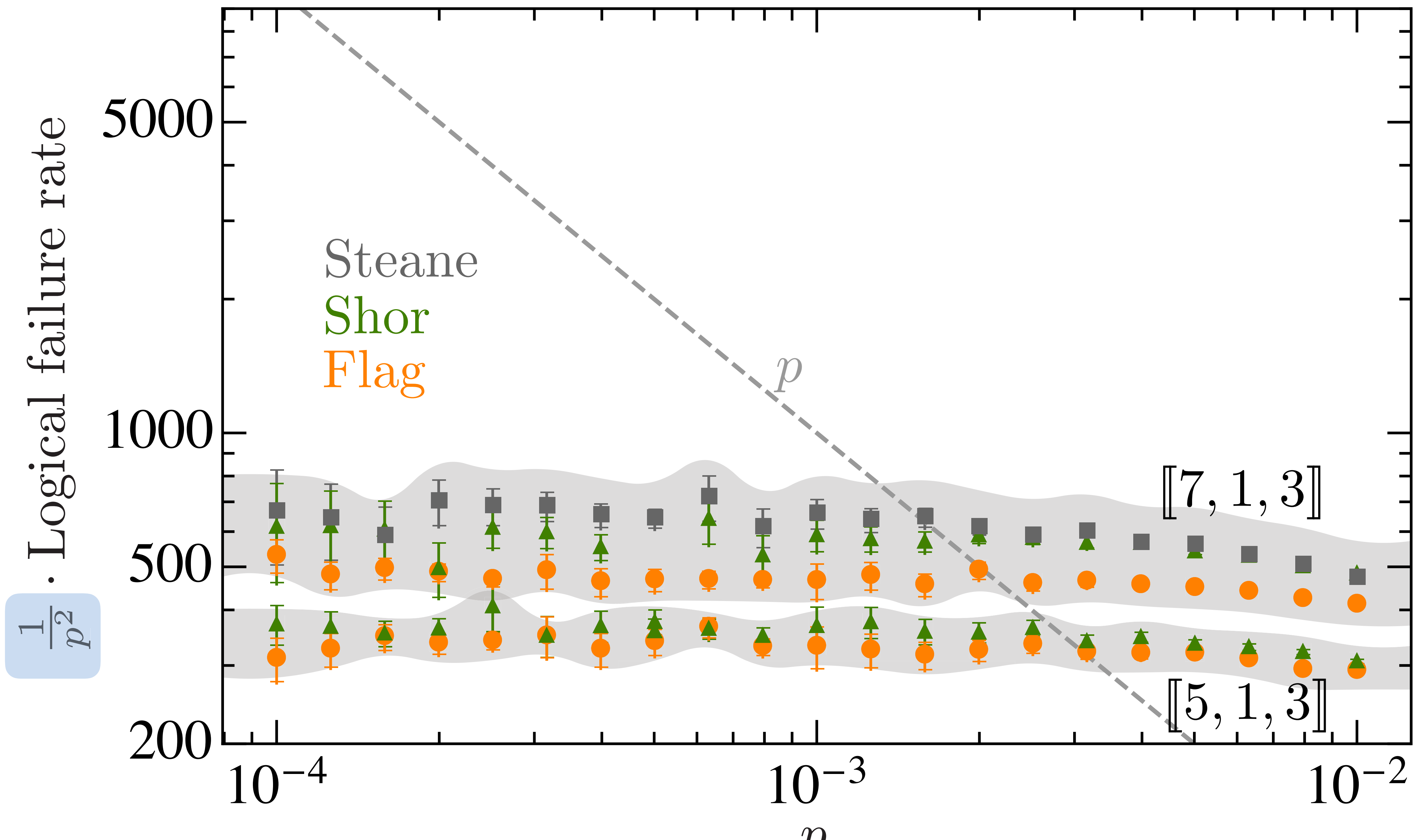
Depolarizing noise, no geometric constraint, no rest error



Morals: Flag is comparable (even slightly better than Shor)

Simulations

Depolarizing noise, no geometric constraint, no rest error



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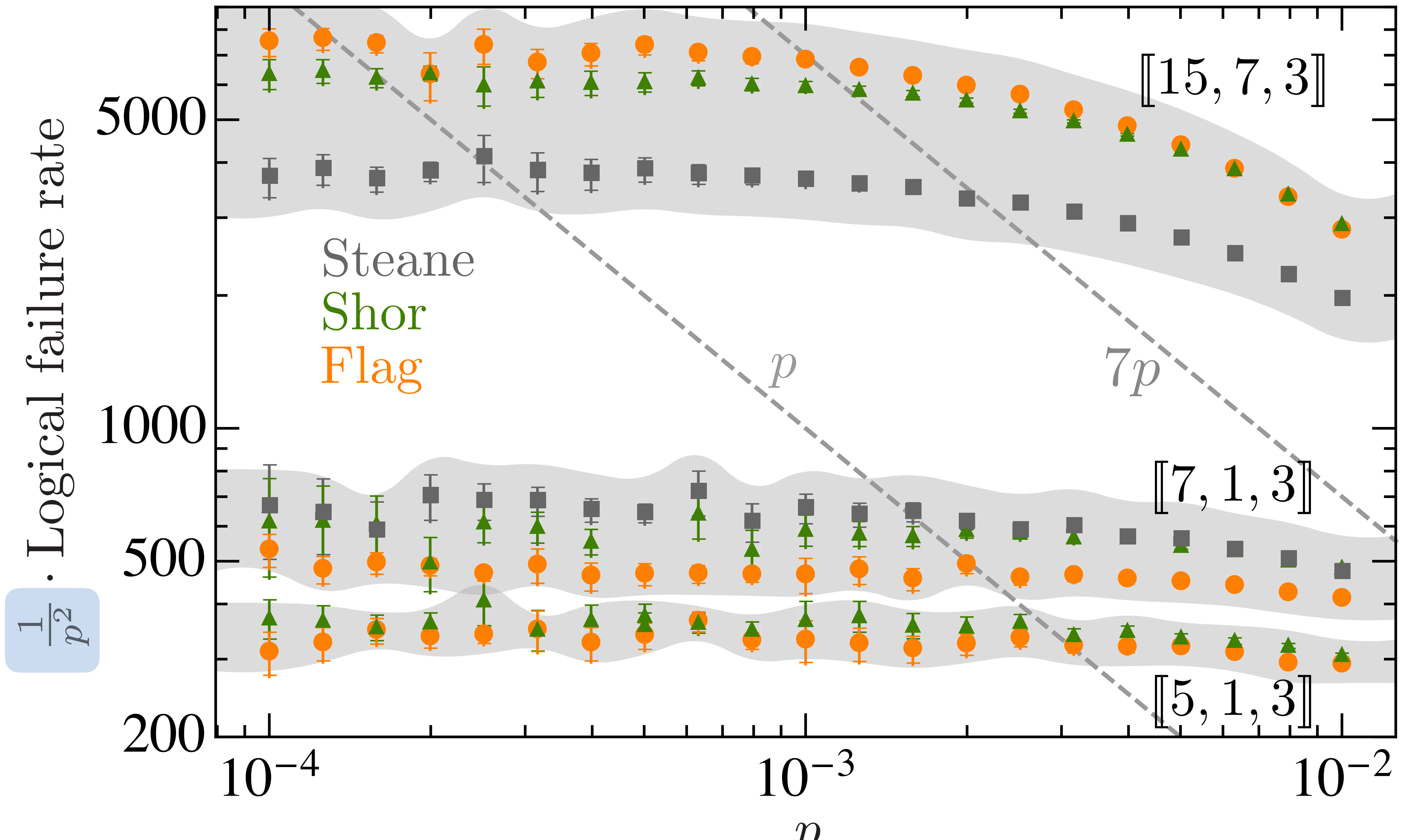
Simulations

