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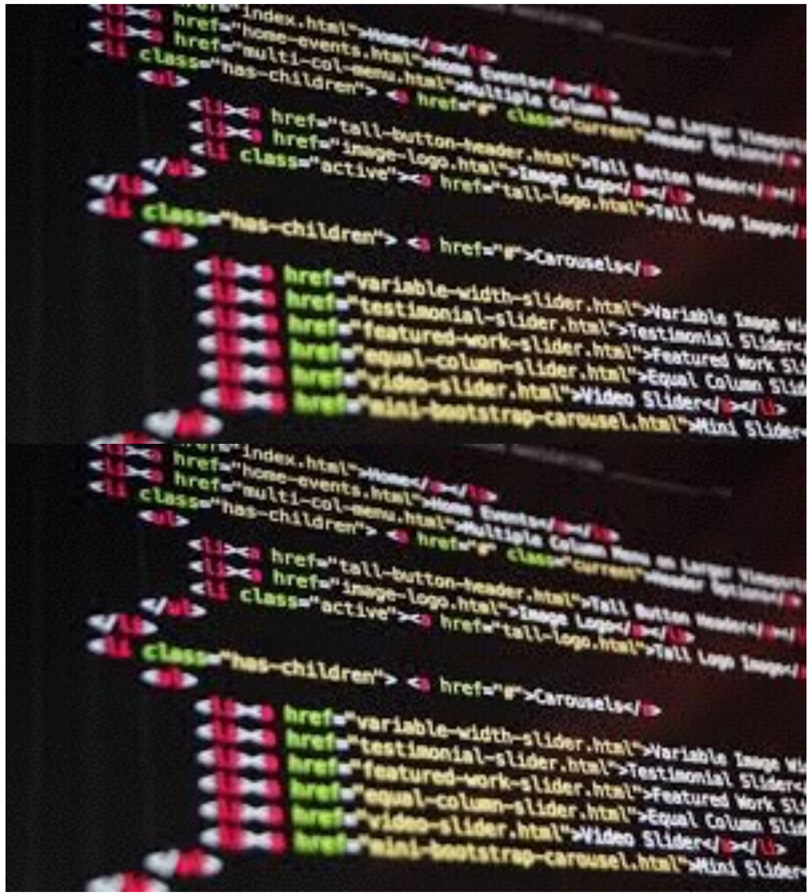
SHORTER GATE SEQUENCES FOR QUANTUM COMPUTING BY MIXING UNITARIES

PHYSICAL REVIEW A **95**, 042306 (2017)
ARXIV:1612.02689

BIG PICTURE



better compiling

Can Randomisation help?

if  then do **X**, but if  then do **Y**

OUTLINE

1. Gate-synthesis / compiling
2. Measuring noise and coherence
3. The RESULT
4. The PROOF

Gate-synthesis /compiling

Gate set $\mathcal{G}$

A collection of unitaries used to build circuits

e.g. from $\mathcal{G} = \{A, B, C\}$ we can build $ABC, ABBC, CBC,$ etc

Universality - Informal statement

$\mathcal{G}$ is **universal** if it can implement any unitary (upto finite precision)

Universality - Formal statement

$\mathcal{G}$ is **universal** if for any target unitary V and $\epsilon > 0$ there exists a finite circuit $U \in \langle \mathcal{G} \rangle$ such that $d(U, V) \leq \epsilon$

Example

Clifford+T or Clifford+Toffoli

Cost model $\mathfrak{C} : \mathcal{G} \rightarrow \mathbb{R}_{+}$

Each elementary gate given a positive valued “cost”

This induces a circuit cost

if $U = \prod_i G_i$ then $\mathfrak{C}(U) = \sum_i \mathfrak{C}(G_i)$

Example

Uniform cost model:

$\mathfrak{C}(G) = 1$ for all $G \in \mathcal{G}$

Magic state cost model / T-count:

