Approximate Quantum Error Correction Revisited: Introducing the Alpha-bit

ArXiv:1706.09434

Patrick Hayden and Geoffrey Penington, Stanford University

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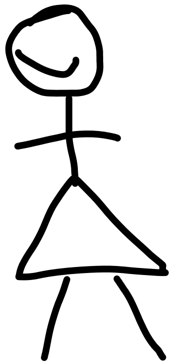
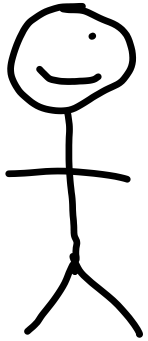
- ❑ It proves P≠NP

Quantum Communication Resource Inequalities





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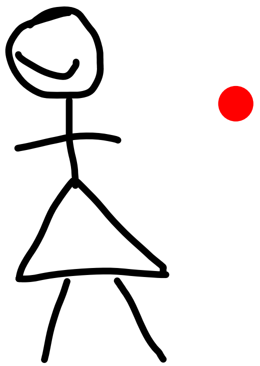
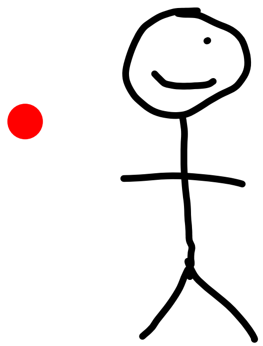
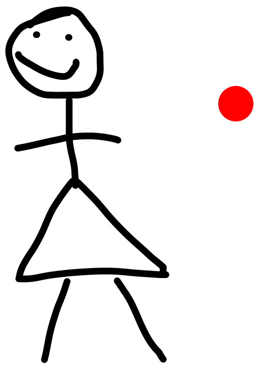
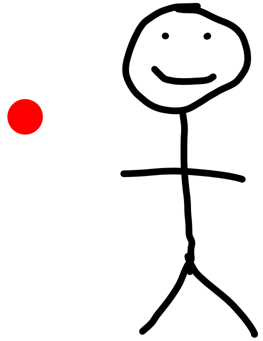




Quantum Communication Resource Inequalities

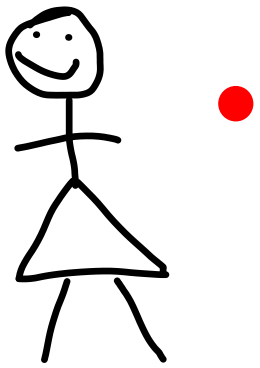
$$1 \ \text{qubit} \geq 1 \ \text{ebit}$$

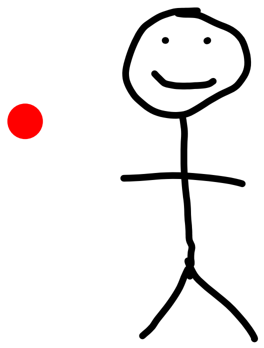




Quantum Communication Resource Inequalities

$$1 \ \text{cbit} \leq 1 \ \text{qubit} \geq 1 \ \text{ebit}$$

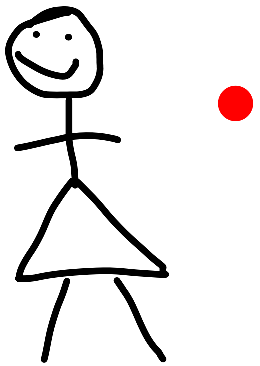


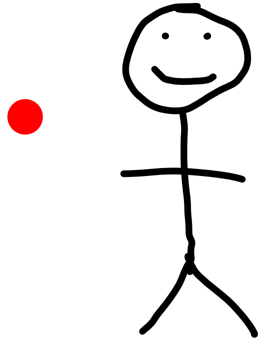


Quantum Communication Resource Inequalities

$$1 \text{ cbit} \leq 1 \text{ qubit} \geq 1 \text{ ebit}$$

$$1 \text{ ebit} + 2 \text{ cbits} \geq 1 \text{ qubit}$$





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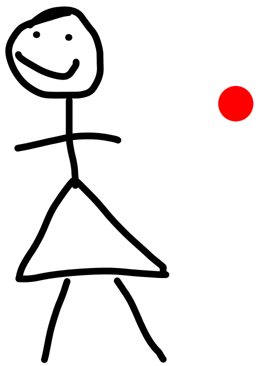

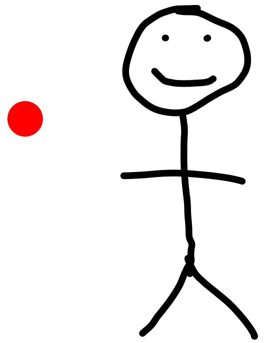

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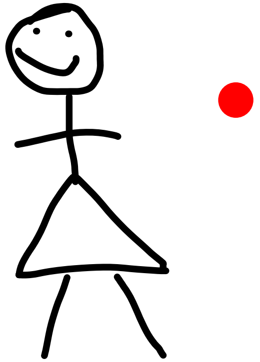

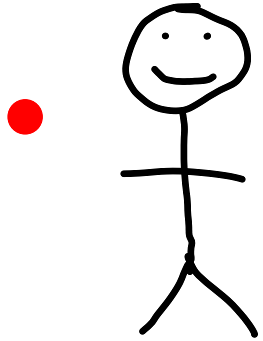

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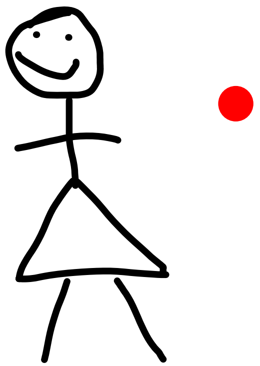
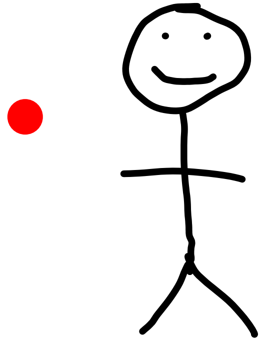
weakened version of qubits





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weakened version of qubits

$$m \ \text{qubits} \geq 2m \ \text{zero-bits} \ ?$$

asymptotic

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Quantum Communication Resource Inequalities

$$1 \;\; \text{cbit} \; < 1 \;\; \text{qubit} \; > \;\; 1 \;\; \text{ebit}$$

$$\underbrace{1\,\text{ebit}}_{\text{coherence}} + \underbrace{2 \text{ zero-bits}}_{\text{communication}} \overset{(a)}{=} 1 \;\; \text{qubit}$$

$$m \text{ qubits} \geq 2m \text{ zero-bits } ?$$

$$1 \;\; \text{cbit} \overset{(a)}{>} 1 \text{ zero-bit} ?$$

What are zero-bits?

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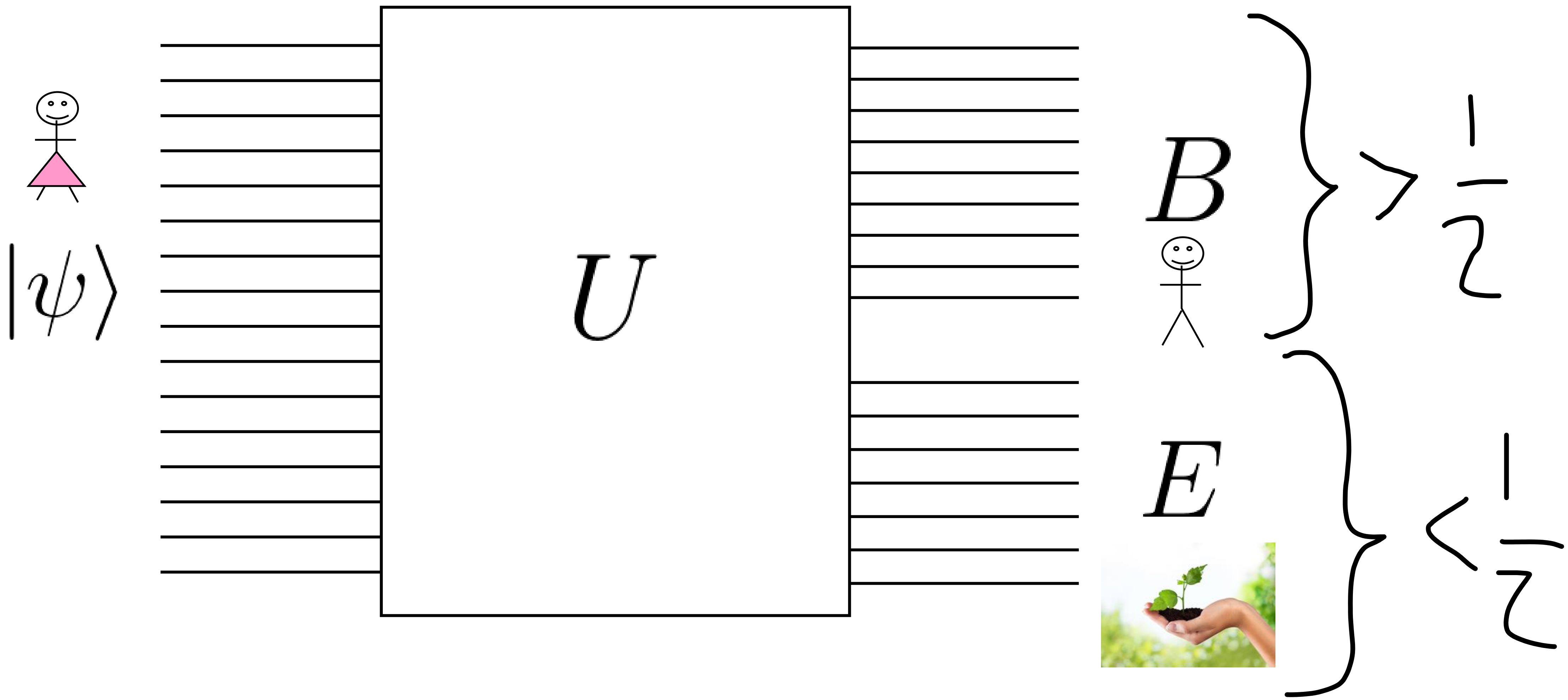
$$1 \text{ zero-bit } = 1 \; \alpha\text{-bit} \quad \text{with } \alpha = 0$$

What are zero-bits?