Depolarizing noise, no geometric constraint, no rest error

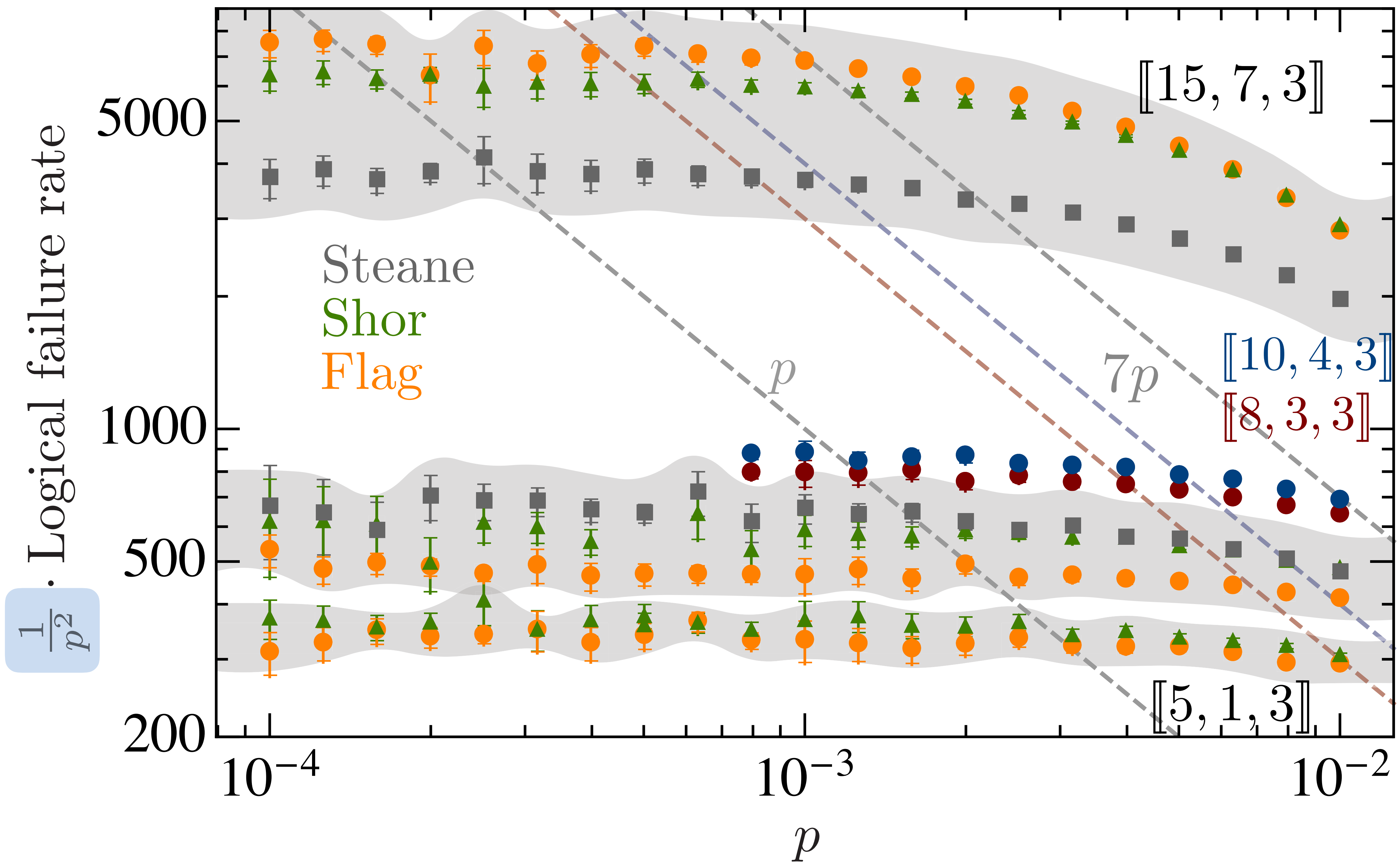


Morals: Flag is comparable (even slightly better than Shor)

For large codes, Steane is probably better (it extracts multiple syndromes at once)