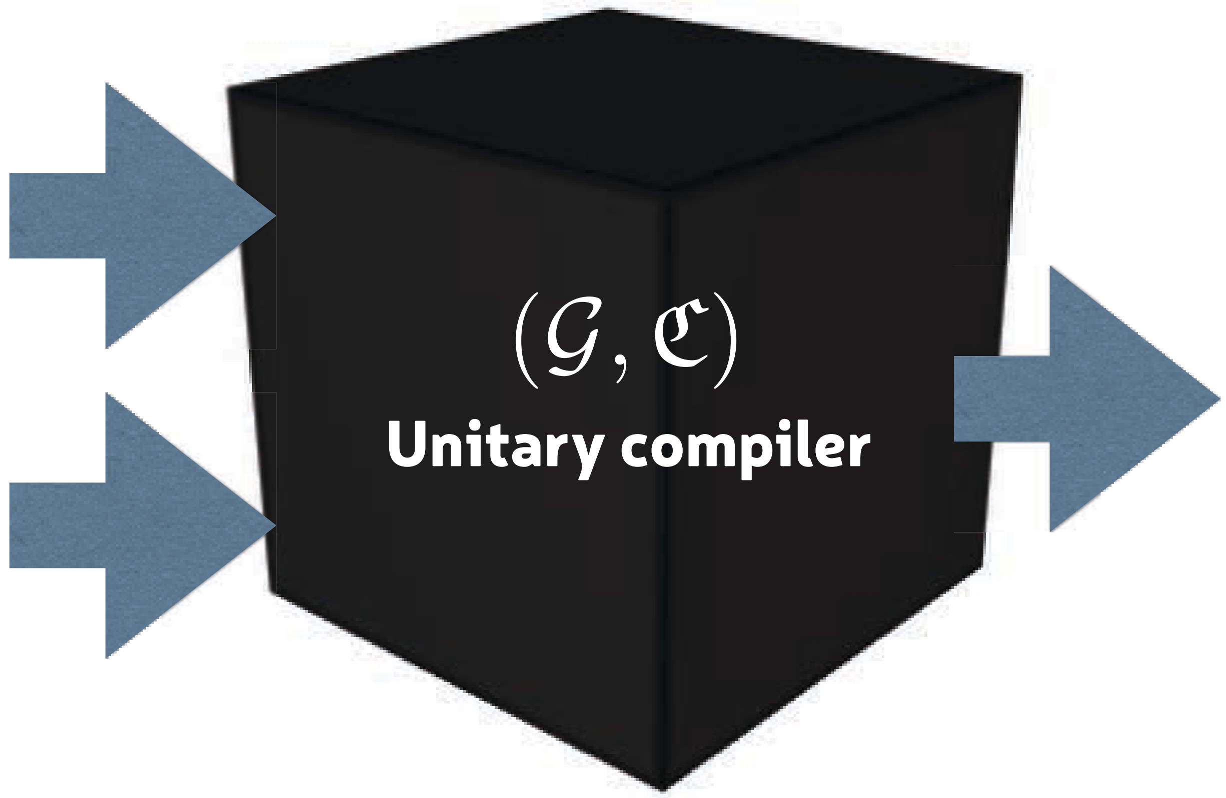
$\mathfrak{C}(T) = 1$ and $\mathfrak{C}(C) = 0$ for all C in the Clifford group.

GATE SYNTHESIS / COMPILING

blackbox software

V

$\epsilon > 0$



$U \in \langle \mathcal{G} \rangle$

with $||U - V||_\infty \leq \epsilon$

usually with some promise
$\mathfrak{C}(U) \leq f(\epsilon)$

For ***efficient*** compilers The promise function $f(\epsilon)$ is often polylog $f(\epsilon) \leq A \log(1/\epsilon)^\gamma$

An ***optimal*** compiler will have the lowest possible $\mathfrak{C}(U)$ and $f(\epsilon)$

Solovay-Kitaev

Consider any universal gate set $\mathcal{G}$ (generating a group) with uniform cost.

We can efficiently solve the compiling problem using $\mathfrak{C}(U) \leq O(\log(1/\epsilon)^{\gamma})$ where γ is a constant dependent on $\mathcal{G}$

Comments:

For Clifford+T, the Solovay-Kitaev method gives $\gamma \leq 3.97$
See: Dawson & Nielsen *QIC*, **6** 81 (2006)

Solovay-Kitaev is efficient but not optimal.

Modern methods

Let $\mathcal{G}$ be Clifford+T and consider the T-count.

We can efficiently solve the compiling problem using $\mathfrak{C}(U) \leq O(\log(1/\epsilon))$

Refs:
Kliuchnikov, Maslov, Mosca, *QIC* **13** 607 (2013) – arXiv:1206.5236
Ross and Selinger *QIC* ***16*** 901 (2016) – arXiv:1403.2975

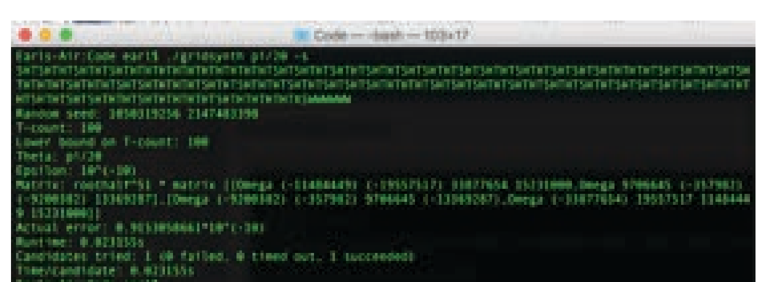
Comments:

Solver is optimal (under mild assumptions)

Asymptotically $\mathfrak{C}(U) \leq O(\log(1/\epsilon))$ is much better than $\mathfrak{C}(U) \leq O(\log(1/\epsilon)^{3.97})$

Practically means 100s of gates rather than 10,000s of gates

Convenient command line tool



Random compiling problem

Given a V and $\epsilon > 0$ output a probability distribution of circuits, realising

$$\mathcal{E}(\rho) = \sum_i p_i U_i \rho U_i^\dagger$$

such that $d(\mathcal{E}, \mathcal{V}) \leq \epsilon$ and minimise $\mathfrak{C}(\mathcal{E})$