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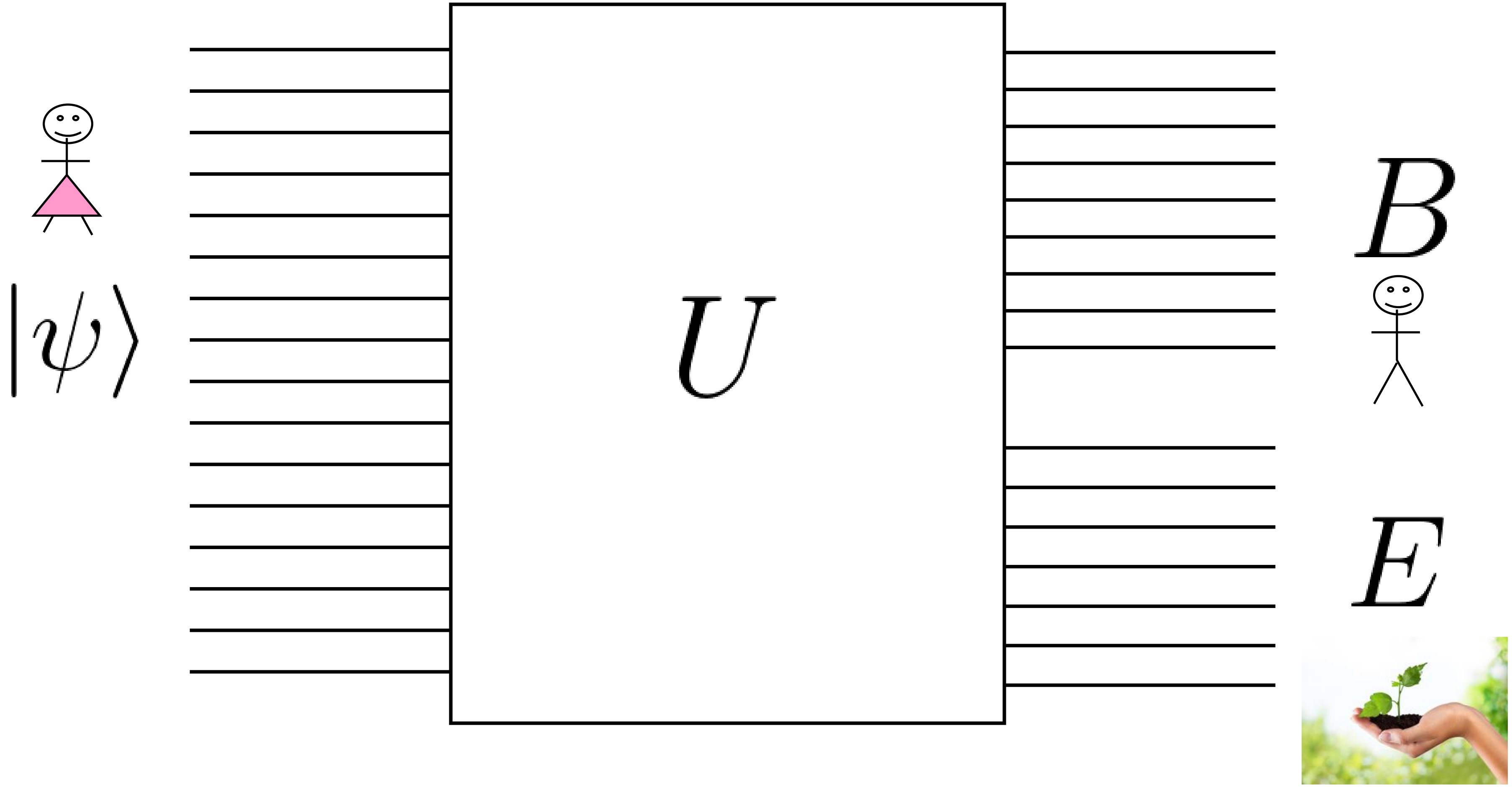
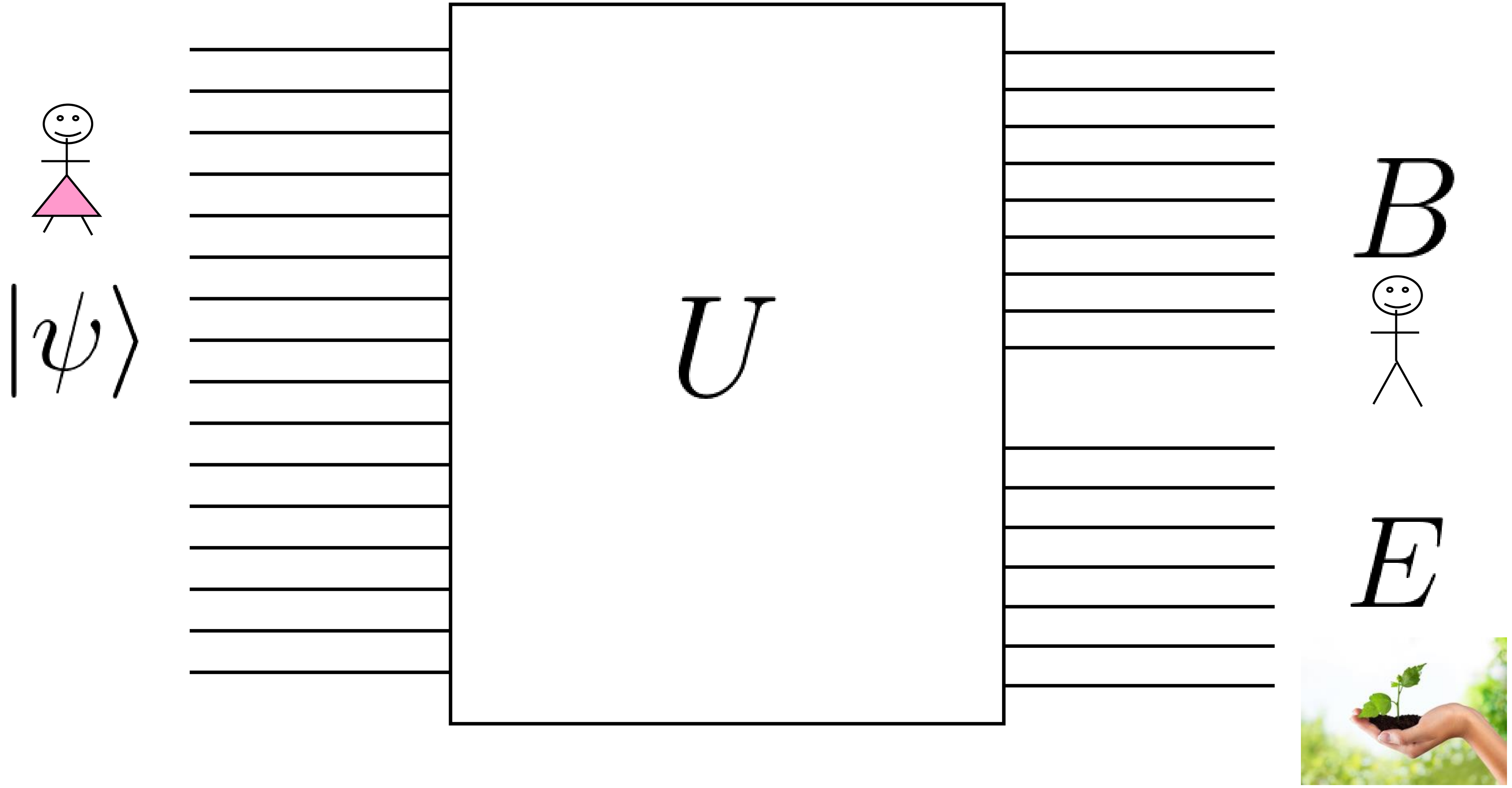


$$\psi^E \approx \omega^E$$

What are zero-bits?

$d_B >> d_E$



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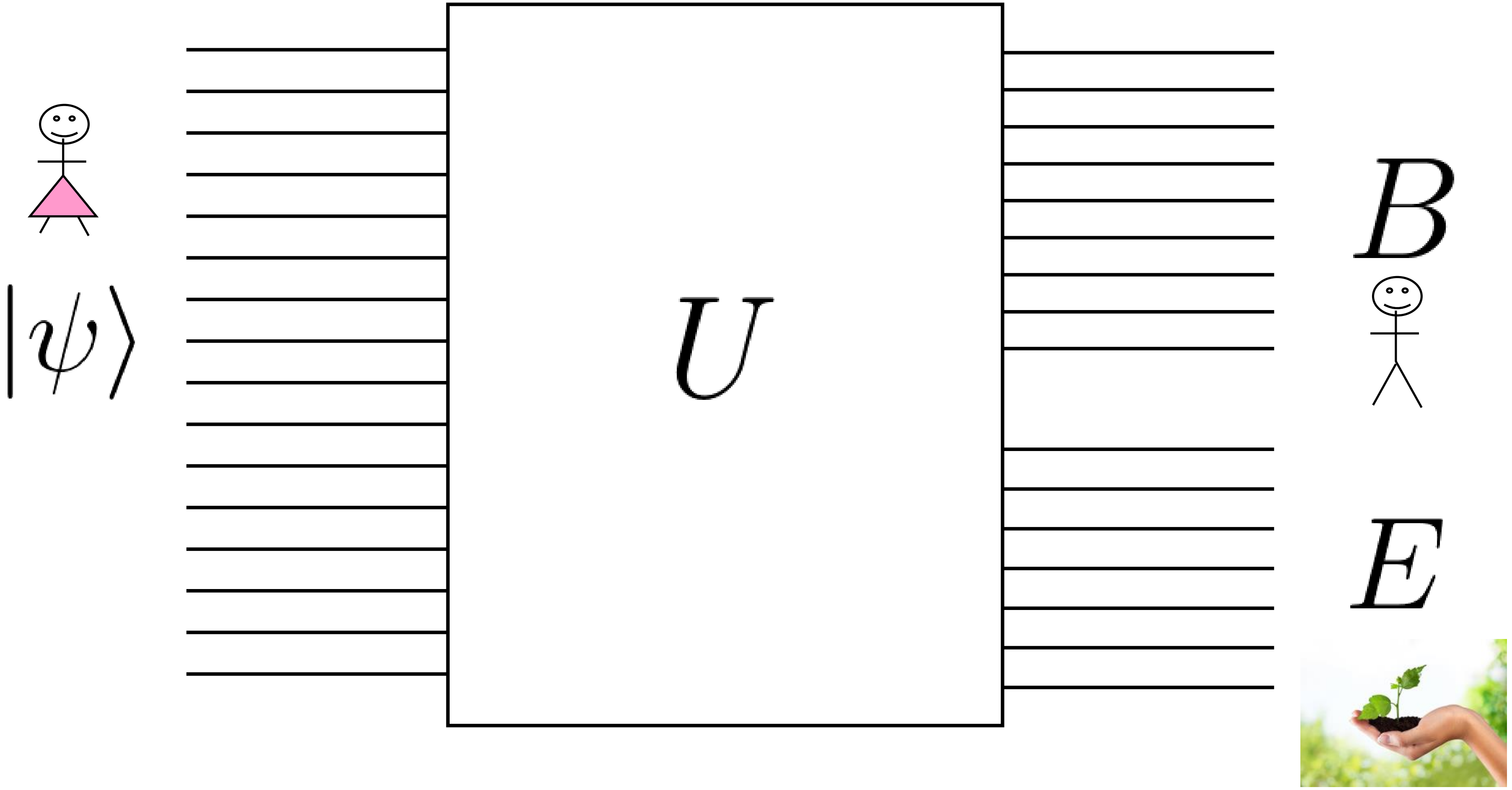

$$\|\psi_1^B - \psi_2^B\|_1 \approx \|\psi_1 - \psi_2\|_1$$

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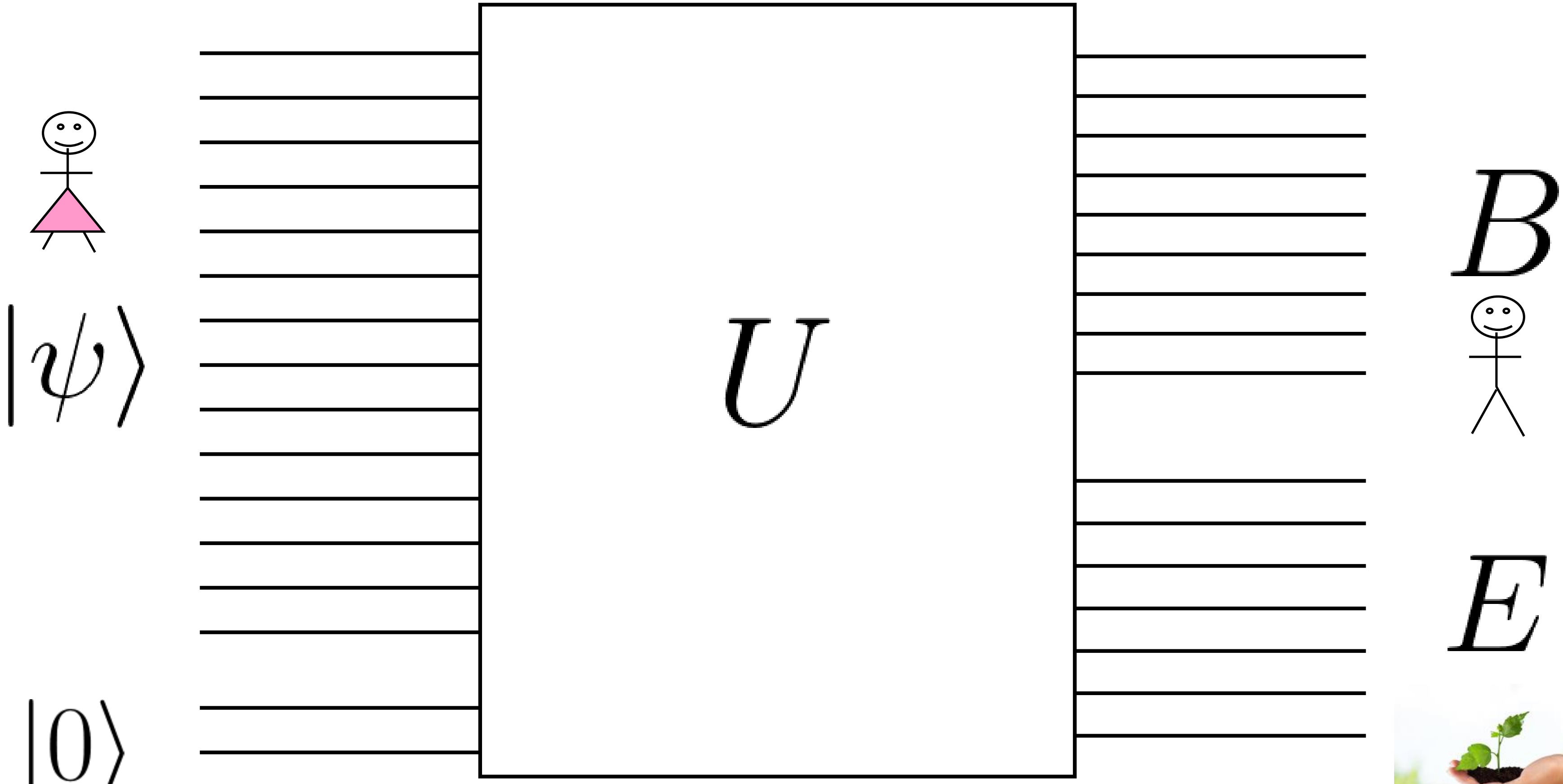