Simulations

Depolarizing noise, no geometric constraint, no rest error

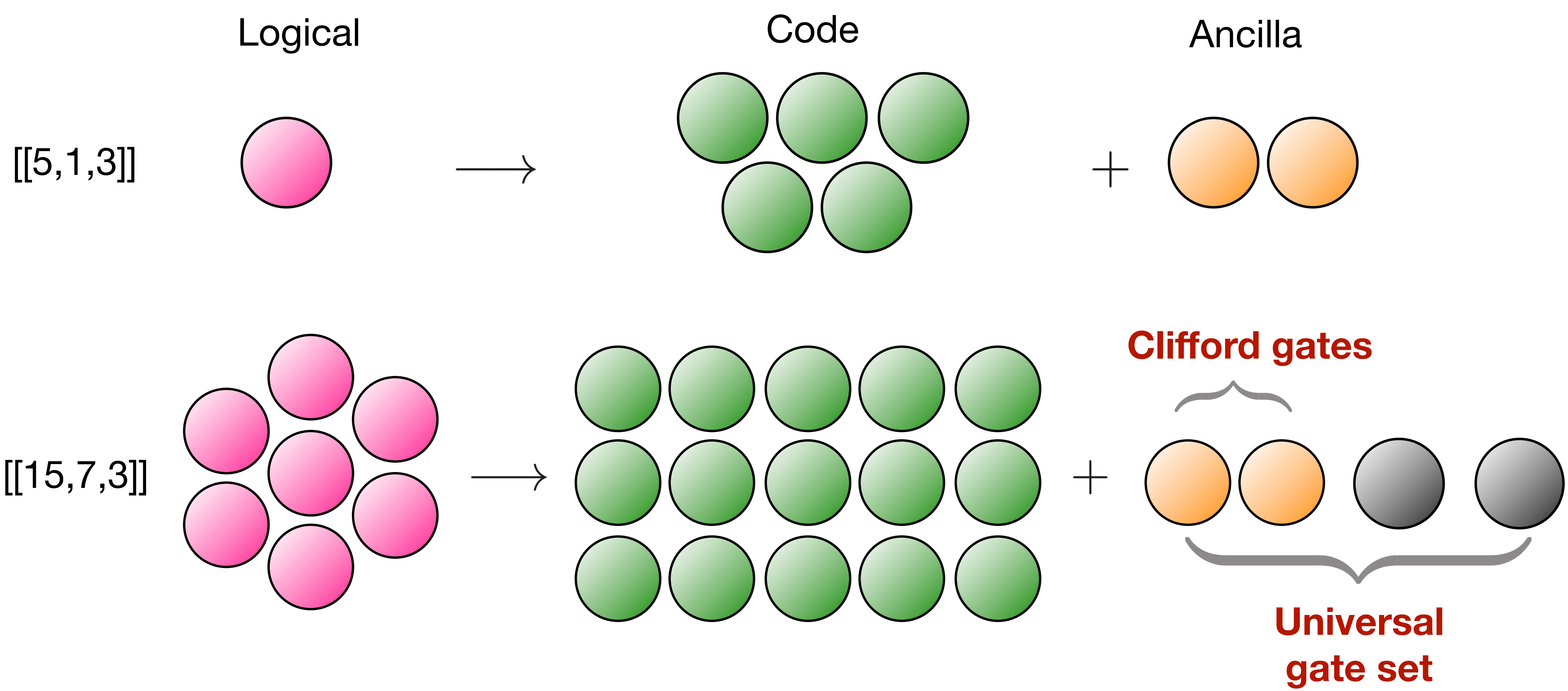


Morals: Flag is comparable (even slightly better than Shor)

For large codes, Steane is probably better (it extracts multiple syndromes at once)

Our results

★ **Very efficient fault-tolerant error correction**



★ **Very efficient fault-tolerant computation**

Main idea: Flags to catch “bad” errors

Fault-tolerant computation

Two conceptual steps

1. **Find logical gates**

- Permute qubits (with a symmetry) [Harrington, Reichardt '11] [Grassl et al., arXiv:1302.1035]
- Round-robin design [Yoder et al., arXiv:1603.03948]
- Build CZ circuits based on permutation symmetries

Fault-tolerant computation

Two conceptual steps

1. Find logical gates

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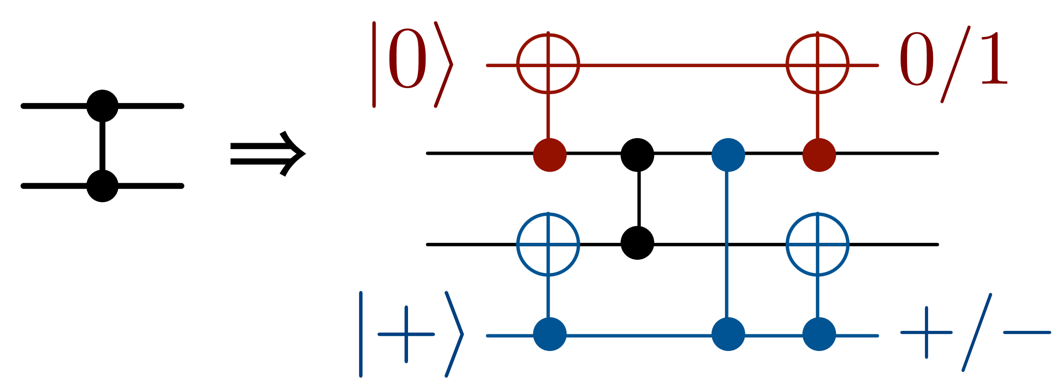
- Build CZ circuits based on permutation symmetries

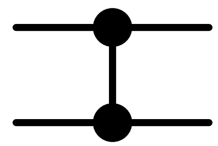
2. Make it fault tolerant

Use flag gadget to catch bad errors



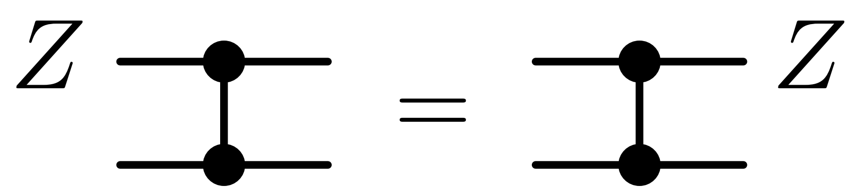
CZ gate

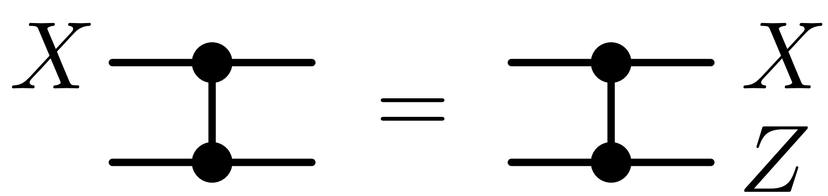
- symmetrical



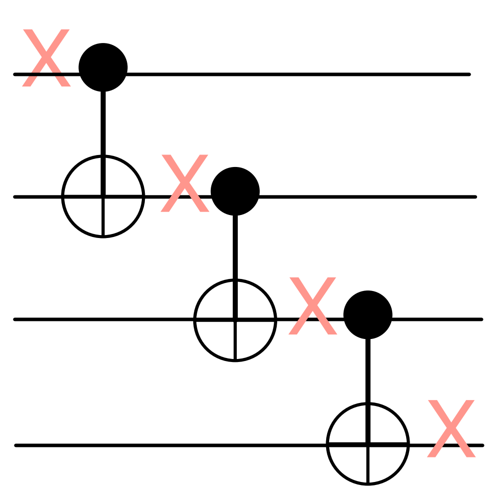
$$= \begin{array}{cccc} 00 & 01 & 10 & 11 \\ \end{array}$$
$$\begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & -1 \end{pmatrix}$$