Def:
$\mathcal{U}(\cdot) := U \cdot U^\dagger$

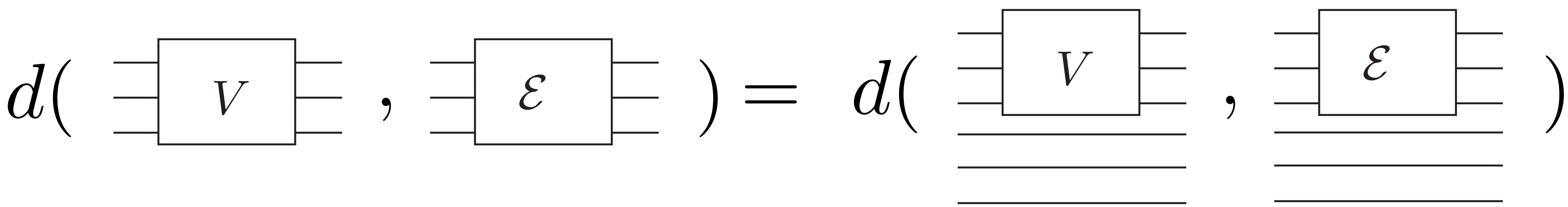
Cost models for random circuits

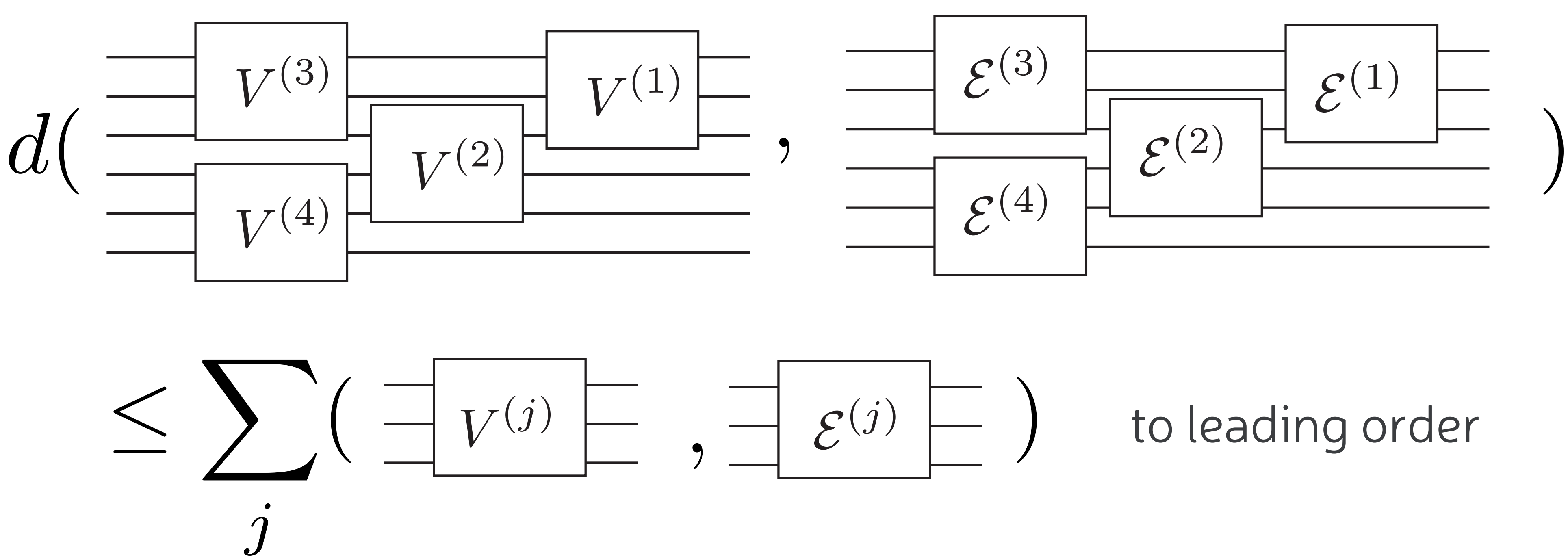
Our results for worst-case $\mathfrak{C}(\mathcal{E}) := \max(\mathfrak{C}(U_i))$

But average cost also interesting $\mathfrak{C}(\mathcal{E}) := \sum_i p_i \mathfrak{C}(U_i)$

Measuring noise & Coherence

Desiderata for $d(\mathcal{E}, \mathcal{V})$ **:** must compose nicely





What distance we use matters!

Average infidelity

$$d_F(\mathcal{E}, U) := 1 - \int_\psi F(\mathcal{E}(\psi), U|\psi\rangle) = 1 - \int_\psi \langle\psi|U^\dagger \mathcal{E}(|\psi\rangle\langle\psi|)U|\psi\rangle$$

- looks simple,
- often used to report experimental results.
- **fails** our desiderata!

What distance we use matters!

Diamond norm distance

$$d_\diamond(\mathcal{E},\mathcal{U}) := \frac{1}{2}||\mathcal{E} - \mathcal{U}||_\diamond = \frac{1}{2}\max_\rho \frac{||(\mathcal{E}\otimes\mathbb{I})(\rho) - (\mathcal{U}\otimes\mathbb{I})(\rho)||_1}{||\rho||_1}$$

where $||X||_1 := \mathrm{Tr}[\sqrt{X^\dagger X}]$ is the Schatten 1-norm

- composed nicely (**yes** to desiderata);
- due to nice properties, used in proofs, e.g. of threshold theorem.
- So <1% threshold statements refer to diamond distance!

Algorithm level "coherent noise":

Consider gate set $\mathcal{G} = \{A, B, C\}$

each individual gate is perfect

When we compile a target gate V

we perform $U \sim ABACABABABA = Ve^{i\delta}$

The approximation error (due to finite sequence length) is a coherent error $e^{i\delta}$.

For coherent noise

$$d_\diamond(\mathcal{U}, \mathcal{V}) \sim \sqrt{d_F(\mathcal{U}, \mathcal{V})}$$

so choice of distance measures is important!

Contrast physical level coherent noise

Consider a scenario where we
want to implement sequence from gate set $\mathcal{G} = \{A, B, C\}$
but our experiment systematically performs $\tilde{\mathcal{G}} = \{\tilde{A}, \tilde{B}, \tilde{C}\}$

we have $\tilde{A} = Ae^{i\delta_A}$, $\tilde{B} = Be^{i\delta_B}$...

where δ are small noise terms.