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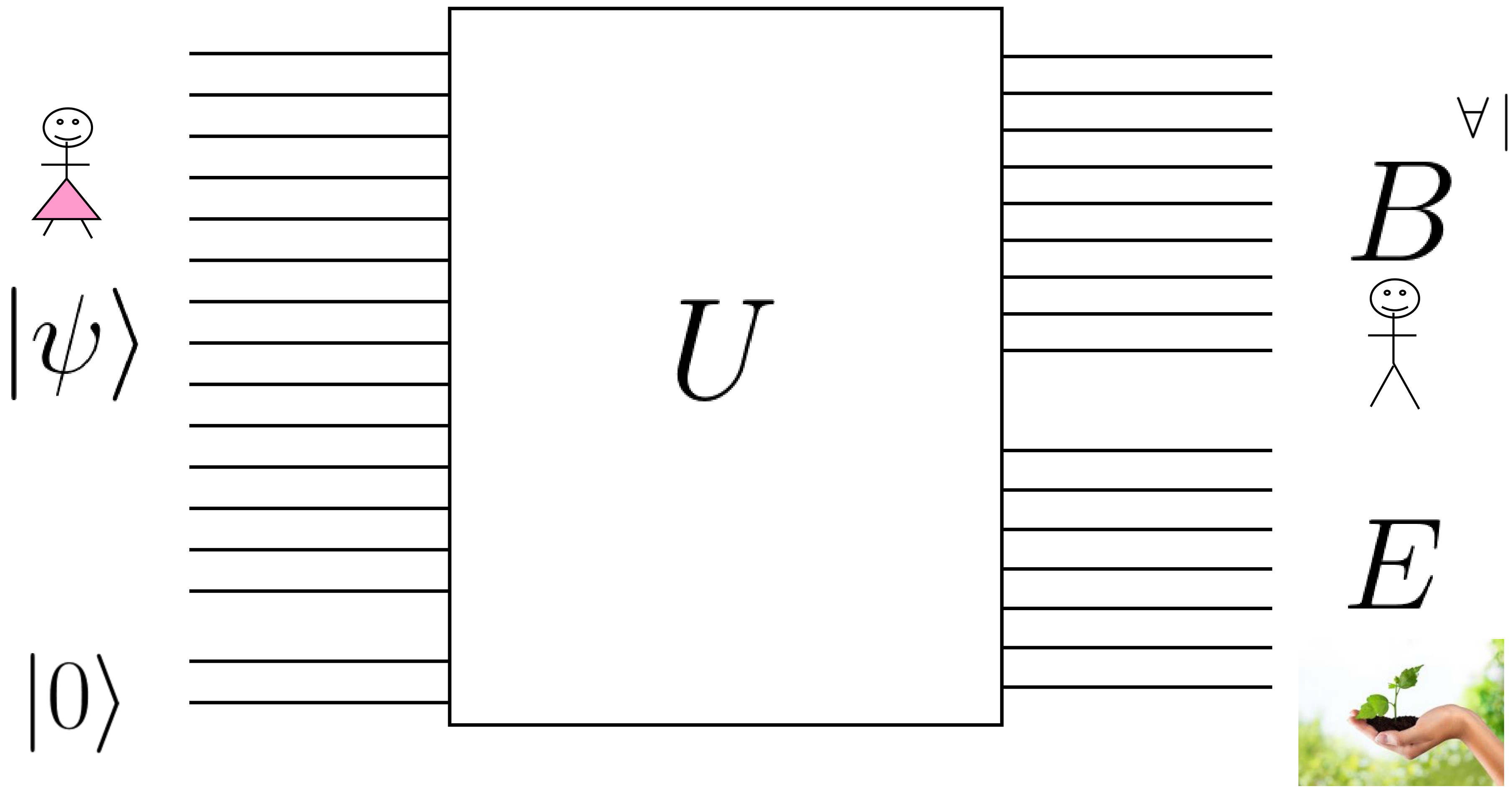

$\forall \, |\psi_1\rangle, |\psi_2\rangle,$

$$\left\| \psi_1^B - \psi_2^B \right\|_1 \approx \left\| \psi_1 - \psi_2 \right\|_1$$

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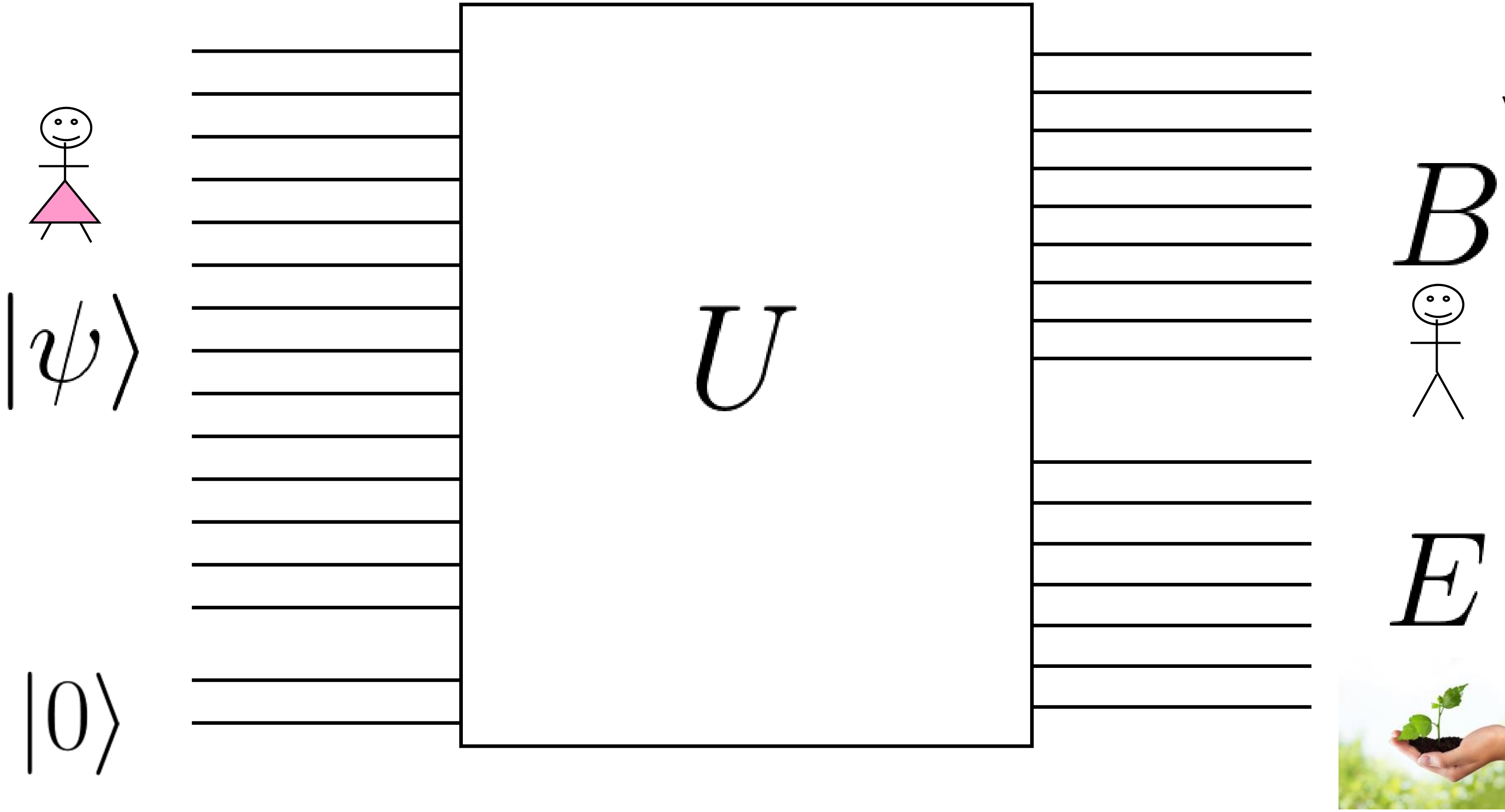

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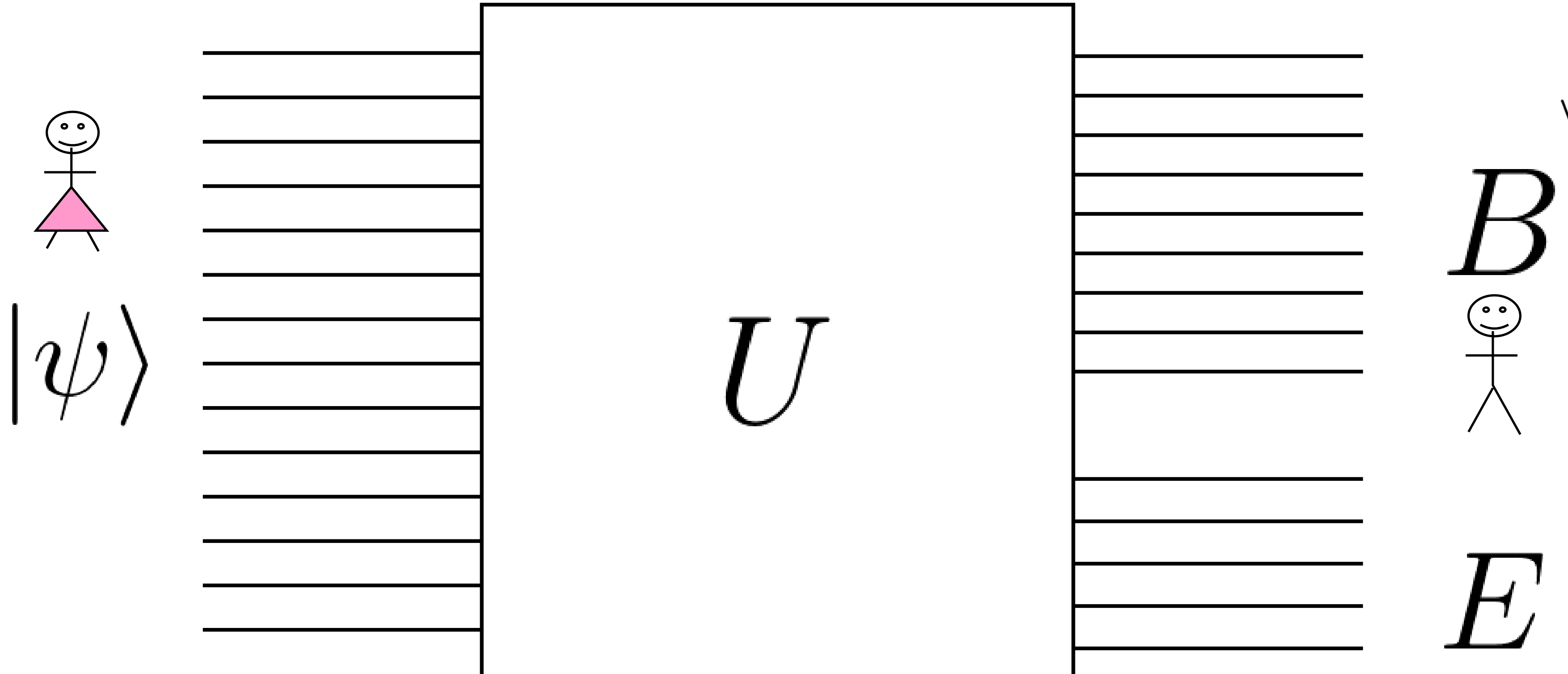

isometric

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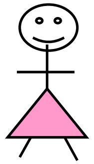
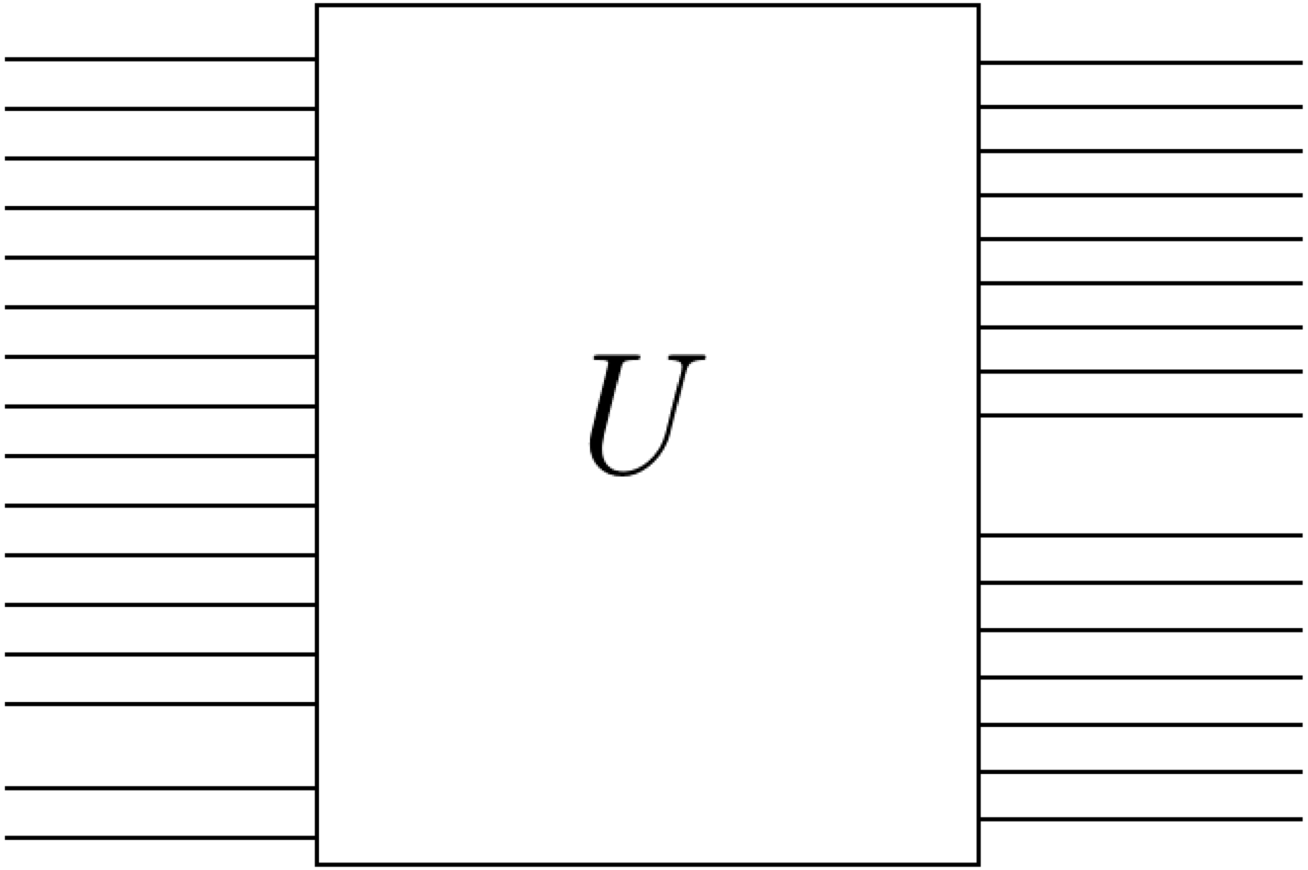
B encodes the zero-bits of $|\psi\rangle$