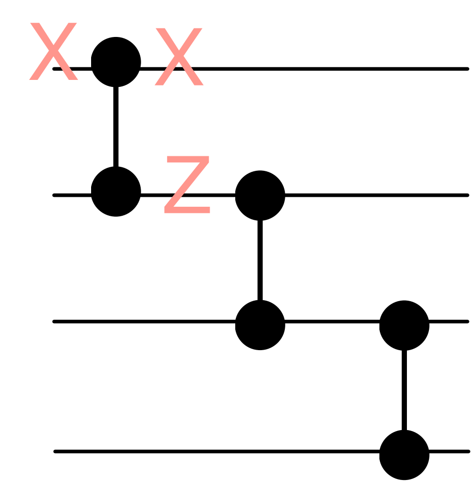
- nice for CSS codes





- errors don't spread more than once



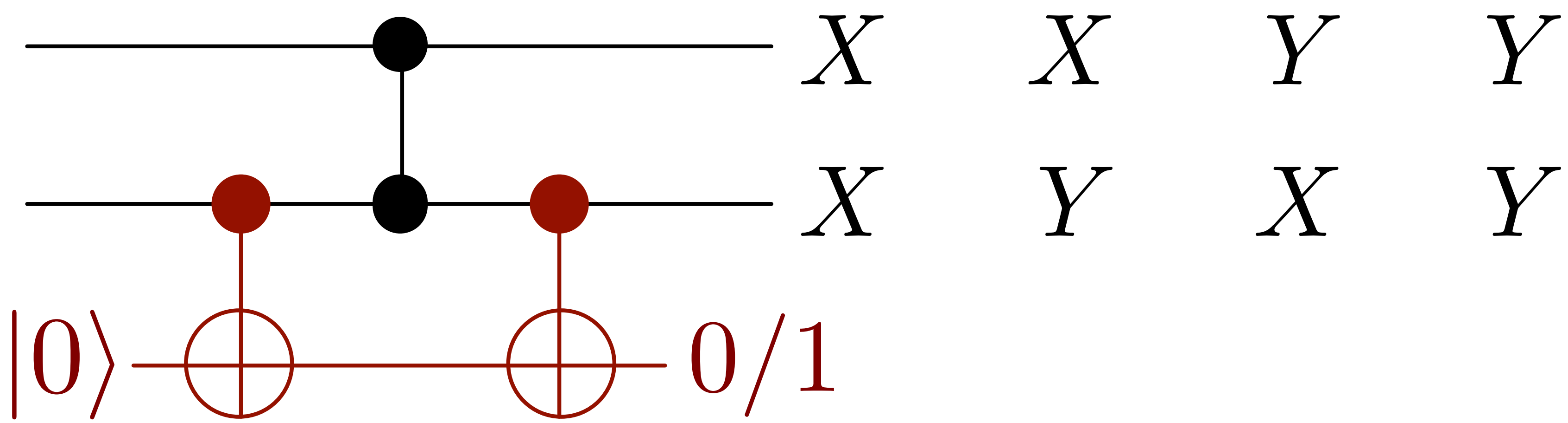


Flag gadget

X gadget

applies CZ

catches XX,XY,YX,YY

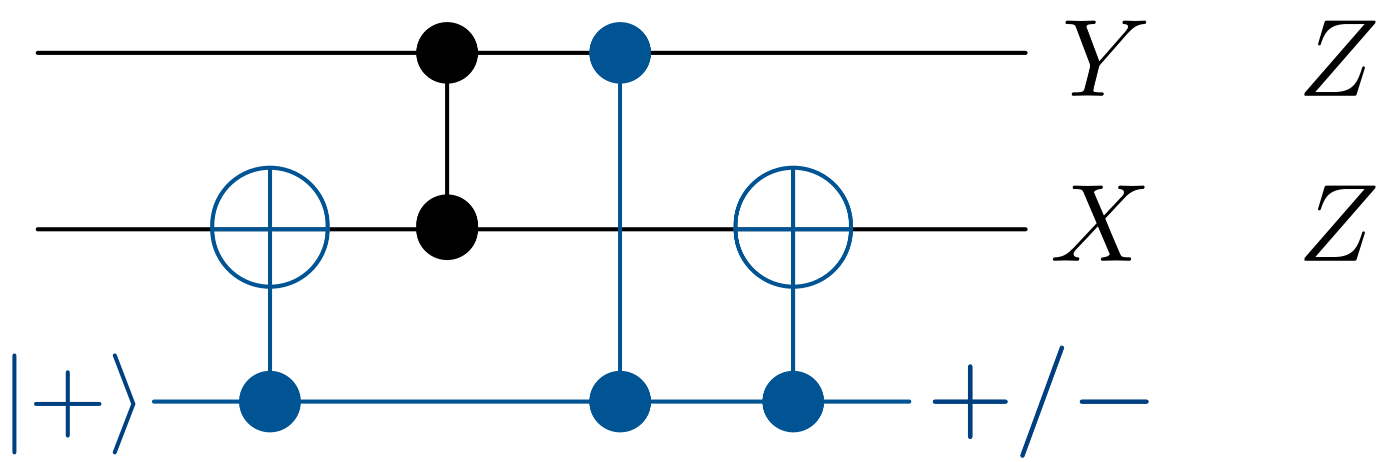


Flag gadget

Z gadget

applies CZ

catches YX, ZZ

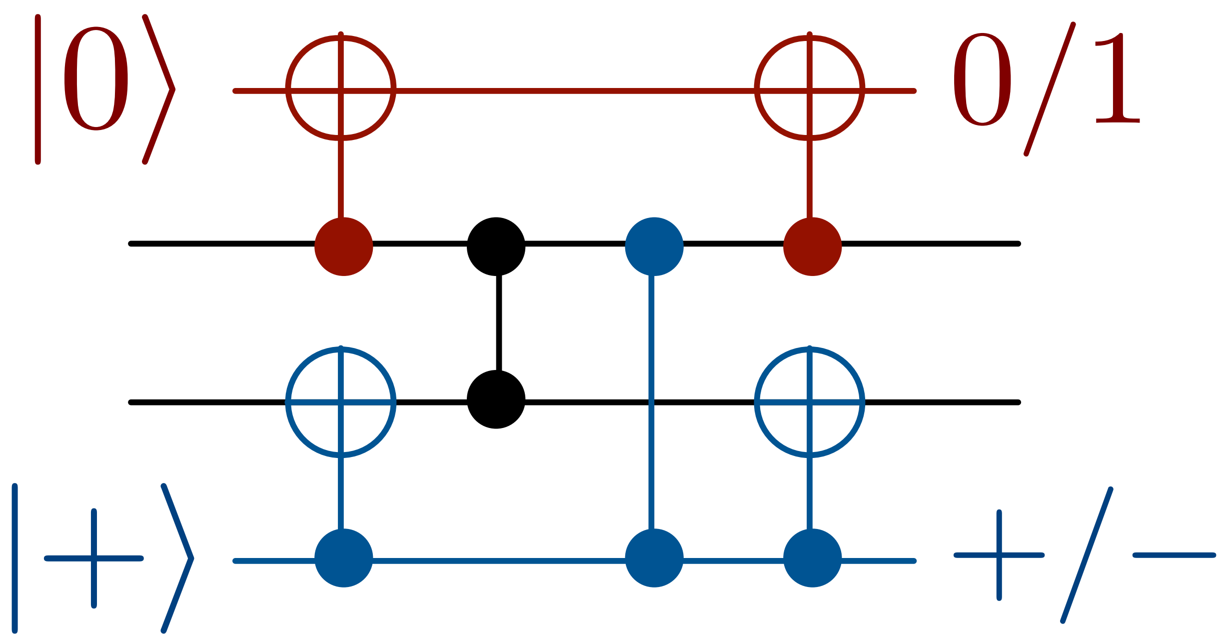


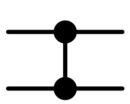
Flag gadget

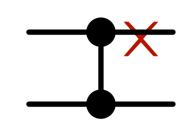
combine: CZ gadget

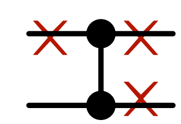
applies CZ

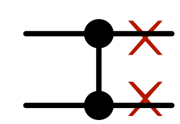
catches all true 2-qubit failures



all possible ways  can fail

