Protocols for communication over quantum networks

Anurag Anshu[1], Rahul Jain[1,2], Naqueeb Ahmad Warsi[1,3]

1. Centre for Quantum Technologies, NUS, Singapore
2. MajuLab, UMI 3654, Singapore.
3. IIITD, Delhi.

Building blocks for communication over quantum networks
Quantum compression protocols over quantum networks

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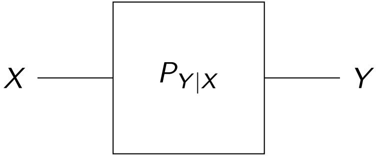
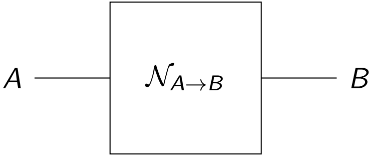
Outline for section 1

Point to point classical channel



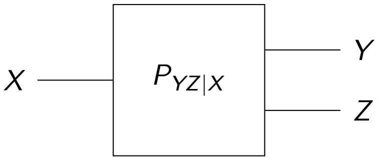
Shannon [Bell. Sys. Tech. Jour., 1948]

Point to point quantum channel



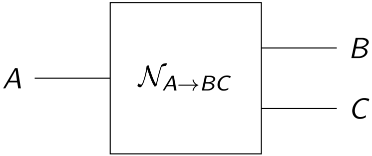
Holevo [IEEE TIT, 1998], Schumacher-Westmoreland [Phys Rev. A., 1997], Lloyd [Phys. Rev. A., 1997], Shor [2002], Devetak [IEEE TIT, 2005], Bennett, Shor, Smolin, Thapliyal [IEEE TIT, 2002].

Broadcast classical channel



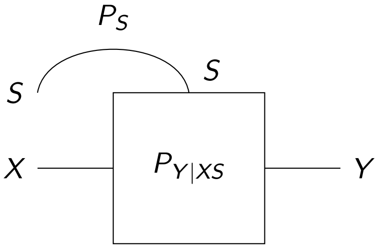
Marton [IEEE TIT, 1979]

Broadcast quantum channel



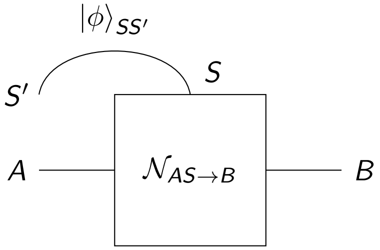
Allahverdyan-Saakian [1998], Yard-Hayden-Devetak [IEEE TIT, 2011], Dupuis' thesis [2010]

Gelf'and-Pinsker classical channel



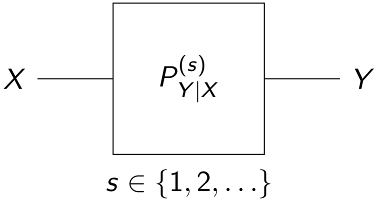
Gelf'and-Pinsker [Prob. Cont. Inf., 1980]

Gelf'and-Pinsker quantum channel



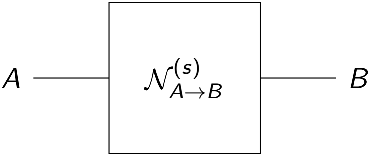
Dupuis' thesis [2010]

Compound classical channel



Wolfowitz [Rat. Mech. Ana.,1959], Blackwell-Breiman-Thomasian [Ann. Math. Stat., 1959]

Compound quantum channel

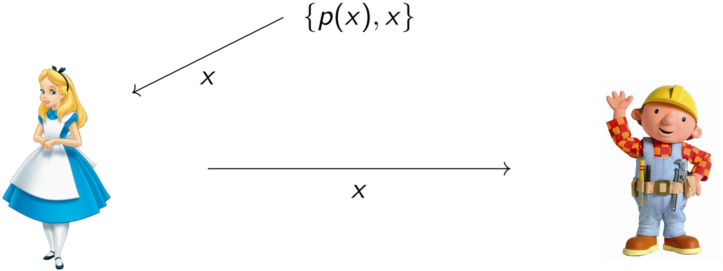


$$s \in \{1, 2, \ldots\}$$

Boche et. al. [2009-2017], Hayashi [Comm. Math. Phys., 2009], Berta-Gharibiyan-Walter [IEEE TIT, 2017]