Randomised compiling can help

E. Knill arXiv:quant-ph/0404104
Wallman & Emerson, *Phys Rev A* **94**, 052325 (2016)

say $\mathcal{U}$ is target unitary
$\tilde{\mathcal{U}}$ is unitary with coherent noise

so that $d_\diamond(\tilde{\mathcal{U}}, \mathcal{U}) = \epsilon$ and $d_F(\tilde{\mathcal{U}}, \mathcal{U}) \sim \epsilon^2$

then there exist twirling schemes $\mathcal{E}(\rho) = \sum_i p_i U_i \rho U_i^\dagger$ where $U_i = P_i \tilde{U} P_i'$

with $d_\diamond(\mathcal{E}, \mathcal{U}) \sim \epsilon^2$

The RESULT

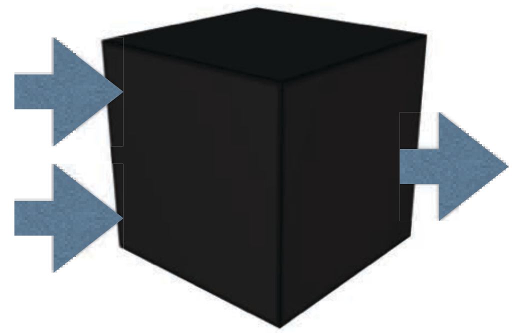
Preamble

Let $\mathcal{G}$ be a universal gate set with cost measure $\mathfrak{C}$

assume some blackbox unitary compiler

V

$0 < \epsilon \leq 0.01$



equipped with promise

$\mathfrak{C}(U) \leq f(\epsilon)$

Theorem

...then there exists a random sequence $\mathcal{E}(\rho) = \sum_j p_j U_j \rho U_j^\dagger$

with $d_\diamond(\mathcal{V}, \mathcal{E}) \leq 10\epsilon^2$ and cost $\mathfrak{C}(\mathcal{E}) \leq f(\epsilon)$

2nd Theorem (paraphrased)

For single qubit axial rotations: *the assumptions can be relaxed and inequalities tightened slightly.*

"same cost gets you better error suppression"

Theorem

...then there exists a random sequence $\mathcal{E}(\rho) = \sum_j p_j U_j \rho U_j^\dagger$

with $d_\diamond(\mathcal{V}, \mathcal{E}) \leq 10\epsilon^2$ and cost $\mathfrak{C}(\mathcal{E}) \leq f(\epsilon)$

"same error suppression for lower cost?"

Corollary

if the unitary cost is polylog $\mathfrak{C}(U) \leq f(\epsilon) = A \log_2(1/\epsilon)^\gamma$

...then there exists a random circuit $\mathcal{E}(\rho) = \sum_j p_j U_j \rho U_j^\dagger$

with $d_\diamond(\mathcal{V}, \mathcal{E}) \leq \epsilon$ and cost $\mathfrak{C}(\mathcal{E}) \leq C^\gamma f(\epsilon) \sim (1/2)^\gamma$

$$C = \left(\frac{1}{2}\right)\left(1 + \frac{\log(A)}{\log(1/\epsilon)}\right) \rightarrow_{\epsilon \rightarrow 0} \left(\frac{1}{2}\right)$$

WE SAVE ~ 2^γ

Known "gamma" values:

For single qubit Clifford+T gate set
and T-count cost metric
$$\mathfrak{C}(U) \leq f(\epsilon) = 9\log(1/\epsilon)$$
so $\gamma = 1$
and roughly 2x saving

	optimal unitary	random compiling
$\epsilon = 10^{-20}$	600	314 or less
$\epsilon = 10^{-10}$	300	165 or less
$\epsilon = 10^{-5}$	150	90 or less

Using Solovay-Kitaev for single qubits $\gamma \sim 3.97$, so save $2^{3.97} \sim 15 - 16$

For optimal multiqubit synthesis $\gamma = ?$ unknown, conjecture $1 \ll \gamma$

Is derandomisation possible?