IX, IY, IZ, XI, YI, ZI, — 1-qubit failure after gate 

XZ, YZ, ZX, ZY — 1-qubit failure before gate 

XX, XY, YX, YY, ZZ — true 2-qubit failure 

No gadgets can do better than this

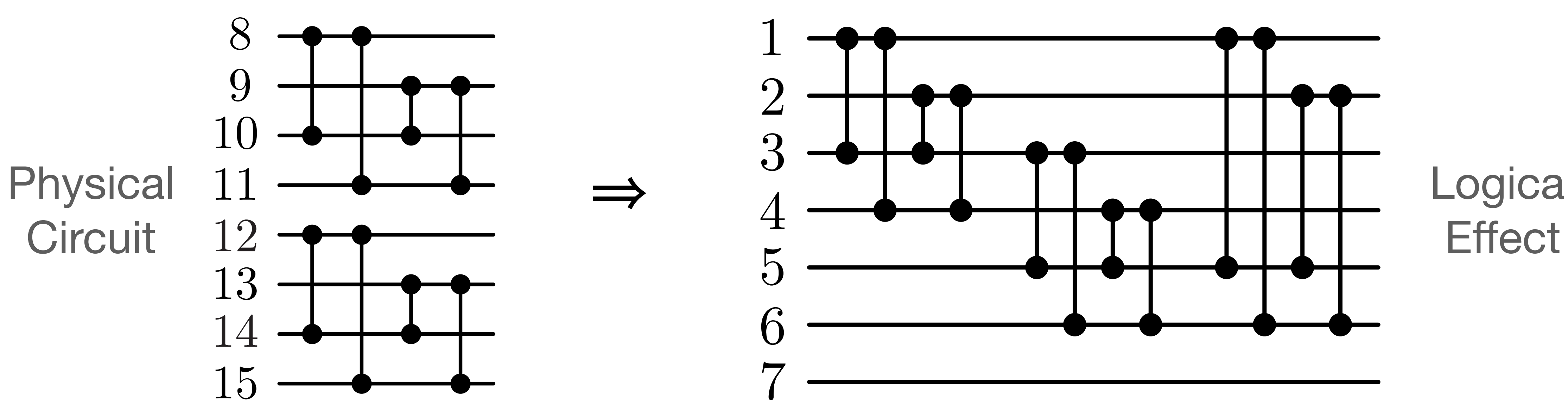
Ex. $[\![15, 7, 3]\!]$ Hamming code

stabilizer generators

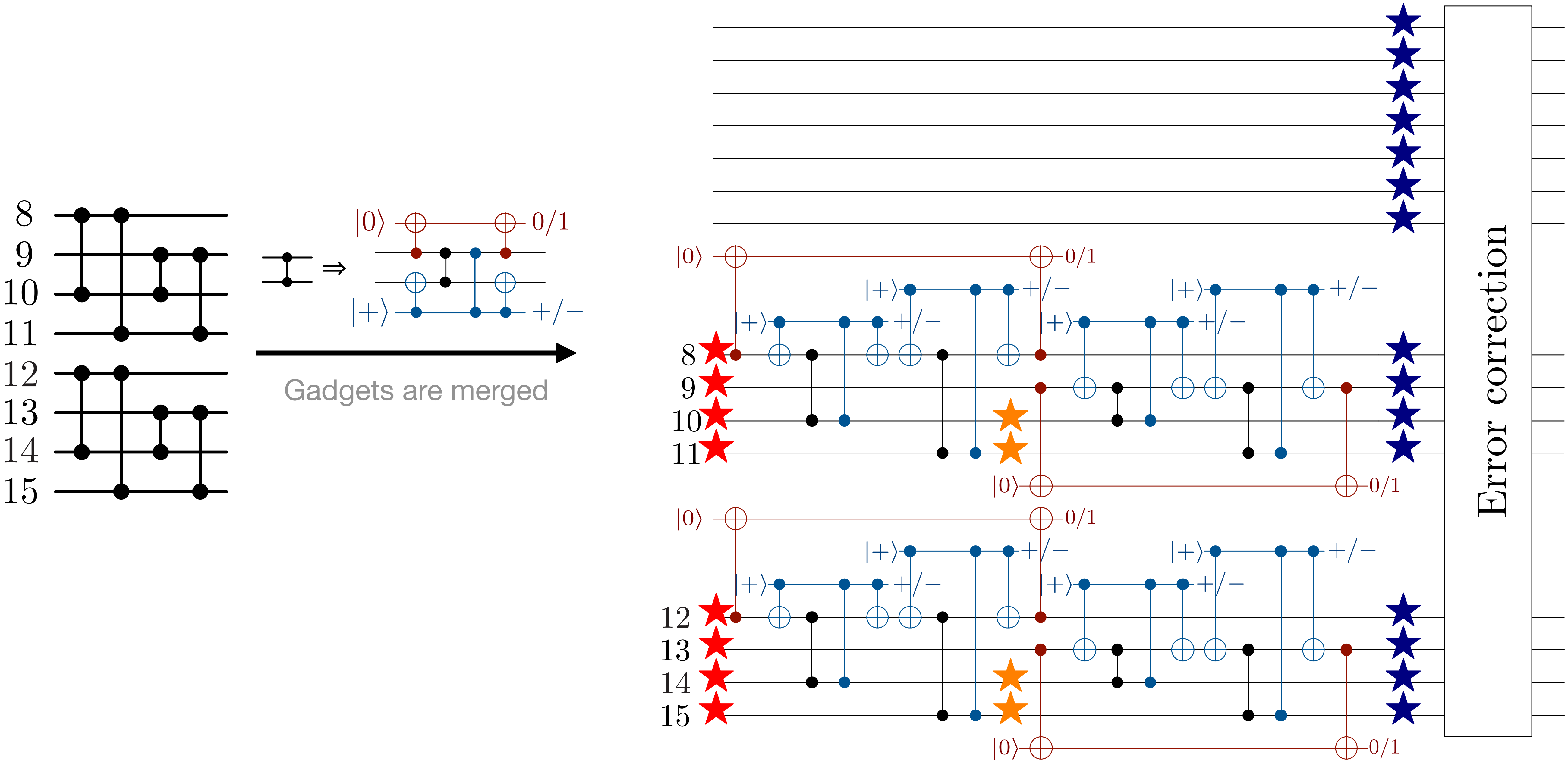
$$\begin{matrix}
0&0&0&0&0&0&0&1&1&1&1&1&1&1&1\\
0&0&0&1&1&1&1&0&0&0&0&1&1&1&1\\
0&1&1&0&0&1&1&0&0&1&1&0&0&1&1\\
1&0&1&0&1&0&1&0&1&0&1&0&1&0&1
\end{matrix}$$

- **Automorphism group:** qubit permutations which fix the codespace
- **New technique:** Build CZ circuits based on permutation symmetries