Outline for section 2

Source compression in classical world



Shannon [Bell Sys. Tech. Jour, 1948]

Source compression in quantum world

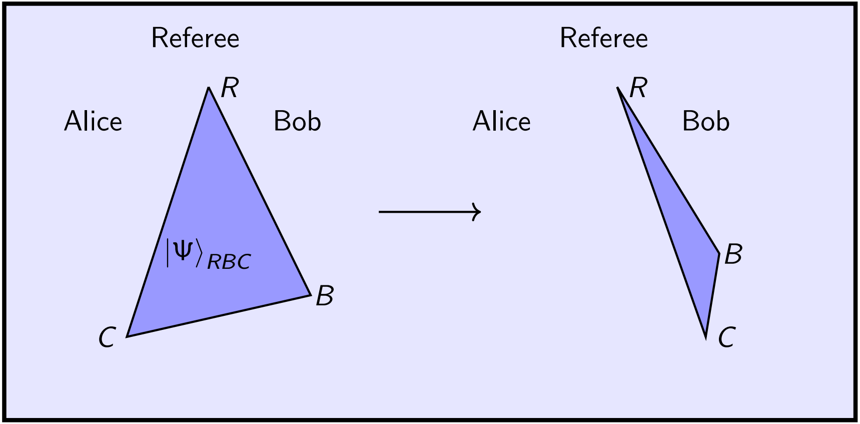
$\{p(x), x\}$

x

$|\Psi_x\rangle$

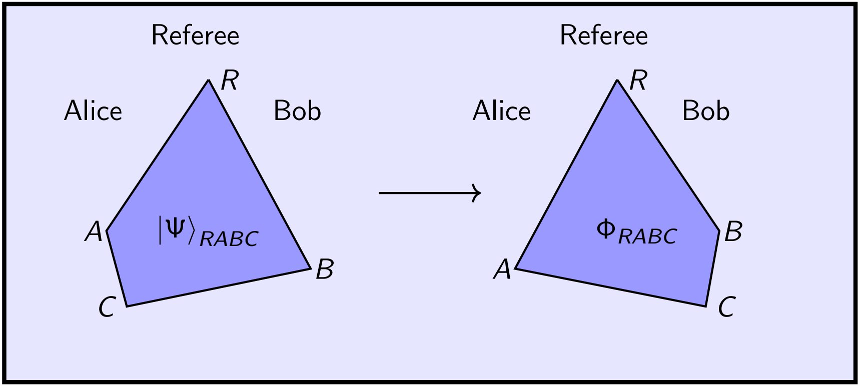
Schumacher [Phys. Rev. A., 1995]

Quantum state merging



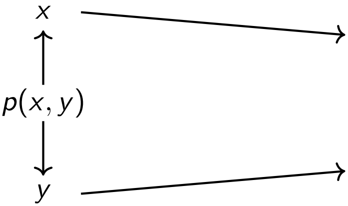
Horodecki, Oppenheim, Winter [Nature, 2005], [Comm. Math. Phys., 2007]

Quantum state redistribution



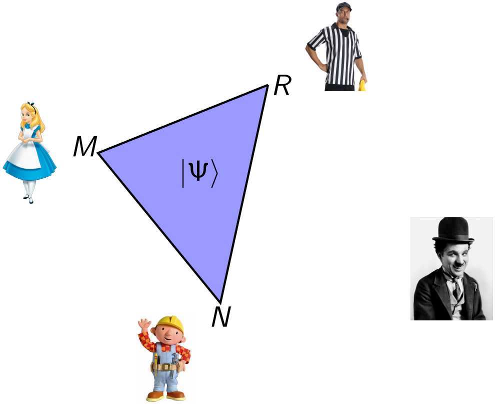
Devetak, Yard [Phys. Rev. Lett., 2008], [IEEE TIT, 2009]

Distributed source compression in classical world



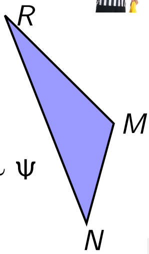
Slepian-Wolf [IEEE TIT, 1973]