isometric

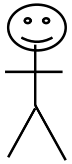


$|\psi\rangle$

$|0\rangle$



B



E

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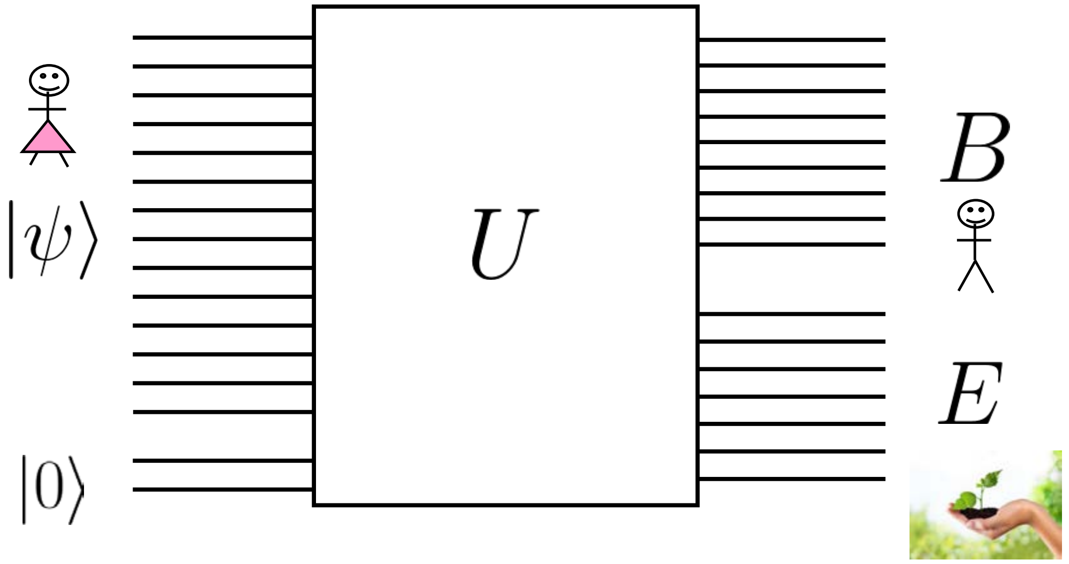
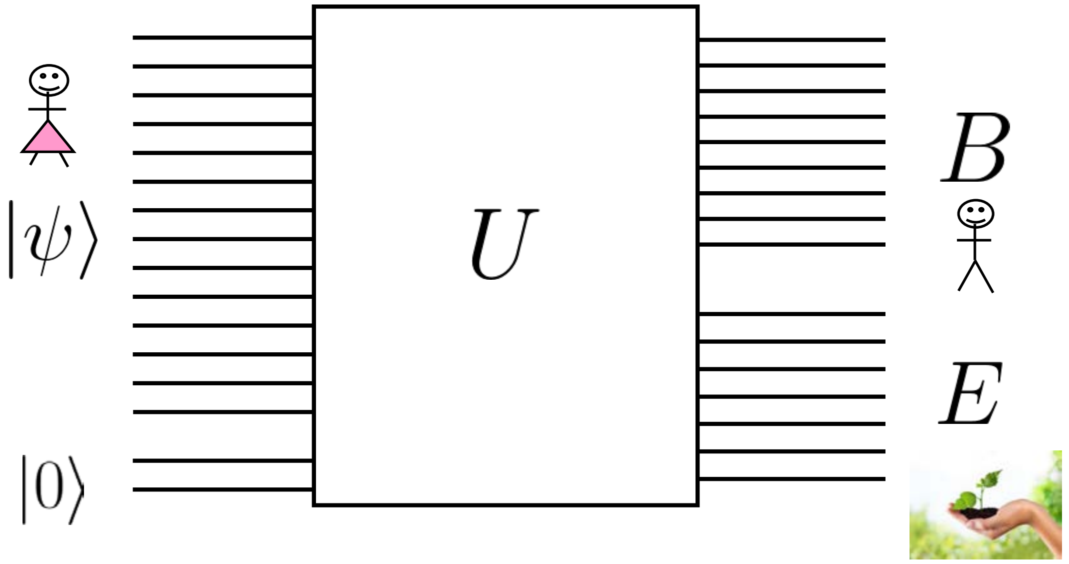


What can you do with zero-bits?

Not possible to recover the state $|\psi\rangle$ from B with no information about the state
- Error correction is not possible



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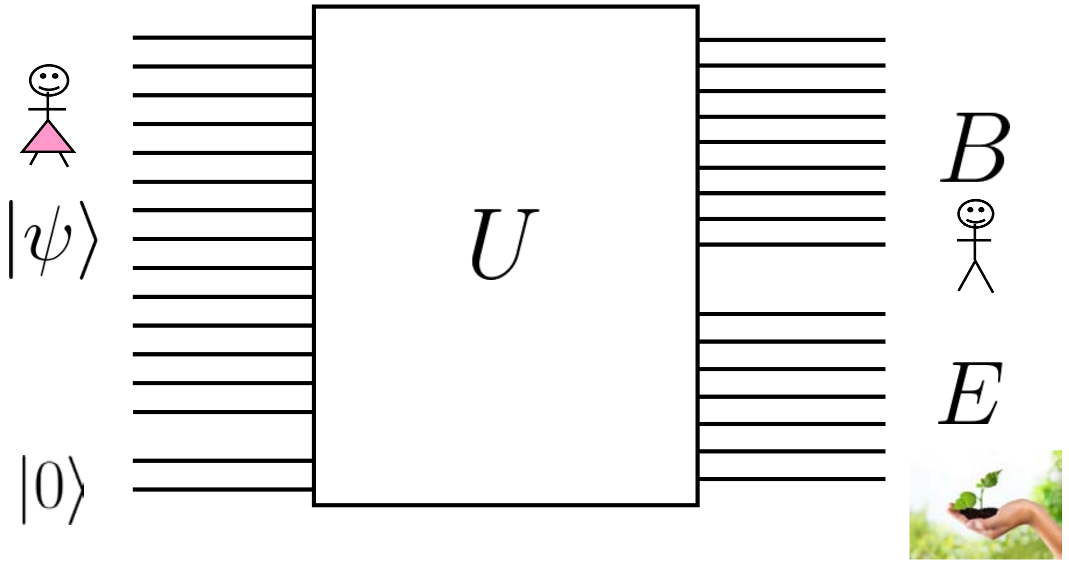

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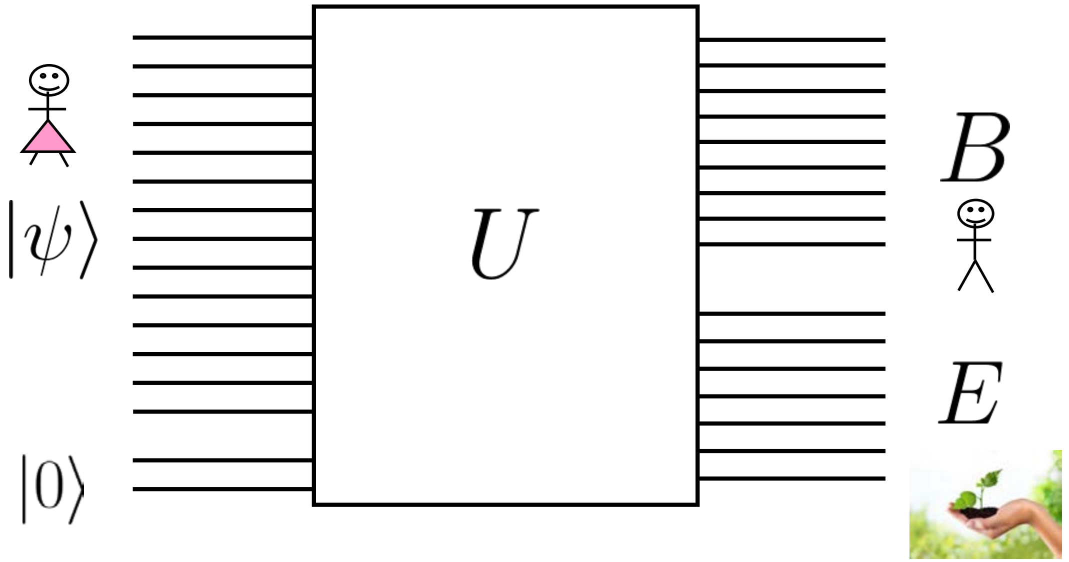
Able to error-correct any two-dimensional subspace S



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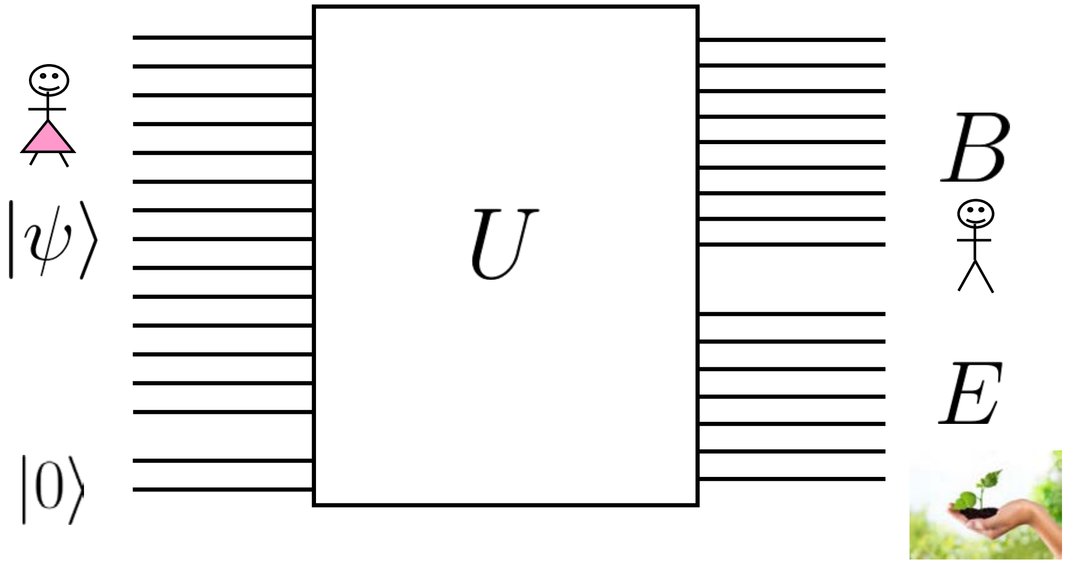

Universal Subspace
Quantum Error Correction

Definition of zero-bits

Definition of qubits

“n qubits”

Definition of qubits

"n qubits"

$$\mathcal{N} : \mathcal{S}(A) \to \mathcal{S}(B)$$

$$d_A = 2^n$$

Definition of qubits

$$\text{“}n\text{ qubits”}$$

$$\mathcal{N} : \mathcal{S}(A) \to \mathcal{S}(B)$$

$$d_A = 2^n$$

What do we need to be true about the channel?

Definition of qubits

"n qubits"

$$\mathcal{N} : \mathcal{S}(A) \to \mathcal{S}(B)$$

$$d_A = 2^n$$

$$\mathcal{N} = \mathrm{Id} \; ?$$

What do we need to be true about the channel?

Definition of qubits

"n qubits"

$$\mathcal{N} : \mathcal{S}(A) \to \mathcal{S}(B)$$

$$d_A = 2^n$$

$$\exists \mathcal{D}$$

$$\mathcal{D} \circ \mathcal{N} = \mathrm{Id}$$

What do we need to be true about the channel?

Bob can always error correct so long as error correction is possible

Definition of zero-bits

"n zero-bits"

$$\mathcal{N} : \mathcal{S}(A) \to \mathcal{S}(B)$$

$$d_A = 2^n$$

$$\forall S \subseteq A \quad d_S = 2$$

$$\exists \mathcal{D}_S$$
$$\mathcal{D}_S \circ \mathcal{N}|_S = \mathrm{Id}_S$$

OK now what about zero-bits?