Ex: (6,7)(8,10,9,11)(12,14,13,15)



Lemma: For a self-dual CSS code, if σ is a qubit permutation that fixes codespace, then circuit with a CZ gate from i to σ(i), for all i≠σ(i), fixes the code space up to Z corrections.



- **All gadgets measure 0:** 1-qubit fault between gadgets
- **Some gadgets measure 1:** Localize and correct

Universal computation

Round-robin construction

(Informal) **lemma:** For any CSS code*, say $\overline{Z_i} = Z_{S_i}, \overline{Z_j} = Z_{S_j}$

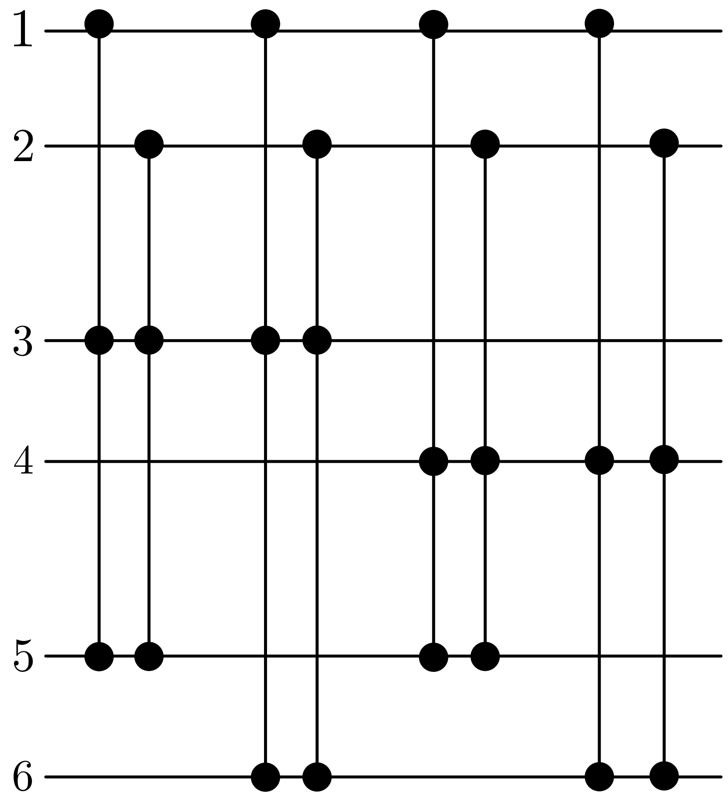
then CZ gates applied to every pair in $S_i \times S_j$ perform $\overline{\mathrm{C}Z_{ij}}$