$$p_1^{(1)} p_1^{(2)} p_1^{(3)} p_1^{(4)} \ldots$$

$$p_2^{(1)} p_1^{(2)} p_1^{(3)} p_1^{(4)} \ldots$$

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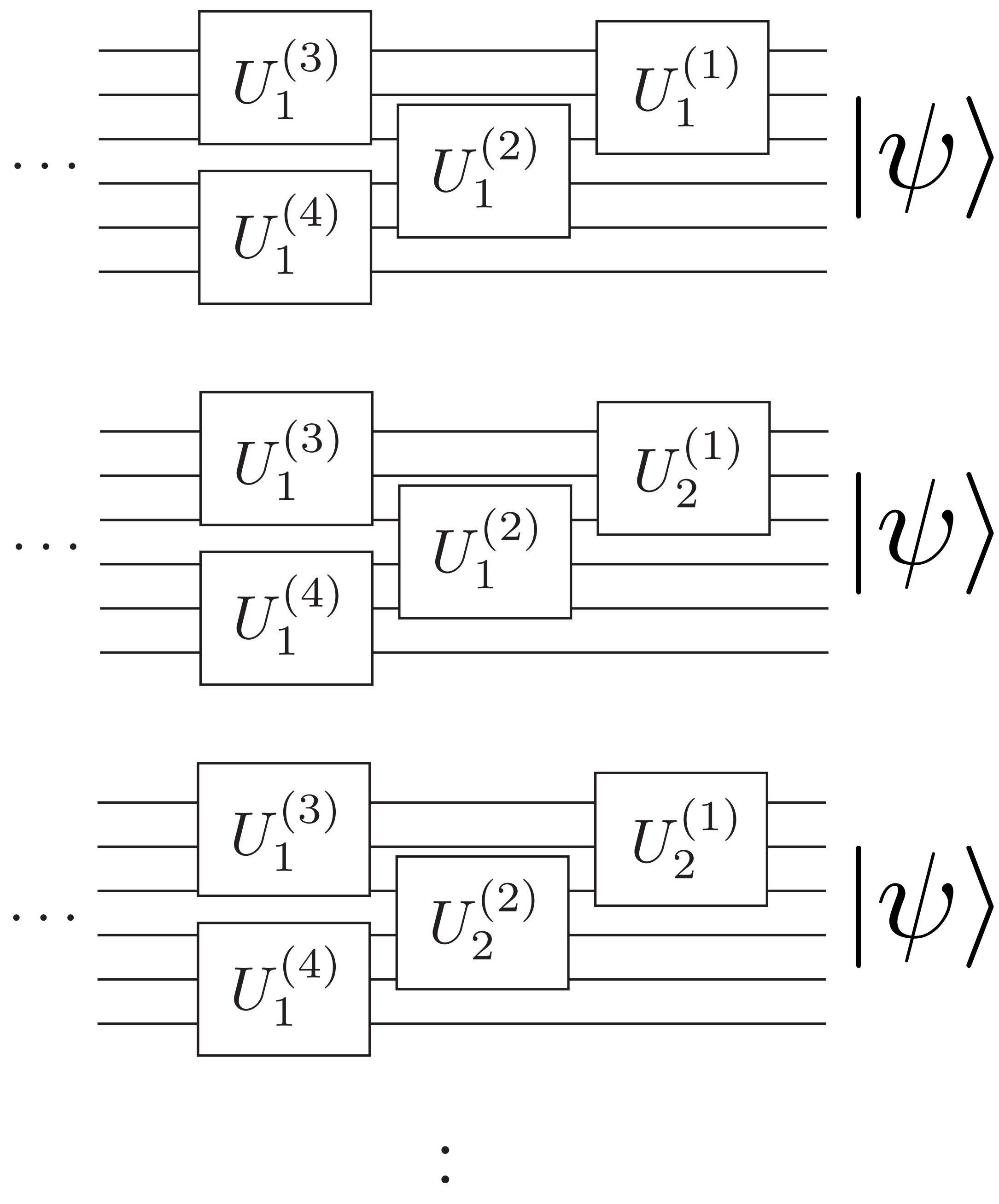


TRUE THAT:
For fixed input there is one "best" choice of unitaries that maximise fidelity

BUT:
- best unitary depends on choice of input state;
- can't be calculated without simulating the whole computation!

The PROOF

STEP 1. Show from “suitable” unitaries/Hamiltonians we can find random circuit with quadratically reduced error suppression.

STEP 2. Give concrete algorithm finding suitable Hamiltonians.

STEP 1. Show from "suitable" unitaries/Hamiltonians we can find random circuit with quadratically reduced error suppression.*

STEP 2. Give concrete algorithm finding suitable Hamiltonians.

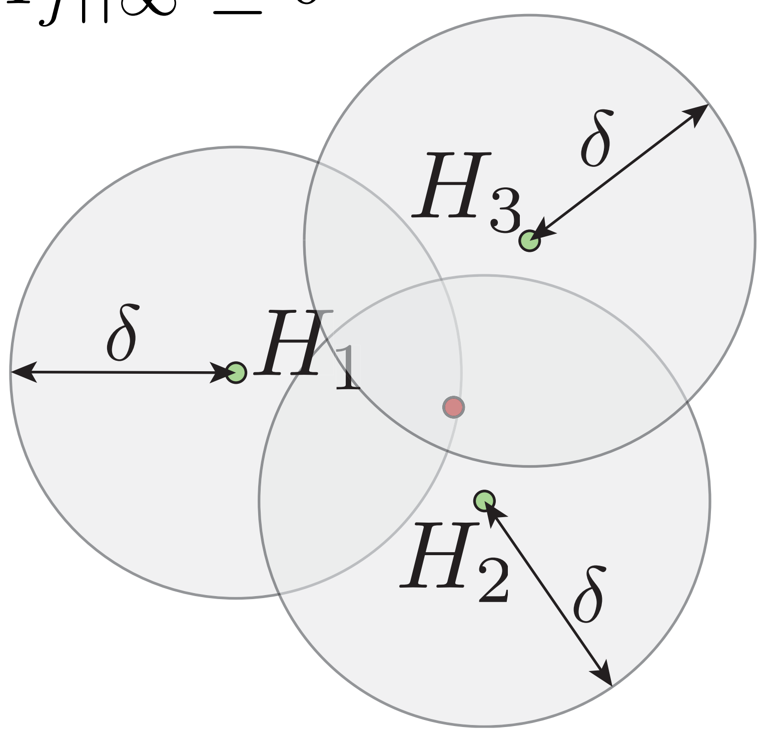
* Similar "step 1" result found simultaneously by Hastings arXiv:1612.01011 QIC **17** 0488 (2017)

STEP 1. Show from "suitable" Hamiltonians we can find random circuit

Lemma. Given target unitary V and a set of unitaries $U_j = V e^{iH_j}$ such that

