A no-go theorem for theories that decohere to quantum mechanics

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Motivation

"The more important fundamental laws and facts of physical science have all been discovered, and these are so firmly established that the possibility of their ever being supplanted in consequence of new discoveries is exceedingly remote"

A. A. Michelson, "Light waves and their uses," 1903.

Motivation

- Quantum theory is the most accurately tested theory in the history of science

- Yet, just as for Michelson, it could turn out to only be an effective description of Nature

Motivation

- If fundamental theory exists, should be some mechanism, akin to decoherence, which suppresses post-quantum effects

Main question: Can a no-go result be established about the existence of theories that *hyperdecohere* to quantum mechanics?

Decoherence: from quantum to classical probability theory

- Quantum system interacts deterministically with environment system, after which environment discarded
- Formalises quantum system irretrievably losing information to environment, leading to effective classical description
- Process induces CPTP map on original quantum system, *decoherence* map $\mathcal{D}(\cdot)$

Decoherence example

1. $U = \sum_i |i\rangle\langle i| \otimes \pi_i$, with $\{|i\rangle\}$ the computational basis and π_i a unitary acting on environment system as $\pi_i|0\rangle = |i\rangle$, $\forall\ i$

2. Decoherence map arising this is:

$$\mathcal{D}(\rho) = \mathrm{Tr}_E \left(U(\rho \otimes |0\rangle\langle 0|_E) U^\dagger \right) = \sum_i \langle i|\rho|i\rangle |i\rangle\langle i|,$$

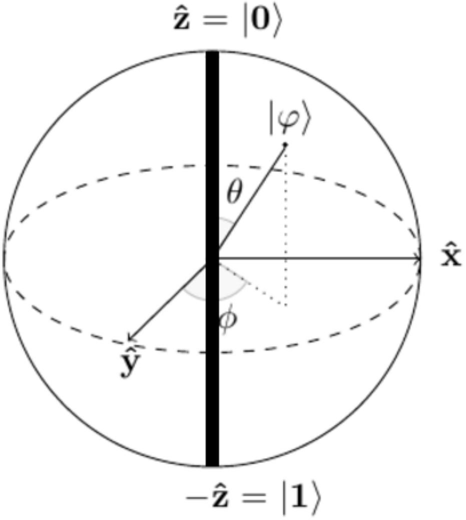
Decoherence

Entirety of classical probability theory arises from $\mathcal{D}$:

- Probability distributions over classical outcomes, $\mathcal{D}(\rho)$
- Stochastic maps acting on said distributions, $\mathcal{D}\left(\mathcal{E}\left(\mathcal{D}(\_)\right)\right)$
- Measurements inferring different outcomes, $\mathrm{Tr}\left(M\mathcal{D}(\_)\right)$

Classical probability theory is a sub-theory of quantum theory, map $\mathcal{D}$ restricts quantum theory to classical sub-theory

Pure classical states are pure quantum states



Three key features of decoherence

1. Trace preserving. "Decoherence is a deterministic process"

2. Idempotent: $\mathcal{D}\left(\mathcal{D}(\rho)\right) = \mathcal{D}(\rho),$ for all ρ. "Classical systems have no more coherence 'to lose' "

3. If $\mathcal{D}(\rho)$ is a pure classical state, then it is also a pure quantum state. "No information lost if decohered state is a state of maximal information"

Generalised theories

- To make progress on main question, need to describe theories other than quantum and classical theory in consistent manner

- Work in generalised probabilistic theory framework developed by Hardy and Chiribella, D'Ariano, & Perinotti, among others

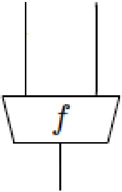
Generalised theories

- Ultimately, any physical theory will be explored by experiments, so should have *operational* description in terms of those experiments

- Theory specifies collection of laboratory devices which can be connected together to form experiments and assigns probabilities to experimental outcomes

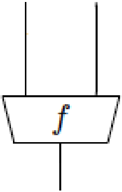
Generalised theories

- A *process* is a particular outcome of a piece of lab equipment, with some number of input/outputs