Distributed source compression in quantum world



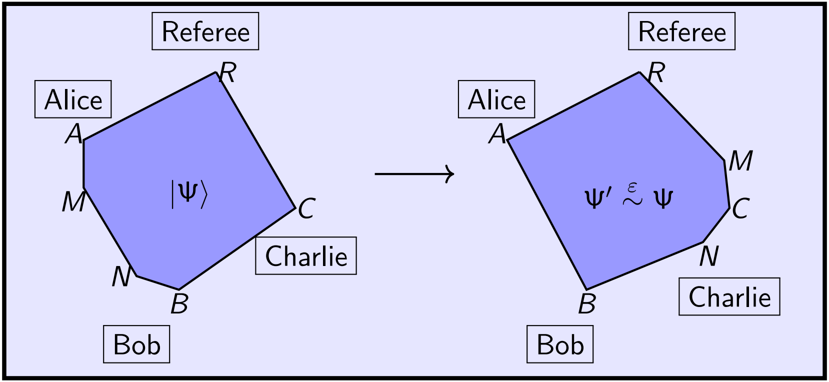
Abeyesinghe, Devetak, Hayden, Winter [Proc. Roy. Soc., 2009], Dutil-Hayden [2010]

Distributed source compression in quantum world...

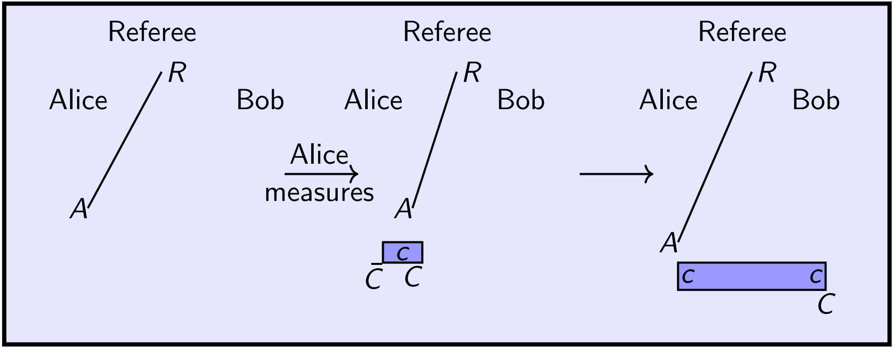


$\Psi' \stackrel{\varepsilon}{\sim} \Psi$

… a generalized quantum Slepian-Wolf

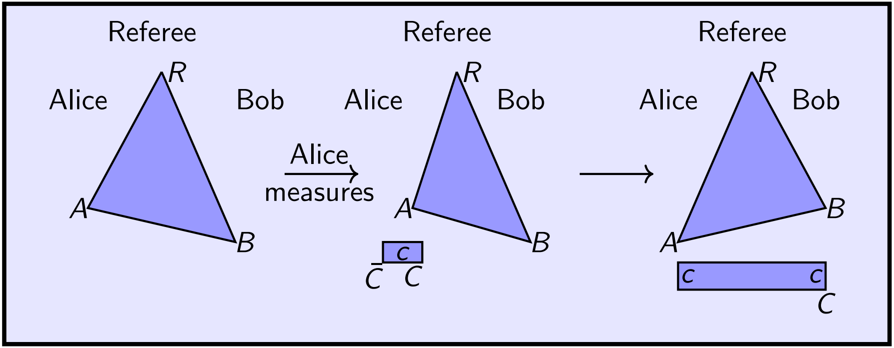


Source compression in classical-quantum world



Winter [Comm. Math. Phys. 2004]

Source compression in classical-quantum world with side information



Wilde, Hayden, Buscemi, Hsieh [J. Phys. A, 2012]

Outline for section 3

Quantum techniques in previous works

- Random codes with pretty good measurement/hypothesis testing
  - Classical capacity of quantum channels [Holevo 1998, Schumacher-Westmoreland 1997, Hayashi-Nagaoka 2002, Renner-Wang 2012].