Now Bob only has to be able to error correct any *two-dimensional subspace*

Definition of zero-bits

“n qubits”

$$\mathcal{N} : \mathcal{S}(A) \to \mathcal{S}(B)$$

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Definition of zero-bits

$$\text{“}n \text{ zero-bits”}$$

$$\mathcal{N} : \mathcal{S}(A) \to \mathcal{S}(B)$$

$$d_A = 2^n$$

$$\forall S \subseteq A \quad d_S = 2$$

$$\exists \mathcal{D}_S$$
$$\|\mathcal{D}_S \circ \mathcal{N}|_S - \mathrm{Id}_S\|_\diamond \leq \delta$$

Need to make definition approximate if zero-bits are to be different from qubits

Definition of zero-bits

"n zero-bits"

$$\mathcal{N} : \mathcal{S}(A) \to \mathcal{S}(B)$$

[Hayden, Winter 2012]

$$d_A = 2^n \qquad\qquad \Leftrightarrow$$

$$\forall S \subseteq A \quad d_S = 2$$

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Definition of zero-bits

"n zero-bits"

$$\mathcal{N} : \mathcal{S}(A) \to \mathcal{S}(B)$$

$$\mathcal{N}^c : \mathcal{S}(A) \to \mathcal{S}(E)$$

[Hayden, Winter 2012]

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Definition of zero-bits

"n zero-bits"

$$\mathcal{N}: \mathcal{S}(A) \to \mathcal{S}(B) \qquad \qquad \mathcal{N}^c: \mathcal{S}(A) \to \mathcal{S}(E)$$

[Hayden, Winter 2012]

$$d_A = 2^n \qquad \Leftrightarrow \qquad \forall\, |\psi\rangle \in A,$$

$$\|\mathcal{N}^c(\psi - \omega)\|_1 \le \varepsilon$$

$$\forall S \subseteq A \quad d_S = 2$$

$$\exists \mathcal{D}_S$$

$$\|\mathcal{D}_S \circ \mathcal{N}|_S - \mathrm{Id}_S\|_\diamond \le \delta$$

Definition of zero-bits

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[Hayden, Winter 2012]

$$\Leftrightarrow$$

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$$\forall |\psi\rangle \in A,$$
$$\|\mathcal{N}^c(\psi - \omega)\|_1 \leq \varepsilon$$

$$\boxed{\frac{1}{16}\delta^2 \leq \varepsilon \leq 8\sqrt{\delta}}$$

Definition of alpha-bits

"n α-bits"

$$\mathcal{N} : \mathcal{S}(A) \to \mathcal{S}(B)$$

$$d_A = 2^n$$

$$\forall S \subseteq A \quad d_S \leq 2^{\alpha n} + 1$$

$$\exists \mathcal{D}_S$$

$$\| \mathcal{D}_S \circ \mathcal{N}|_S - \mathrm{Id}_S \|_\diamond \leq \delta$$

Definition of alpha-bits

"n α-bits"

$$\mathcal{N} : \mathcal{S}(A) \to \mathcal{S}(B)$$

$$d_A = 2^n$$

$$\forall S \subseteq A \quad d_S \leq 2^{\alpha n} + 1$$

$$\exists \mathcal{D}_S \qquad \alpha = 1 \Rightarrow \text{qubits}$$

$$\|\mathcal{D}_S \circ \mathcal{N}|_S - \mathrm{Id}_S\|_\diamond \leq \delta$$

Definition of alpha-bits

"n α-bits"

"Subspace decoupling duality"

$$\mathcal{N} : \mathcal{S}(A) \to \mathcal{S}(B)$$

$$d_A = 2^n$$

$$\forall S \subseteq A \quad d_S \leq 2^{\alpha n} + 1$$

$$\exists \mathcal{D}_S$$

$$\|\mathcal{D}_S \circ \mathcal{N}|_S - \mathrm{Id}_S\|_\diamond \leq \delta$$

$\alpha = 1 \Rightarrow$ qubits

$$\Leftrightarrow$$

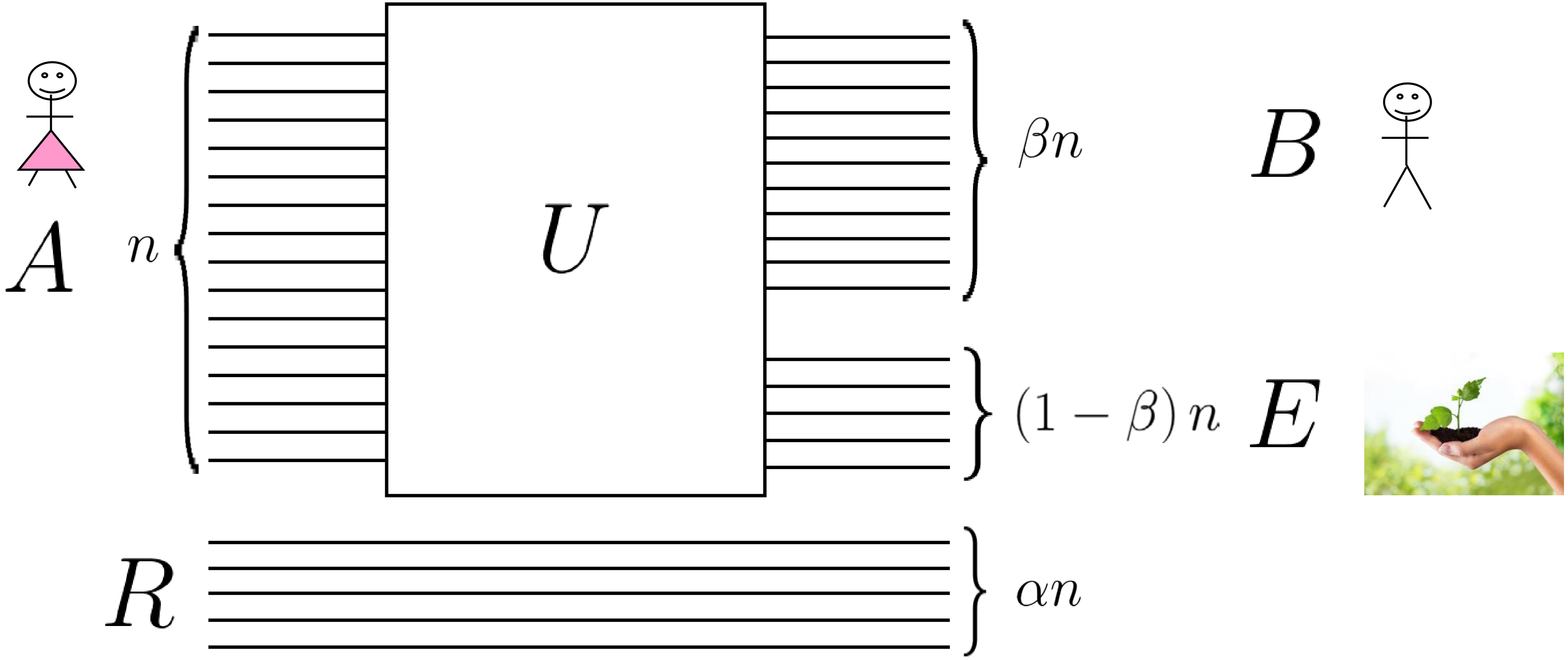
$$\mathcal{N}^c : \mathcal{S}(A) \to \mathcal{S}(E)$$

$$\forall |\psi\rangle \in AR, \quad d_R = 2^{\alpha n}$$

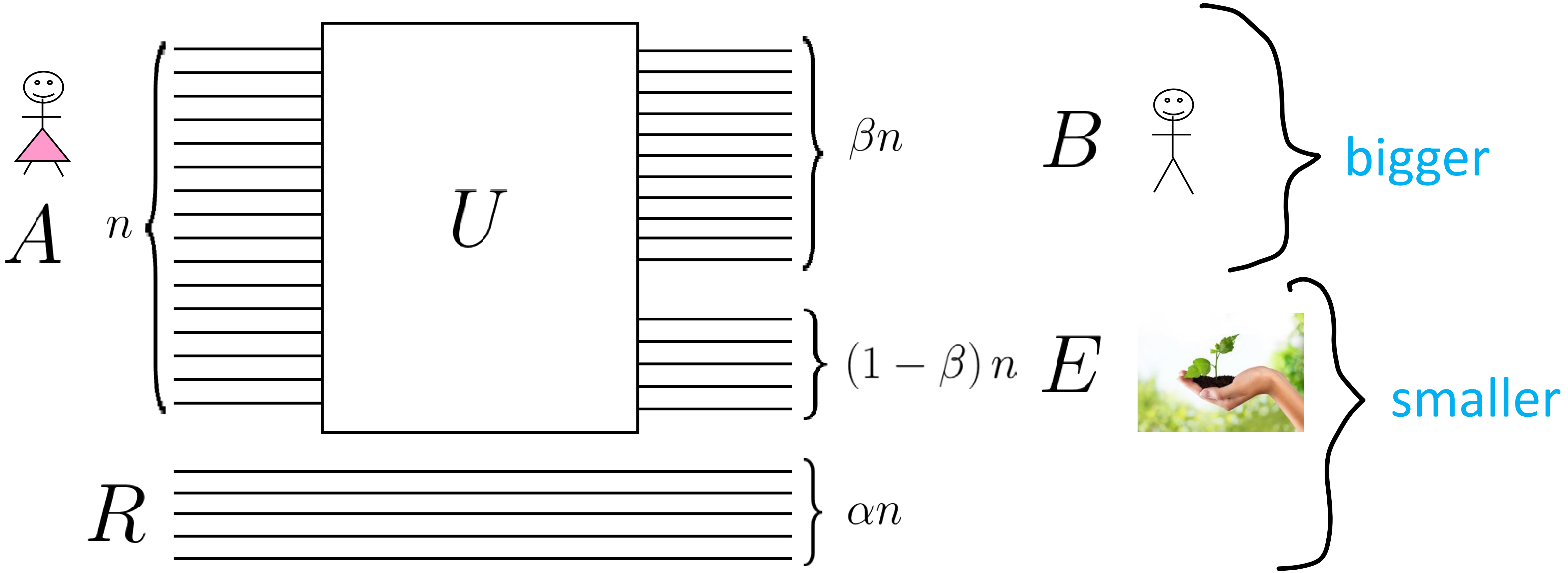
$$\left\|\mathcal{N}^c \otimes \mathrm{Id}_R \left(\psi - \omega \otimes \psi^R\right)\right\|_1 \leq \varepsilon$$

$$\boxed{\frac{1}{16}\delta^2 \leq \varepsilon \leq 8\sqrt{\delta}}$$

Transmitting alpha-bits



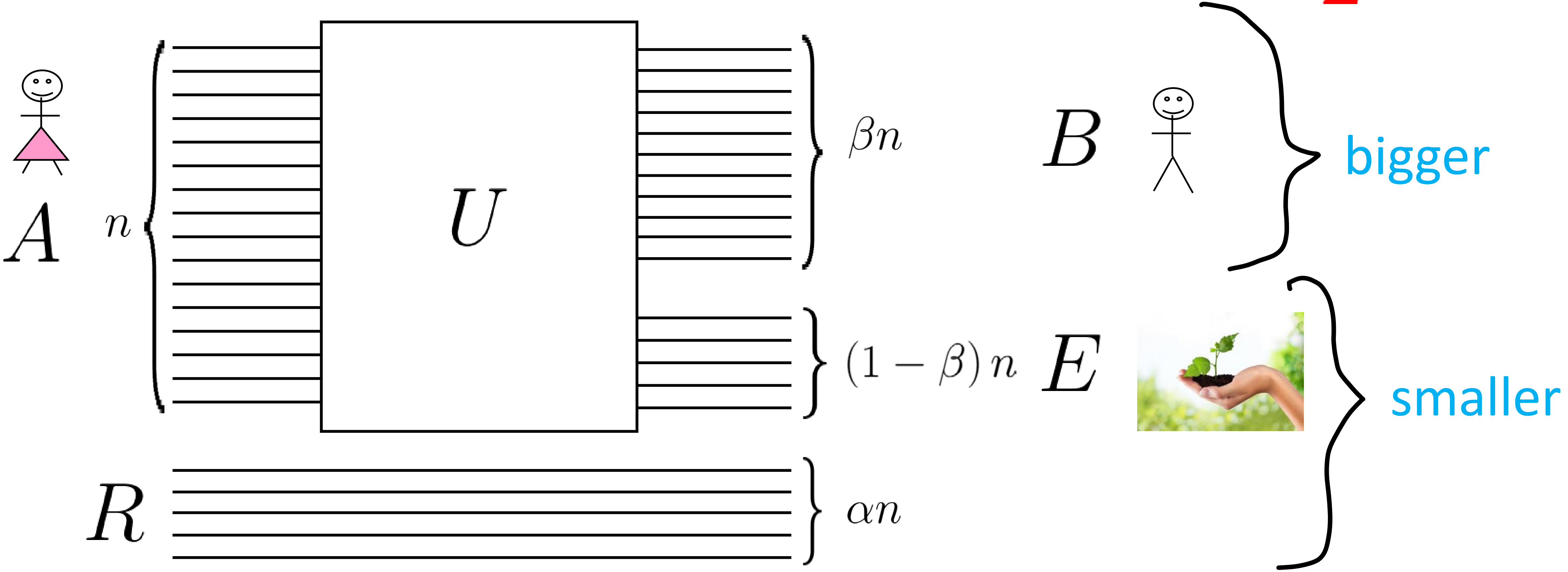
Transmitting alpha-bits



Necessary condition to send alpha-bits. Also sufficient (with some subtleties about needing to use shared randomness and block coding).