Similar for $\overline{\mathrm{CC}Z}$

$\overline{Z_1} = Z_{\{1,2\}}$

$\overline{Z_2} = Z_{\{3,4\}}$

$\overline{Z_3} = Z_{\{5,6\}}$



$\Rightarrow \quad \overline{\mathrm{CC}Z_{\{1,2,3\}}}$

*CSS isn't needed…

Universal computation

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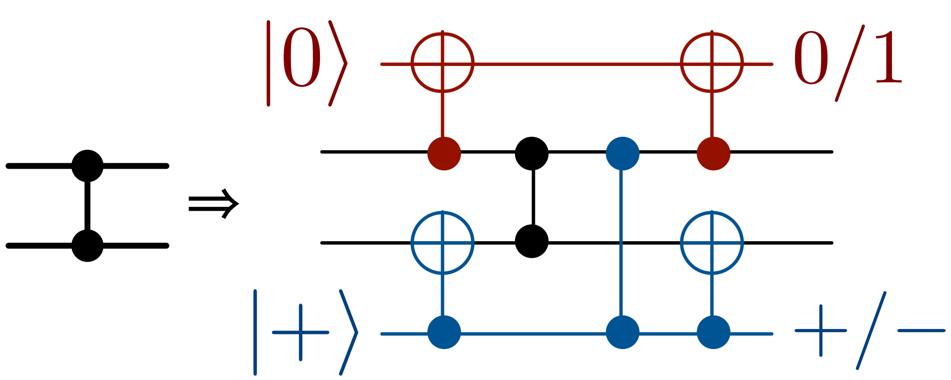
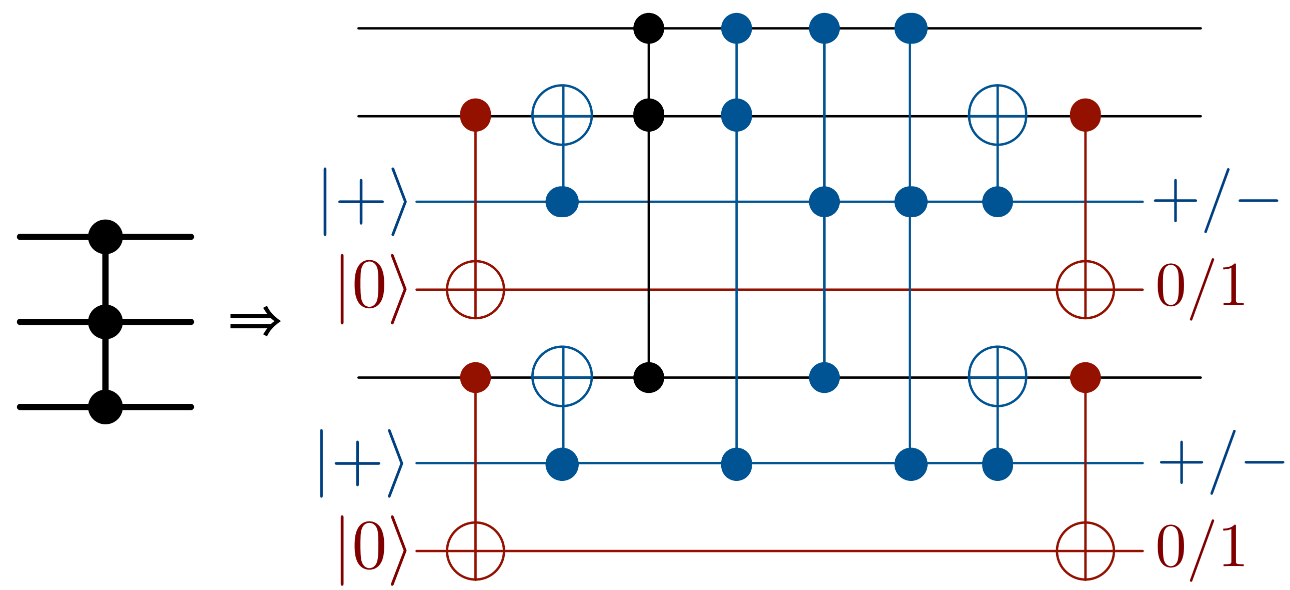
CZ gadget



2 ancillas for Clifford group

CCZ gadget



4 ancillas for universal gate set

*CSS isn't needed…

Summary

Experimentalists need good tests

- to test/demonstrate the theory
- to assess FT schemes' performance in real error models
- to adapt FT schemes to real noise

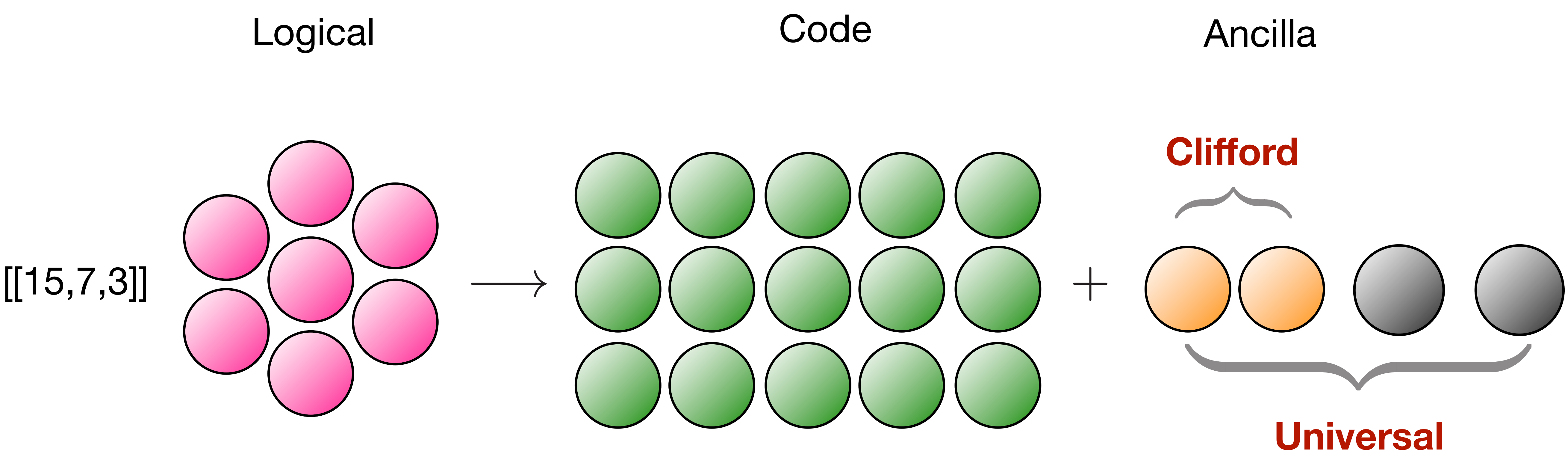
Theorists need to come up with them! **Start from small devices**

Summary

Previously:

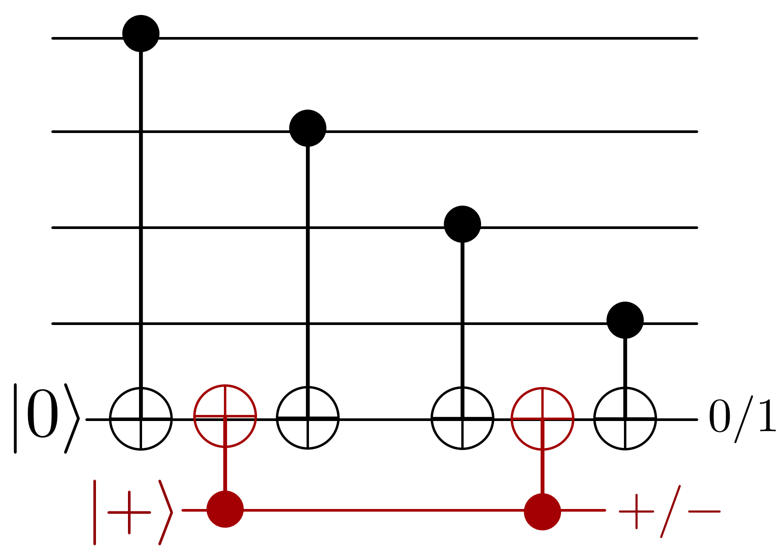
With [[7,1,3]] code, need 53=49+4 qubits to test FT computation on 7 logical qubits