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Quantum techniques in previous works

- Random codes & pretty good measurement/hypothesis testing
- Decoupling via random unitary
  - Quantum state redistribution [Devetak-Yard 2008, Datta-Hsieh-Oppenheim 2014, Berta-Christandl-Touchette 2014]
  - Quantum state merging [Horodecki-Oppenheim-Winter 2004, Abeyesinghe et. al. 2009, Berta 2009, Berta-Christandl-Renner 2011]
  - Quantum capacity of quantum channels, originally proved by Lloyd, Shor, Devetak. [Hayden-Horodecki-Winter-Yard 2007]
  - Entanglement assisted capacities [Dupuis, Datta-Hsieh, Berta-Gharibian-Walter 2016]
  - Distributed source compression [Dutil-Hayden 2009], [Abeyesinghe et. al. 2009]

Quantum techniques in previous works

- Random codes with pretty good measurement/hypothesis testing
- Decoupling via random unitary
- Super-dense coding argument
  - Entanglement assisted capacity [Bennett-Shor-Smolin-Thapliyal 2001]

Quantum techniques in previous works

- Random codes with pretty good measurement/hypothesis testing
- Decoupling via random unitary
- Super-dense coding argument
- Operator-Chernoff bound
  - Strong converse proof [Ahlswede-Winter 2002]
  - Private capacity and quantum capacity [Devetak 2005]
  - Measurement compression [Winter 2004, Wilde-Hayden-Buscemi-Hsieh 2012]

Quantum techniques in previous works

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Our contribution

- A unified method for achieving all of the above results.
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- A unified method for achieving all of the above results.
- In one-shot setting.
- Near-optimal one-shot communication cost for entanglement assisted capacity of point to point and compound channel.
- Protocol for one-shot quantum state redistribution with smaller communication than previous works
- Protocols for generalized quantum Slepian-Wolf without need for time-sharing.

Outline for section 4

Some basic notions

- Max-relative entropy: $\mathrm{D}_{\mathsf{max}}(\rho\|\sigma) : \inf\{\lambda : \rho \preceq 2^{\lambda}\sigma\}$.
- Another interpretation: $\sigma = 2^{-\lambda}\rho + (1-2^{-\lambda})\rho'$.

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$$\mathrm{D}_{\max}(\rho\|\sigma) \text{ ——————— } \mathrm{D}(\rho\|\sigma) \text{ ——————— } \mathrm{D}_{\mathrm{H}}(\rho\|\sigma)$$

Notations

σ_B

Notations

Notations

Ψ_R

Notations

Ψ_{RB}

Notations

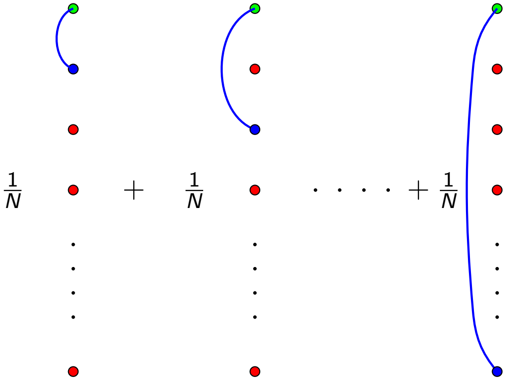
$$\Psi_{B_1} \otimes \sigma_{B_2}$$

Notations

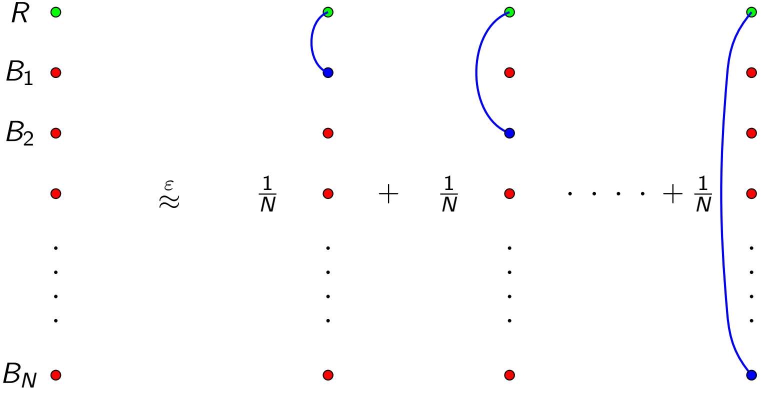
$\frac{1}{2}$  $+$ $\frac{1}{2}$ 

$$\frac{1}{2}\Psi_{B_1} \otimes \sigma_{B_2} + \frac{1}{2}\sigma_{B_1} \otimes \Psi_{B_2}$$

A convex combination of quantum states



Convex-split lemma



If $\log N \geq \mathrm{D}_{\max}(\Psi_{RB} \| \Psi_R \otimes \sigma_B) + \log \frac{1}{\varepsilon}$.