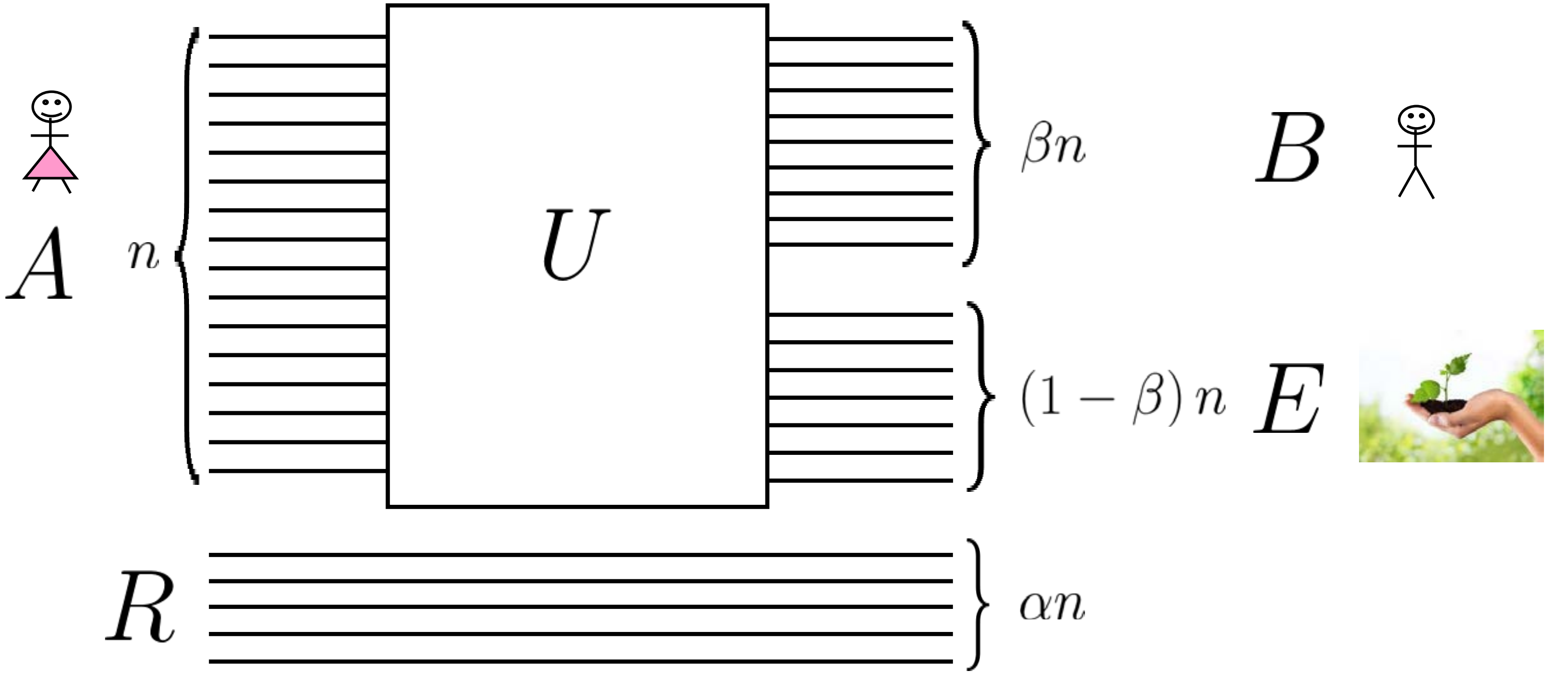
Transmitting alpha-bits

$$\beta > \frac{1+\alpha}{2}$$



Necessary condition to send alpha-bits. Also sufficient (with some subtleties about needing to use shared randomness and block coding).

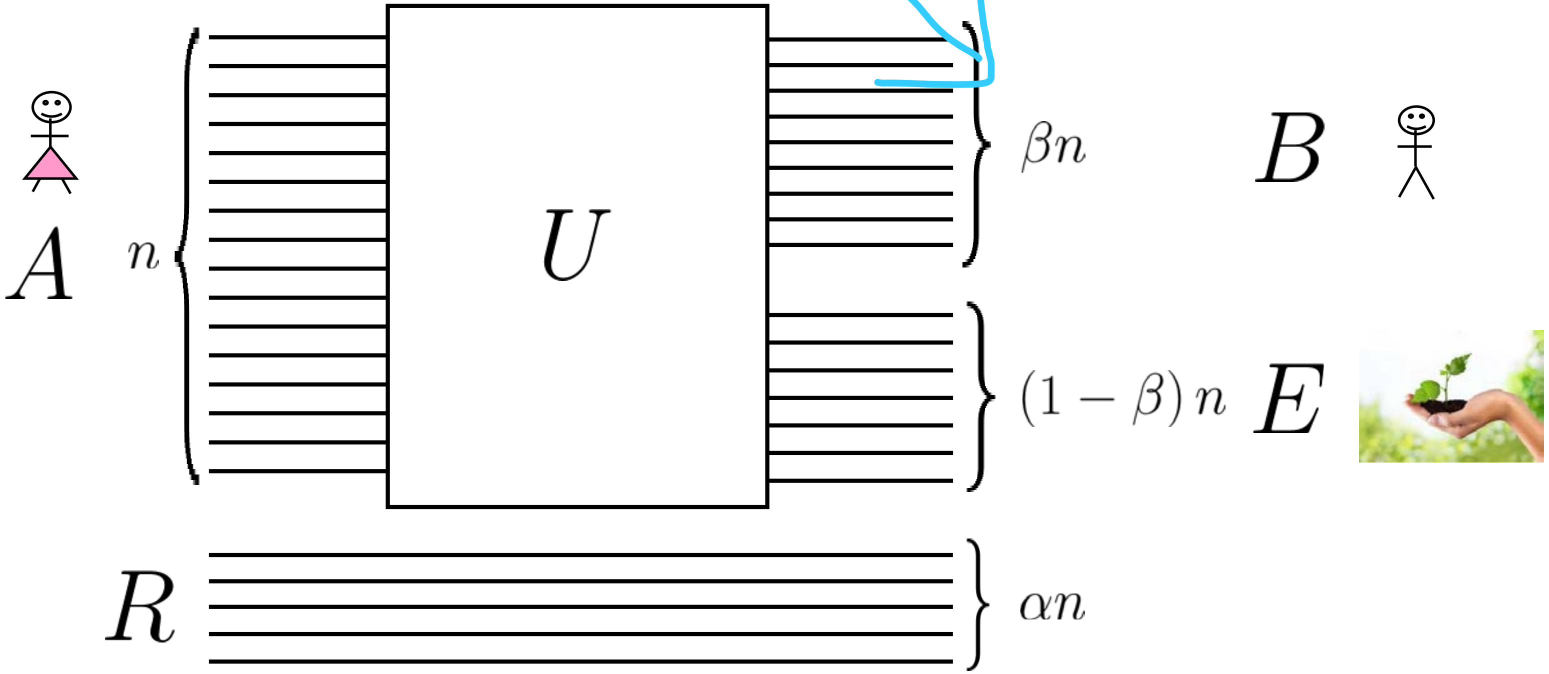
Transmitting alpha-bits



$$\beta > \frac{1+\alpha}{2}$$

Transmitting alpha-bits

$$\frac{1+\alpha}{2}\,n \text{ qubits}$$



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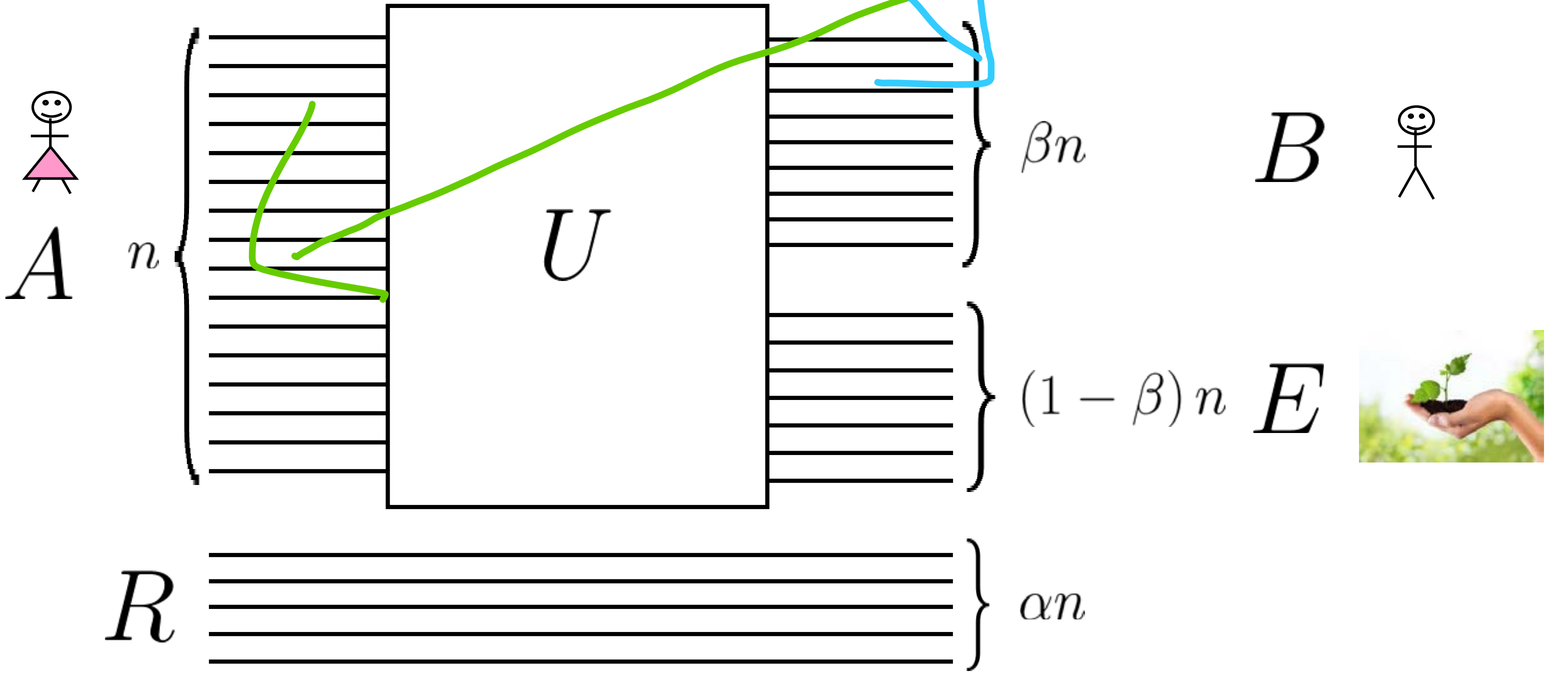
$$\frac{1+\alpha}{2} n \text{ qubits} \geq n\ \alpha\text{-bits}$$