FT universal computation on 7 logical qubits with 19 qubits

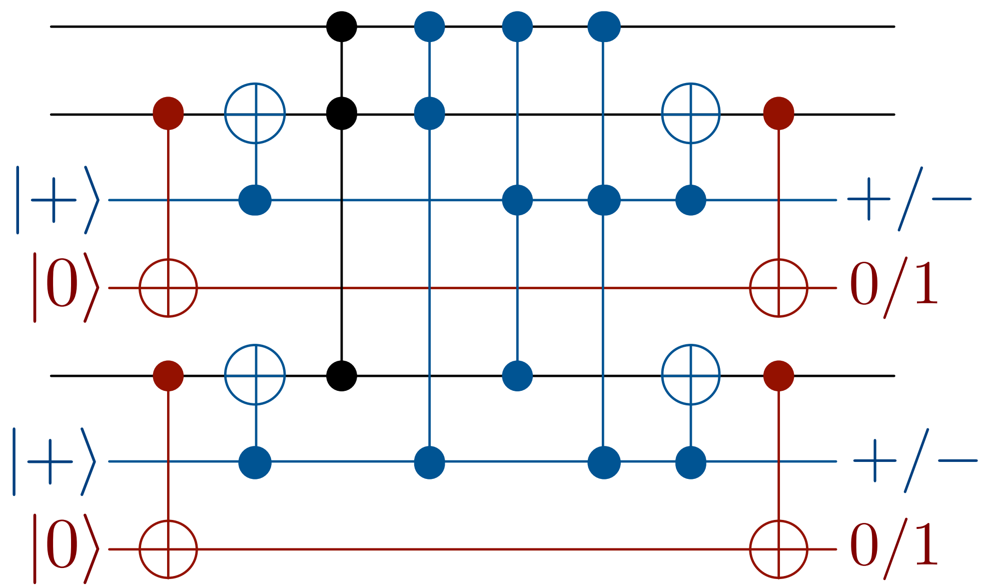


Summary

Main idea: Flags to catch “bad” errors



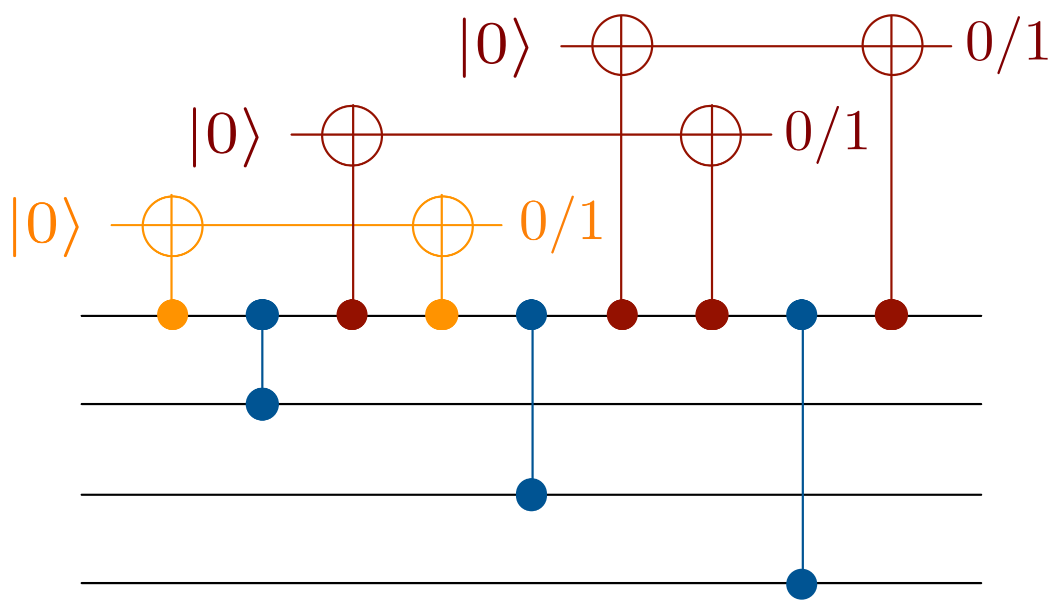
FT EC with 2 ancilla

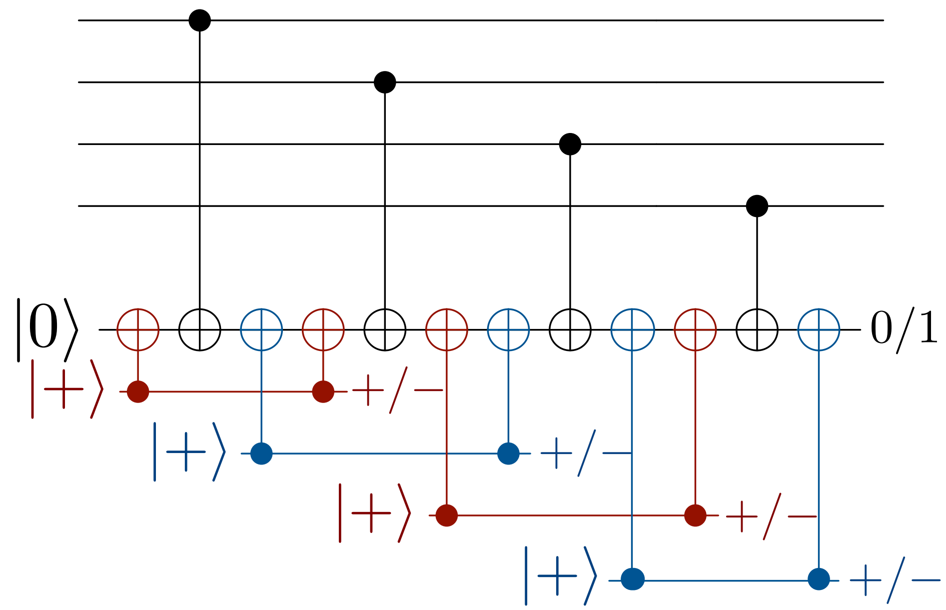


FT computation with 4 ancilla

Summary

Flag paradigm





- **Two ancillas for logical state preparation, measurement and code conversion…**
- **EC for arbitrary distance codes** [C. Chamberland and M. Beverland arXiv:1708.02246]
- **CZ gadget, CCZ gadget, XXX gadget…**