Convex-split Lemma

- A., Devabathini, Jain [Phys. Rev. Lett. 2017, arXiv 2014].
- Gives operational meaning to $\mathrm{D}_{\max}(\Psi_{RB} \| \Psi_R \otimes \sigma_B)$.
- Let $k = \mathrm{D}_{\max}(\Psi_{RB} \| \Psi_R \otimes \sigma_B)$.

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- Let $k = \mathrm{D}_{\max}(\Psi_{RB} \| \Psi_R \otimes \sigma_B)$.
- $$\tau_{RB_1B_2\ldots B_N} = \frac{1}{N}\sum_{j=1}^{N} \Psi_{RB_j} \otimes \sigma_{B_1} \otimes \sigma_{B_2} \ldots \otimes \sigma_{B_{j-1}} \otimes \sigma_{B_{j+1}} \ldots \otimes \sigma_{B_N}$$

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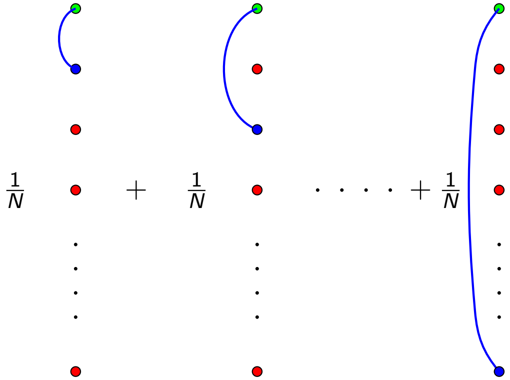
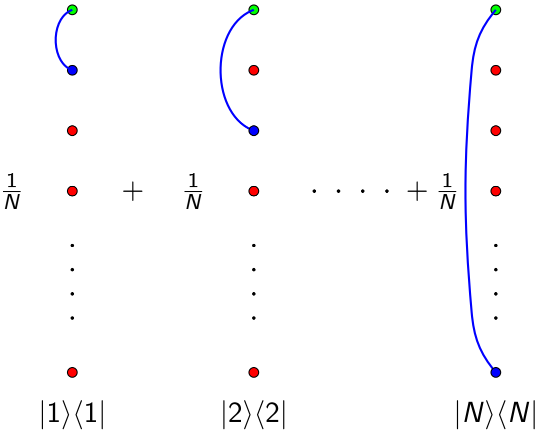
- Then,

$$\mathrm{D}(\tau_{RB_1B_2\ldots B_N} \| \Psi_R \otimes \sigma_{B_1} \otimes \sigma_{B_2} \ldots \otimes \sigma_{B_N}) \leq \frac{2^k}{N}.$$

Position-based decoding



Position-based decoding



Fidelity $1 - \varepsilon$

Position-based decoding

- Distinguishing possible if $N \leq \varepsilon \cdot 2^{\mathrm{D}_{\mathrm{H}}^{\varepsilon}(\Psi_{RB} \| \Psi_R \otimes \sigma_B)}$.

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- Gives operational meaning to $\mathrm{D}_{\mathrm{H}}^{\varepsilon}(\Psi_{RB} \| \Psi_R \otimes \sigma_B)$.
- Proof follows from Hayashi-Nagaoka inequality (Hayashi, Nagaoka [IEEE TIT, 2003]) or Sen's sequential bound (Sen [ISIT, 2012]).
- Alternatively, one can use a sequential version of pretty-good measurement.

Outline for section 5

Task: Quantum state splitting

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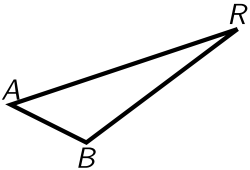
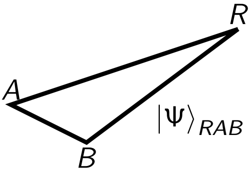