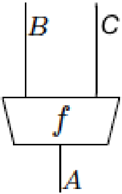
Generalised theories

- Can intuitively think of *physical systems* as passing between input and output ports



Generalised theories

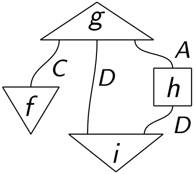
- Systems are labelled by different *types* $A, B, C, \ldots$



Generalised theories

Processes can be connected together to form experiments:

$$\mathbf{Pr}(f, g, h, i) \quad :=$$



i) System types much match, and ii) no cycles can be formed

Generalised theories

- **Tomography**: if two processes give same probabilities in all experiments, they are the same

$$f = g \iff \forall X, \ \ \mathbf{Pr}(f, X) = \mathbf{Pr}(g, X)$$

- **Convexity**: probabilistic mixtures of allowed processes are allowed processes, $h = \sum_i p_i f_i$

- A state is *pure* if it is not a convex combination of other states

Example: quantum theory

Quantum Theory:

- Systems are finite dimensional complex Hilbert spaces, system type corresponds to dimension
- Processes with no inputs are density matrices, no outputs POVM elements
- Processes with inputs and outputs are completely positive, trace non-increasing maps

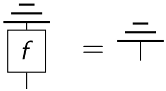
Example: generalised theories

- Classical probability theory is also a generalised theory
- Theory containing PR boxes which maximally violate CHSH inequality without violating no-signalling
- Theory with QT pure states, but different mixed states & measurements [T. Galley, L. Masanes, Quantum 1, 15 (2017)]

Physical principle I: Causality

1. "Future measurement choices do not effect current experiments"

2. Equivalent to existence of a unique discarding measurement

3. A process is *deterministic* if 

Physical principle I: Causality

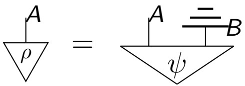
- Quantum theory satisfies Causality, with  $= \mathrm{Tr}_A(\_)$

- There exist generalised theories which violate Causality

Physical principle II: Purification

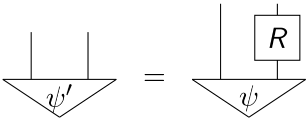
1. "Each state of incomplete information arises in an essentially unique way due to a lack of information of environment"

2. For every state ρ_A, there exists a pure state ψ_{AB}, such that ρ_A arises as marginalisation of ψ_{AB}:



Physical principle II: Purification

3. Two pure states ψ_{AB} and ψ'_{AB} which both purify ρ_A are connected by a reversible transformation



Quantum theory satisfies Purification, but there exist theories that violate it

Hyperdecoherence

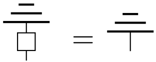
- **Informal**: A post-quantum theory is a generalised theory which *hyperdecoheres* to quantum theory

- Hyperdecoherence map restricts systems in a generalised theory to quantum systems

Hyperdecoherence assumption I

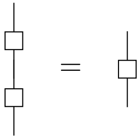
- Hyperdecoherence map: deterministic interaction with environment system, after which environment is discarded

- "Irretrievable loss of information to an environment"

- Determinism implies: 

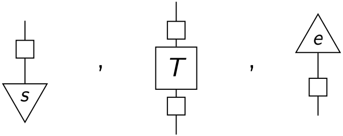
Hyperdecoherence assumption II

- Hyperdecohering twice same as hyperdecohering once, as hyperdecohered system has no more “post-quantum coherence” to lose
- Hyperdecoherence map should be idempotent:



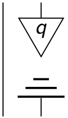
Hyperdecoherence

Density matrices, completely positive trace non-increasing maps, and POVM elements are elements of a *sub-theory*:



Hyperdecoherence

Need further constraints beyond determinism and idempotence to capture 'sensible' hyperdecoherence. Let $q = \sum_i p_i |i\rangle\langle i|$, consider