(1) All Hamiltonians within δ of origin

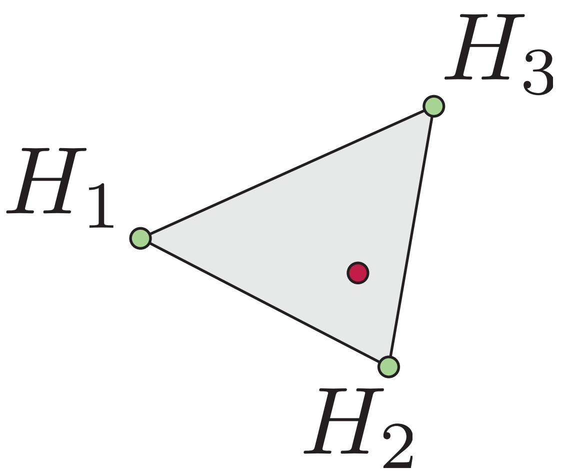
$$||H_j||_\infty \leq \delta$$



(2) Hamiltonians enclose the origin

$$\exists p_j \quad 0 \leq p_j \quad \sum p_j = 1$$

$$\sum_j p_j H_j = 0$$



then $\mathcal{E}(\rho) = \sum_j p_j U_j \rho U_j^\dagger$ ***satisfies*** $d_\diamond(\mathcal{E}, \mathcal{V}) \leq 1.1 * \delta^2$

Sketch

start with

$$d_\diamond(\mathcal{E},\mathcal{V}) = d_\diamond(\mathcal{V}^\dagger \circ \mathcal{E}, \mathrm{identity})$$

and $$\mathcal{V}^\dagger \circ \mathcal{E}(\rho) = \sum_j p_j e^{-iH_j} \rho e^{iH_j}$$

Expand exponential and find all first order terms cancel due to $\sum_j p_j H_j = 0$

$$\mathcal{V}^\dagger \circ \mathcal{E}(\rho) = \rho + O(H_j^2, \rho)$$

Subtracting identity gives

$$\mathcal{V}^\dagger \circ \mathcal{E}(\rho) - \rho = O(H_j^2, \rho)$$

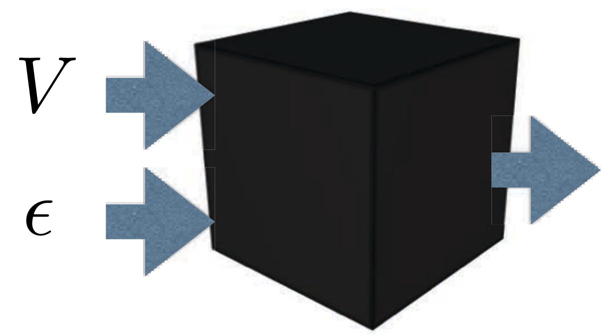
Leaving H_j^2 and higher order terms

Take 1-norm, use triangle inequality, Hölder's inequality, carefully bound higher order terms...

$$||\mathcal{V}^\dagger \circ \mathcal{E}(\rho) - \rho||_1 = O(||H_j||_\infty^2) = O(\delta^2)$$

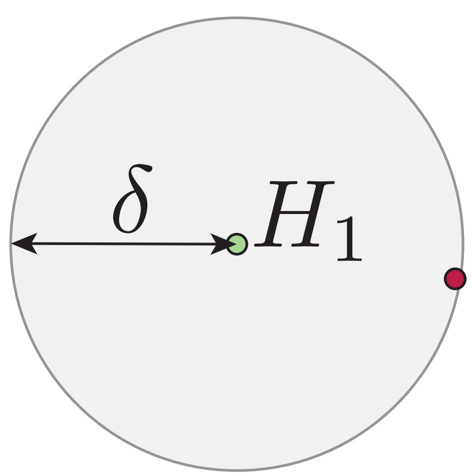
STEP 2. Give concrete algorithm finding suitable Hamiltonians.

assume some blackbox unitary compiler



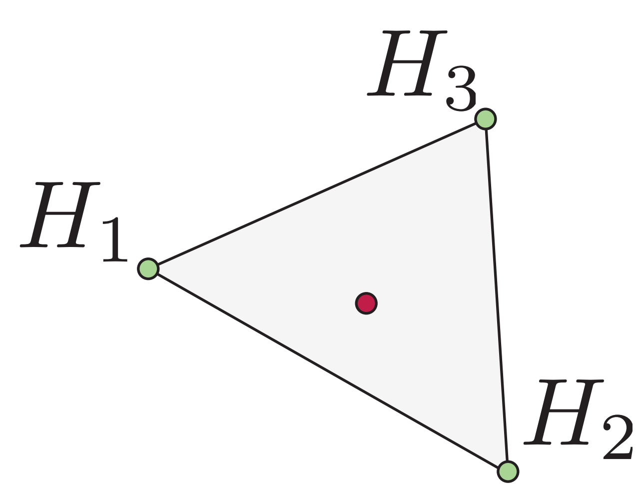
U equipped with promise $\mathfrak{C}(U) \leq f(\epsilon)$

It follows $U_1 = V e^{iH_1}$ where $||H_j||_\infty \leq \delta = 3\epsilon + 7\epsilon^2$