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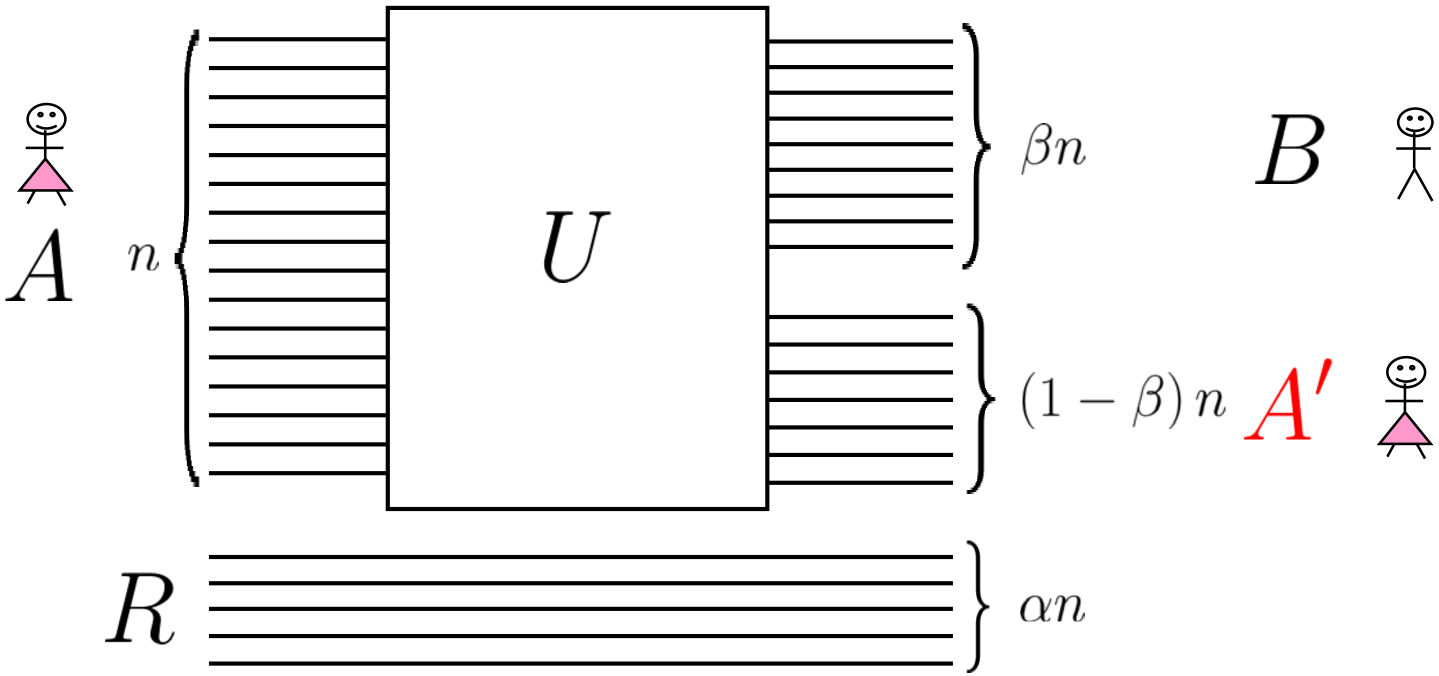
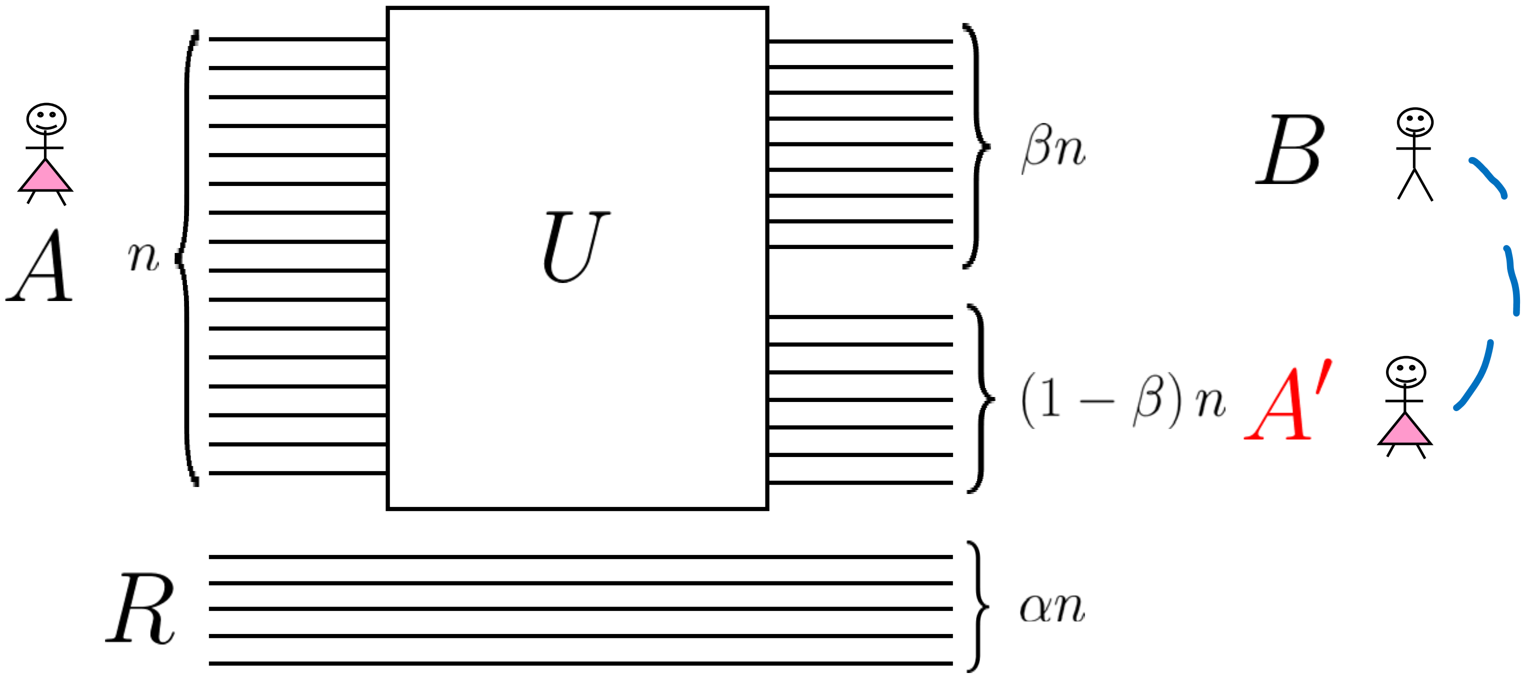


$$\beta > \frac{1+\alpha}{2}$$

Transmitting alpha-bits

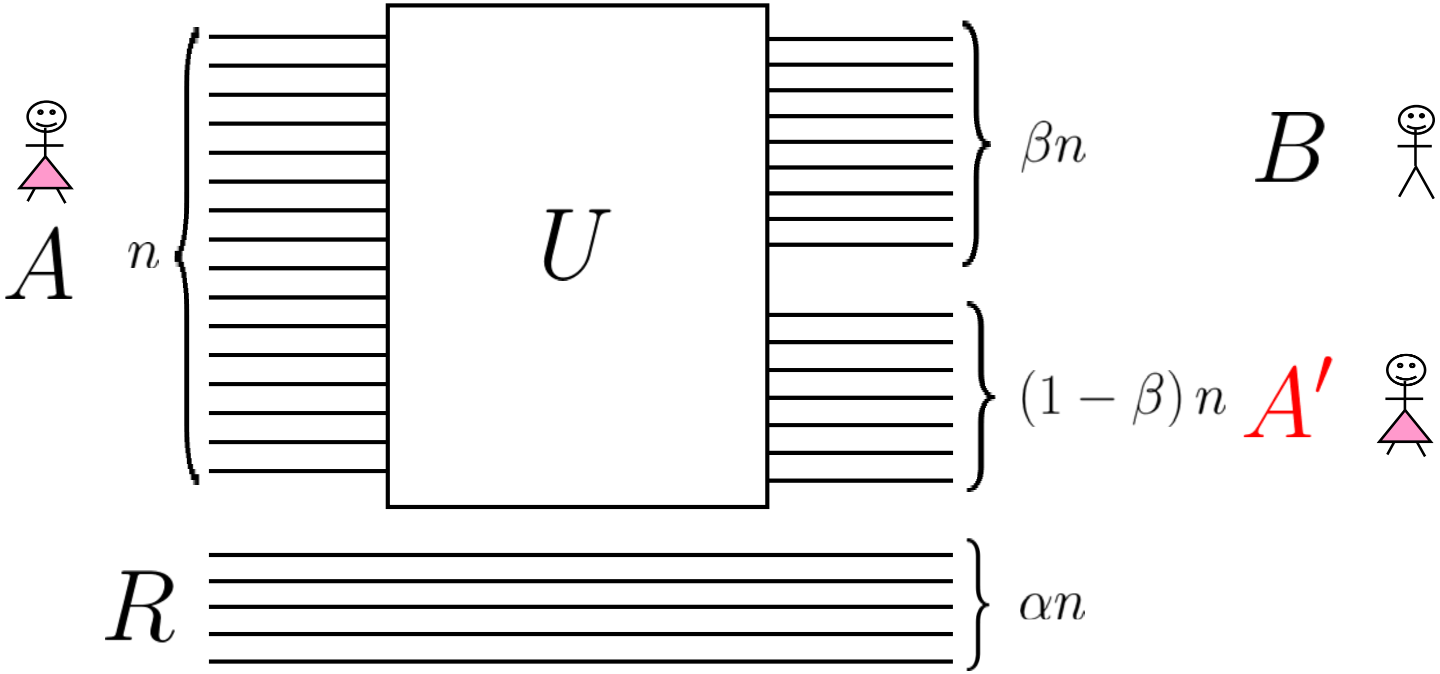
$$\frac{1+\alpha}{2} n \text{ qubits} \geq n \; \alpha\text{-bits} + \frac{1-\alpha}{2} n \text{ ebits}$$



$$\beta > \frac{1+\alpha}{2}$$

Transmitting alpha-bits

$$(1+\alpha)\ \text{qubits} \overset{(a)}{\geq} 2\ \alpha\text{-bits}\ +\ (1-\alpha)\ \text{ebits}$$



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Alpha-bit resource equalities

$$(1+\alpha)\ \text{qubits} \overset{(a)}{=} 2\ \alpha\text{-bits}\ +\ (1-\alpha)\ \text{ebits}$$

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To show RHS $\geq$ LHS:

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To show RHS $\geq$ LHS:

To do...

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(Coherent) super-dense coding

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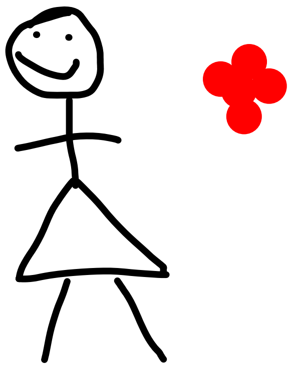
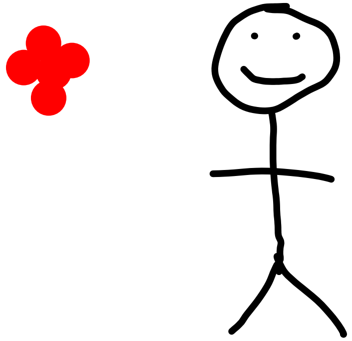
1. Alice and Bob share n ebits

$$\sum_{k=0}^{2^n-1} |k\rangle_A |k\rangle_B$$

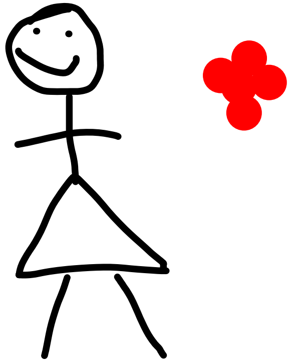




(Coherent) super-dense coding

1. Alice and Bob share n ebits
2. Alice applies an operation to her qubits

$$\sum_{k=0}^{2^n-1} e^{\frac{2\pi\, k\, r\, i}{2^n}} \, |k \oplus s\rangle_A \, |k\rangle_B$$



$$0 \leq r < 2^n$$
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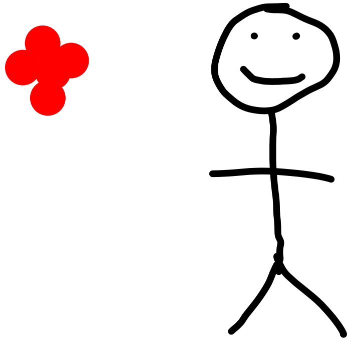

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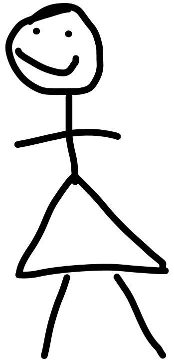

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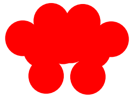

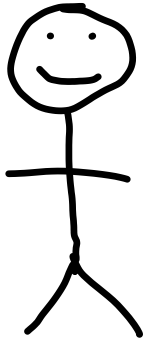

(Coherent) alpha-bit super-dense coding

1. Alice and Bob share n ebits
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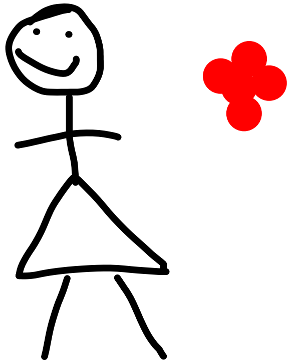

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$$0 \leq r < 2^n$$
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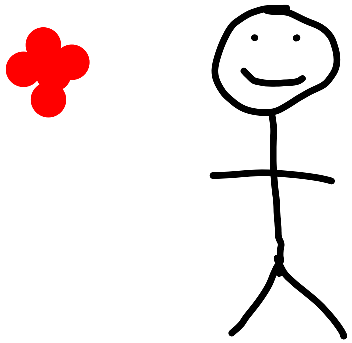

(Coherent) alpha-bit super-dense coding

1. Alice and Bob share n ebits
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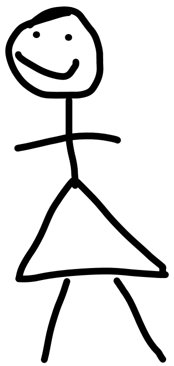
$$\sum_{k=0}^{2^n-1} e^{\frac{2\pi\, k\, r\, i}{2^n}} |k \oplus s\rangle_A |k\rangle_B$$



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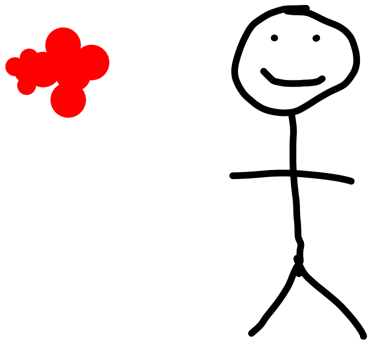

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Done ✓

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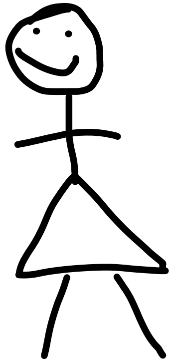

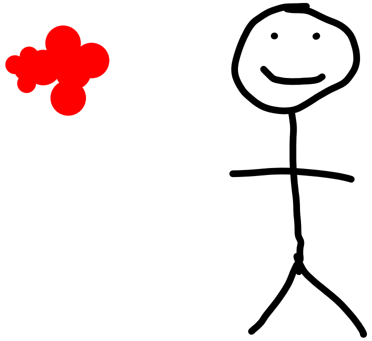

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Zero-bits and ebits as fundamental resources