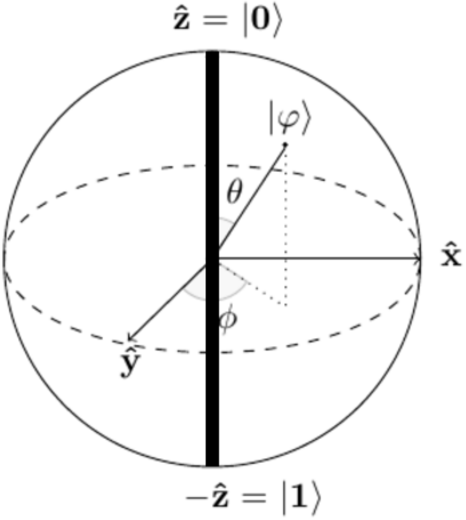
$$= \quad \mathbb{1} \otimes (\mathbf{q} \circ \mathrm{Tr}(\_))$$

Above is deterministic and idempotent, but allows for quantum system to "decohere" to itself

Hyperdecoherence assumption III

- A state is *pure in the sub-theory* if cannot be written as convex combination of other states *from the sub-theory*.
- We demand that pure states in the sub-theory are pure in the post-quantum theory
- "If hyperdecohered state is a state of maximal information, then no information should have been lost "

Hyperdecoherence assumption III

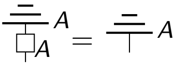
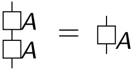


Hyperdecoherence assumption III

- This assumption rules out previous example
- Is the minimal assumption needed? Seemingly weaker preservation of information dimension also rules them out
- Can derive pure quantum states are pure from preservation of information dimension

Post-quantum theory

Post-quantum theory: for each system type A, there exists a hyperdecoherence map $\square_A$ satisfying:

1. $\square_A$ is deterministic: 
2. $\square_A$ is idempotent: 
3. Pure states in the sub-theory are pure states.

Sub-theory defined by collection $\left\{\square_A\right\}$ is quantum theory and at least one of the hyperdecoherence maps must be non-trivial.

Main result

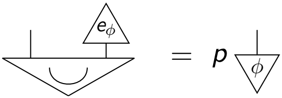
Theorem

There is no post-quantum theory satisfying both causality and purification.

Proof idea

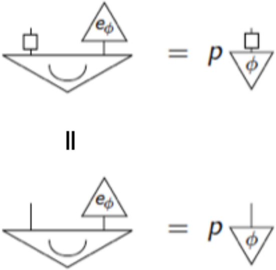
1. Assume toward contradiction that post-quantum theory satisfies causality and purification

2. Can prove that by performing post-quantum measurement on quantum Bell state and post-selecting an outcome, any post-quantum state can be steered to:



Proof idea

1. As quantum states are left invariant by the hyperdecoherence, so are all post-quantum states:



2. Hence hyperdecoherence map is identity—contradiction

Discussion of assumptions

To supersede quantum theory, must abandon purification, causality, or assertion quantum states are fundamentally pure

Discussion of assumptions

1. **Purification**: lack of conservation of information also suggested by Black Hole Information problem

2. **Causality**: indefinite causal structure also suggested by insights from quantum gravity

3. **Pure states**: quantum gravity insights also suggest pure quantum states may become "fuzzy" at Planck length

Thank you!

Pure quantum states are pure

Definition (Information dimension)

The information dimension of a system is the number of states in a maximal set that are all pairwise perfectly distinguishable.

Definition (Strong purification)

1. Every mixed state of system A can be purified to a state of system AA
2. If a state ρ of system A is pure, then it has trivial purifications on all systems. That is, it has a purification ψ on system AB which factorises as $\psi = \rho \otimes \chi$, where χ is a state of B, for all systems B.

Pure quantum states are pure

1. Every quantum pure state is an element of a maximal set of pairwise perfectly distinguishable quantum states. Assume at least one quantum state is mixed in the post-quantum theory, and decompose it as a convex combination of post-quantum states.

2. Every post-quantum state in this decomposition is perfectly distinguishable from any state the original quantum state is distinguishable from.

3. Using strong purification, we show that there must be a pair of perfectly distinguishable post-quantum states in this decomposition. Hence, we have at least an information dimension of $d_Q + 1$, where d_Q is the quantum information dimension.