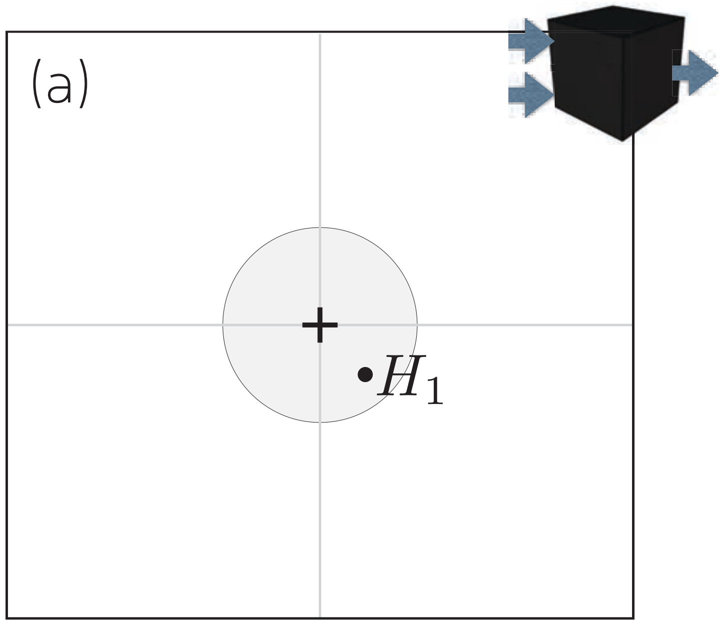


This gives 1 nearby Hamiltonian,

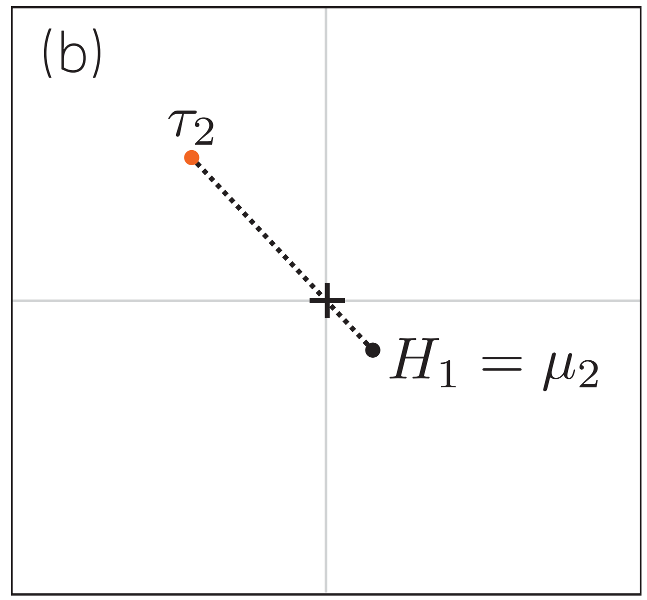
but in a D variable problem we typically need D+1 points to enclose the origin! e.g. For 2 variable problems we need 3 points/Hamiltonians



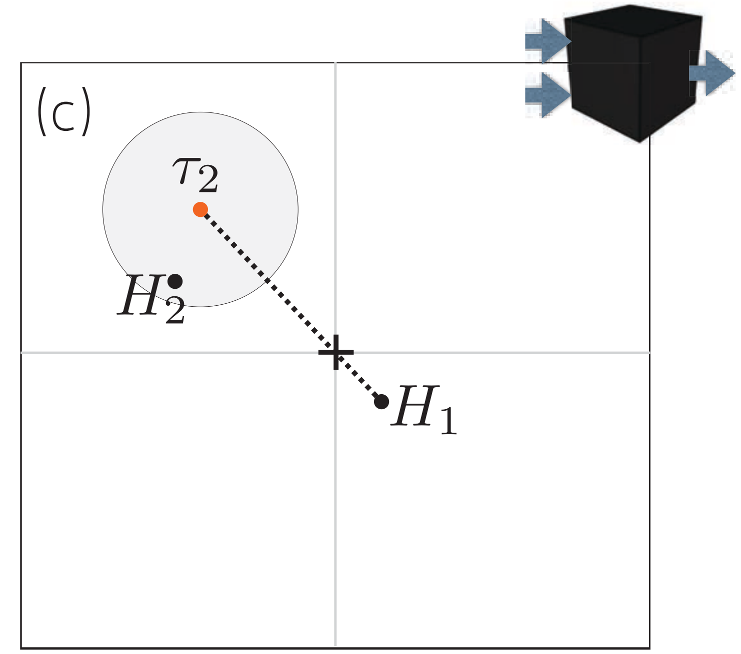
STEP 2. Give concrete algorithm finding suitable Hamiltonians.