All noiseless quantum resources (qubits, α-bits, cobits. . .) can be rewritten in terms of zero-bits and ebits

$$\text{e.g.} \quad 1\ \alpha\text{-bit} \overset{(a)}{=} (1+\alpha)\ \text{zero-bits} + \alpha\ \text{ebits}$$

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When rewritten in this basis, the quantum resource ordering becomes the product ordering:

$$(a,b) \geq (a',b') \iff (a \geq a') \wedge (b \geq b')$$

Alpha-bit Capacities

The α-bit capacity of a channel $\mathcal{N} : S(A') \to S(B)$ is given by

$$\mathcal{Q}_\alpha(\mathcal{N}) = \sup_k \frac{1}{k} \sup_{|\psi\rangle \in A'^k A^k} \min\left(\frac{1}{1+\alpha} I(A:B)_\rho, \frac{1}{\alpha} I(A\rangle B)_\rho\right)$$

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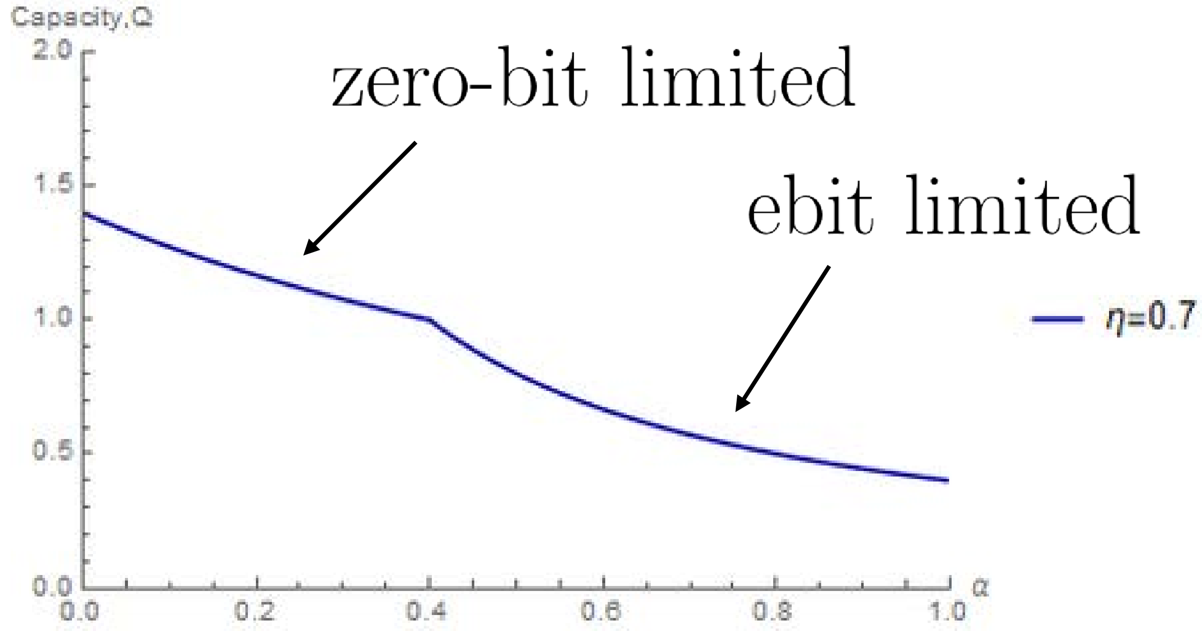

$$1 \ \alpha\text{-bit} \overset{(a)}{=} (1+\alpha) \text{ zero-bits} + \alpha \text{ ebits}$$

Amortised and entanglement-assisted capacities

With entanglement-assistance or an amortised quantum side channel, the capacity is given by

$$\frac{1}{1+\alpha} \sup_{|\psi\rangle \in A'A} I(A:B)_\rho$$

Single letter!

Unconstrained by ebits and so only zero-bits matter. This explains why all entanglement-assisted capacities are proportional to mutual information.

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BUT amortised quantum capacity $=$ quantum capacity

Answer: As $\alpha \to 1$, the size of the side channel diverges

Further Applications

$$\langle \mathcal{N}_{A' \to B} \rangle \overset{(a)}{\geq} I(A \rangle B) \text{ qubits}$$

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Non-additivity of quantum capacity?

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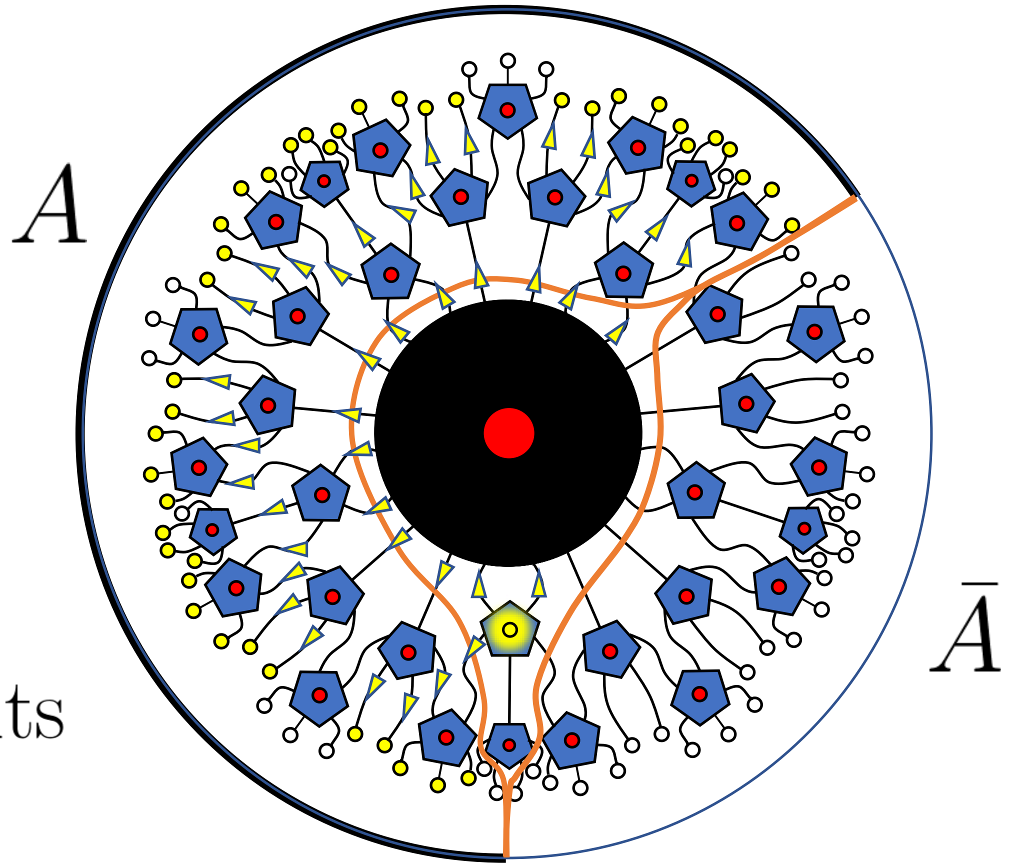
Zero-bits can substitute for classical bits in:

entanglement distillation, state merging
remote state preparation and channel simulation

Optimality follows from optimality of zero-bit teleportation

Alpha-bits and Black Holes

Alpha-bits arise naturally when studying black holes in AdS/CFT



Boundary subregion may encode α-bits of a bulk region

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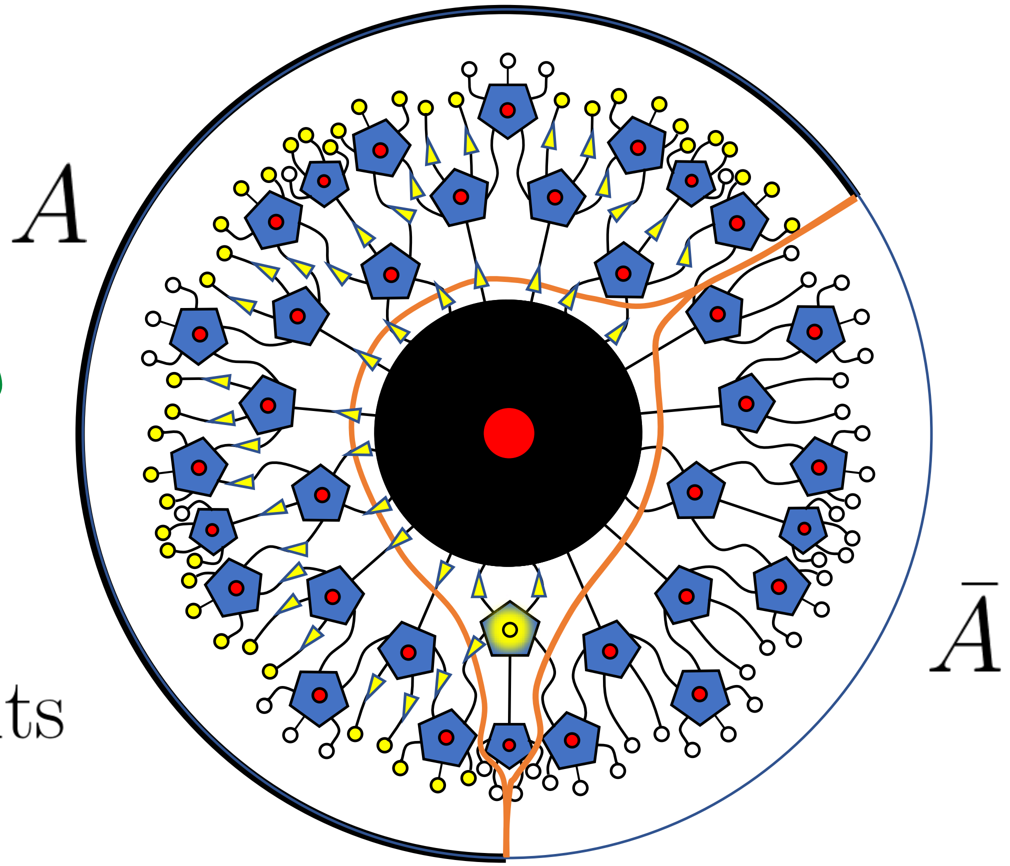
Tensor network toy model of AdS/CFT. Alpha-bits of region between orange lines encoded in A



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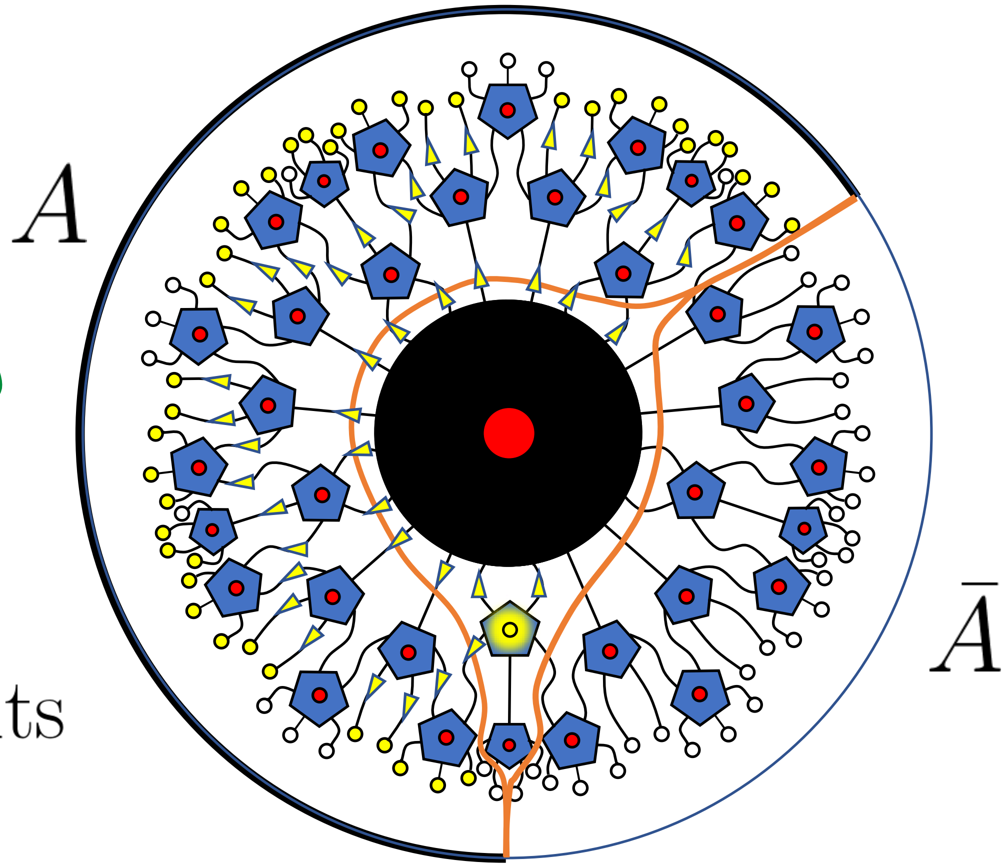

Boundary subregion may encode α-bits of a bulk region

Implications: Error-correction is only approximate, reconstructed operators are state-dependent

Thank you