Pre-shared Entanglement

Task: Quantum state splitting



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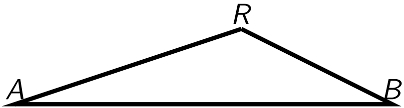
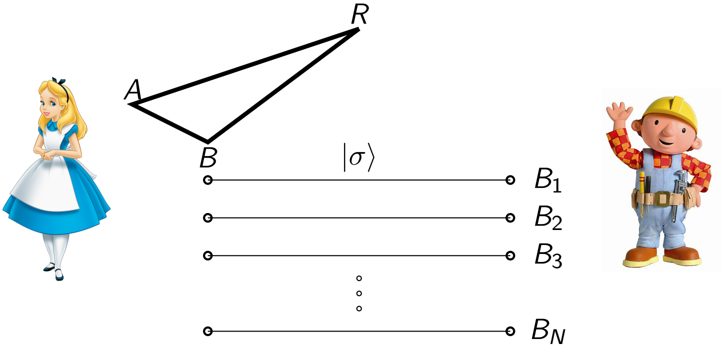
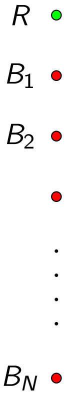


Entanglement

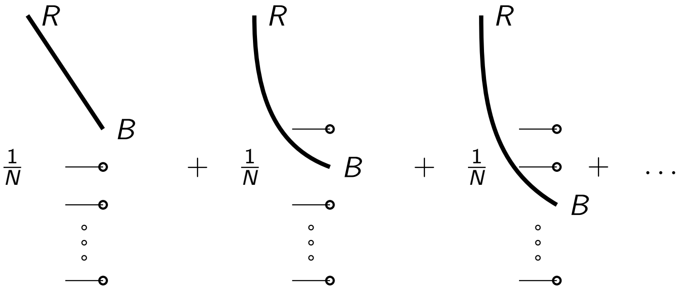
Our protocol: form of pre-shared entanglement



Quantum state with Reference and Bob



Alice sees the following state



$$\log N = \mathrm{D}_{\max}(\Psi_{RB} \| \Psi_R \otimes \sigma_B) + \log \frac{1}{\varepsilon}.$$

Resulting protocol

- If two quantum states are close, there exist equally close purifications. (Uhlmann [Rep. Math. Phys., 1976])

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Resulting protocol

- If two quantum states are close, there exist equally close purifications. (Uhlmann [Rep. Math. Phys., 1976])
- Alice uses this fact. Measures and communicates $\log N = \mathrm{D}_{\max}(\Psi_{RB} \| \Psi_R \otimes \sigma_B) + \log \frac{1}{\varepsilon}$.
- Optimize over σ_B to achieve:

$$\mathrm{I}_{\max}(R : B) = \inf_{\sigma_B} \mathrm{D}_{\max}(\Psi_{RB} \| \Psi_R \otimes \sigma_B).$$

Resulting protocol

- We achieve $\mathrm{I}^{\varepsilon}_{\max}(R : B) + \log \frac{1}{\varepsilon}$ for error 2ε.

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- Quantum state merging $\stackrel{timereverse}{=}$ Quantum state spitting.