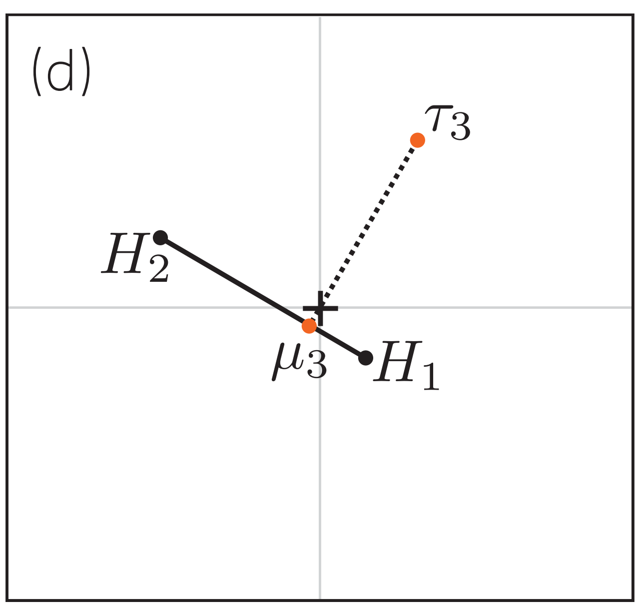
Find H_1 near origin



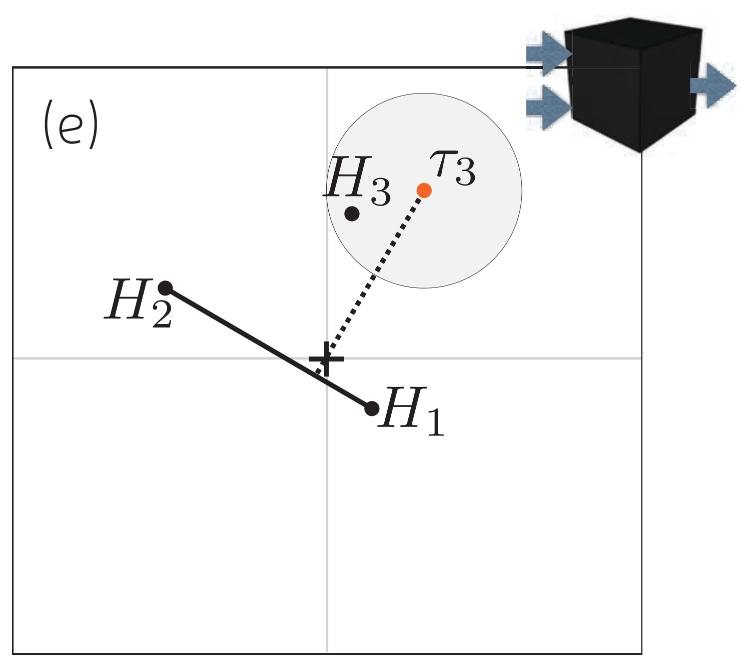
Select new target
$\tau_2 \propto -H_1$



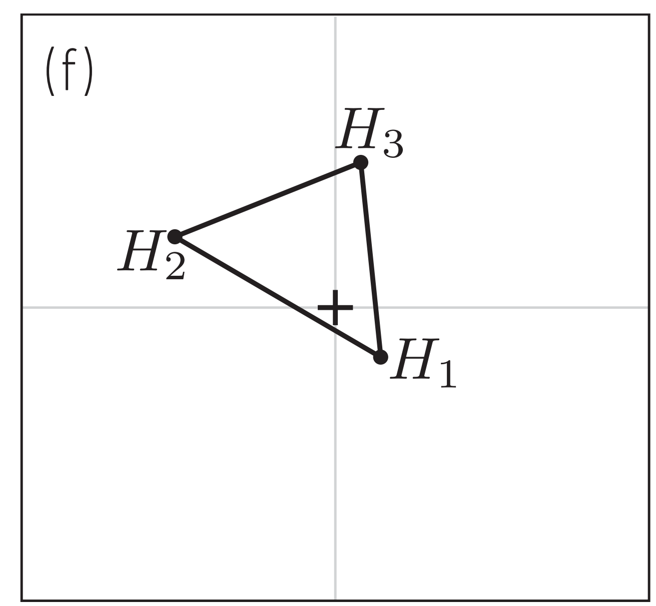
Find H_2 near τ_2



Find $\mu_3 = p_1 H_1 + p_2 H_2$
near origin and $\tau_3 \propto -\mu_3$



Find H_3 near τ_3



Find μ_4 near origin
EXIT when origin enclosed
or continue looping

STEP 2. Give concrete algorithm finding suitable Hamiltonians.

We prove that:

- Each iteration give a new H_j within $3\epsilon + 7\epsilon^2$ of origin.
- Each oracle call gives a new mixture

$$\mu_n = \sum_{j=1}^{n} p_j H_j$$

- When $||\mu_n||_\infty = 0$ the program EXITs, but can also EXIT when $||\mu_n||_\infty \ll \epsilon^2$
- and convergence is exponentially fast $||\mu_n||_\infty \leq 6\epsilon e^{-0.62n}$

Furthermore (unproven),

- Geometric intuition hints that algorithm terminates in constant number of steps

STEP 1. Show from "suitable" Hamiltonians we can find random circuit

showed $\mathcal{E}(\rho) = \sum_j p_j U_j \rho U_j^\dagger$ ***satisfies*** $d_\diamond(\mathcal{E}, \mathcal{V}) \leq 1.1 * \delta^2$

STEP 2. Give concrete algorithm finding suitable Hamiltonians.

found unitaries with $||H_j||_\infty \leq \delta = 3\epsilon + 7\epsilon^2$

COMBINED RESULT $d_\diamond(\mathcal{E}, \mathcal{V}) \leq 10\epsilon^2$

The END bit

We reviewed how

- Unitary compilers have coherent noise on the algorithm level.
- Coherent noise at the physical level can be quadratically reduced using random circuits.

This work showed

- Our random circuits give free quadratic error suppression;
- Free quadratic error suppression can be swapped for shorter gate sequences;
- For single qubit gates we save a factor 2x;
- Savings could be larger for multiqubit unitaries.

Future work

- Implement and numerically test.
- Classical runtime convergence in constant time?
- Better algorithms?

HIRING
4 POSTDOCS

QCDA - Quantum Code Design & Architecture

www.qcda.eu

Sheffield - Earl Campbell
London - Dan Browne
Paris - Anthony Leverrier & Jean Pierre Tillich
Delft - Barbara Terhal, Koen Bertels & Carmina Garcia Almudever
Munich - Robert Koenig

Funded though QUANTERA

THANK YOU!