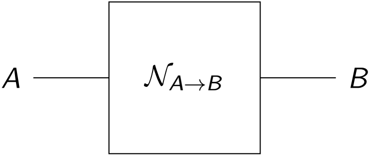
Point to point quantum channel coding

A — $\boxed{\mathcal{N}_{A\to B}}$ — B

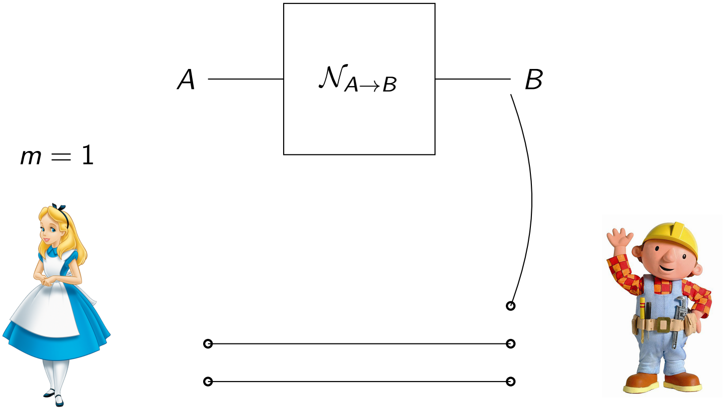
$m \in [1 : 2^R]$

Entanglement

Protocol



Protocol



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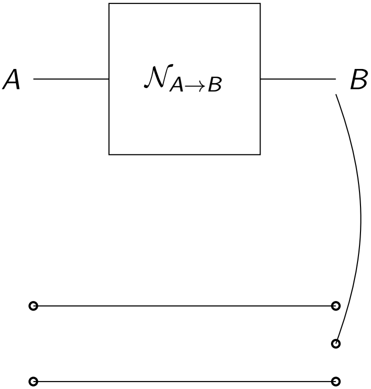
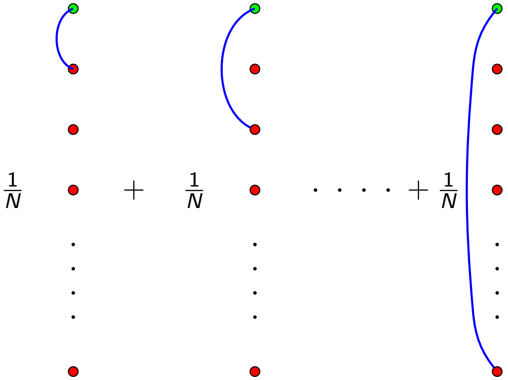


$m = 2$

Quantum state with Bob for uniform input



Achievable rate

- Reliable communication with error $2\varepsilon + \delta$ possible if $R \leq \mathrm{D}_{\mathrm{H}}^{\varepsilon}(\Phi_{A'B} \| \Phi_{A'} \otimes \Phi_B) + O(\log \delta)$, $\Phi_{A'B} = \mathcal{N}_{A \to B}(\Psi_{AA'})$.
  - arXiv:1702.01940

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- A nearly matching upper bound of $\mathrm{D}_{\mathrm{H}}^{2\varepsilon}(\Phi_{A'B} \| \Phi_{A'} \otimes \Phi_B)$ known from (Wehner, Matthews [IEEE TIT, 2012]).
- Error dependence of 2ε can be reduced to $\varepsilon + \delta$, as pointed out by (Wilde, Qi, Wang [2017]).

Achievable rate

- Recovers the result of Bennett, Shor, Smolin, Thapliyal [IEEE TIT, 2002] for entanglement assisted quantum capacity:

$$\max_{\Psi_{AA'}} \mathrm{I}(A' : B)_{\mathcal{N}_{A\to B}(\Psi_{AA'})}.$$

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- Restricting $\Psi_{AA'}$ to classical-quantum states recovers the result of (Wang, Renner [Phys. Rev. Lett., 2012]) in one-shot and of (Schumacher, Westmoreland [Phys Rev A, 1997], Holevo [IEEE TIT, 1998]) in the asymptotic and i.i.d. setting.

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- It is also possible to reduce the amount of pre-shared entanglement to near-optimum in the asymptotic and i.i.d. setting.

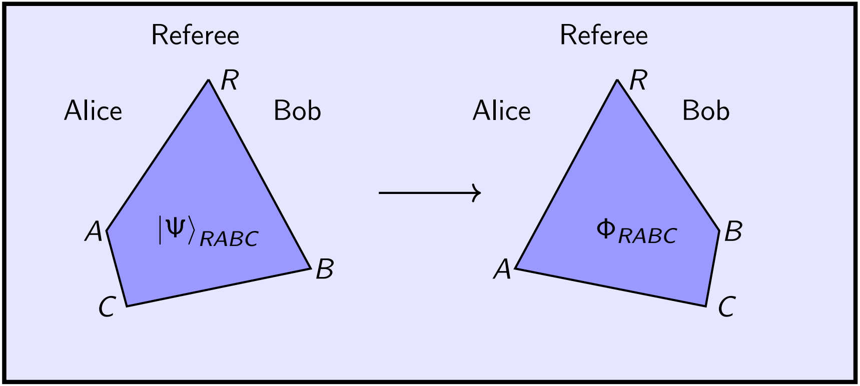
Comparison with earlier works

- Datta, Hsieh [IEEE TIT, 2013] obtained upper bound of the form $H^{\varepsilon}_{min}(A') - H^{\varepsilon^{1/8}}_{max}(A'|B)$ and lower bound of the form $H^{\varepsilon^4}_{min}(A') - H^{\varepsilon^4}_{max}(A'|B)$, using decoupling theorem.

Comparison with earlier works

- Datta, Hsieh [IEEE TIT, 2013] obtained upper bound of the form $H^{\varepsilon}_{min}(A') - H^{\varepsilon^{1/8}}_{max}(A'|B)$ and lower bound of the form $H^{\varepsilon^4}_{min}(A') - H^{\varepsilon^4}_{max}(A'|B)$, using decoupling theorem.
- Datta, Tomamichel, Wilde [Quant. Inf. Proc., 2016] obtained a one-shot version of argument from Bennett, Shor, Smolin, Thapliyal [IEEE TIT, 2002]: $\mathrm{D}^{\varepsilon}_{\mathrm{H}}(\Phi_{A'B} \| \tau_{A'B})$, where $\Phi_{A'B} = \mathcal{N}_{A\to B}(\Psi_{AA'})$ and $\Psi_{AA'}, \tau_{A'B}$ are special class of states.

Quantum state redistribution



Quantum state redistribution

- Introduced by Devetak and Yard [Phys. Rev. Lett., 2008].
- Communication cost captured by $\mathrm{I}(R : C|B)$.
- Relevant to quantum communication complexity (Touchette [STOC, 2015]).

Quantum state redistribution

- One-shot/ second order versions considered in Berta, Christandl, Touchette [IEEE TIT, 2016] and Datta, Hsieh, Oppenheim [Jour. Math. Phys. 2016].

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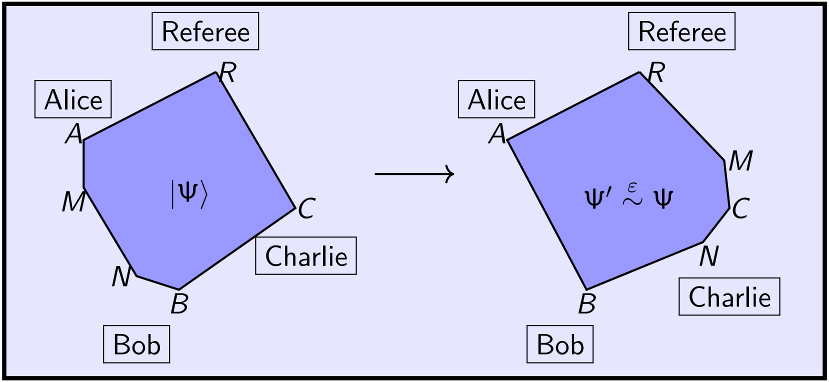
- Communication cost smaller than those obtained in earlier works.

A generalized quantum Slepian-Wolf



Earlier works

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- Used time sharing method to reduce the problem to two-party quantum state splitting problem.
- Cannot be extended to one-shot setting as time sharing method works only in asymptotic and i.i.d. setting.

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- Considered the entanglement consumption of the task.
- Hsieh, Watanabe [ITW, 2015] considered the case where register A is trivial.
- Considered trade-off between entanglement consumption and communication cost.

Our results

- We obtain one-shot 'rate regions' by simple extension of convex-split lemma to bipartite setting.
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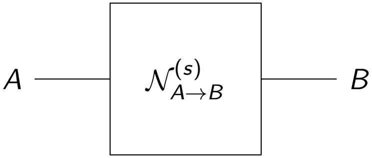
- When register C is non-trivial, problem is to satisfy max-relative entropy constraints on overlapping registers.

Compound quantum channel



$s \in \{1, 2, \ldots\}$

Boche et. al. [2009-2017], Hayashi [Comm. Math. Phys., 2009], Berta-Gharibiyan-Walter [IEEE TIT, 2017]

Earlier works

- Entanglement assisted capacities studied in Berta, Gharibiyan, Walter [IEEE TIT, 2017] and Boche, Jansen, Kaltenstadler [Quant. Inf. Proc., 2017] in the asymptotic and i.i.d. setting.
- Berta, Gharibiyan, Walter [IEEE TIT, 2017] also obtained one-shot bounds in terms of conditional min-max relative entropies.

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- Problem: Given projectors Π_1, Π_2, find a projector Π^* such that for a quantum state ρ
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- Considered by Aaronson [CCC, 2006] and Harrow, Lin, Montanaro [SODA, 2017].

Quantum OR bound

- The result in Harrow, Lin, Montanaro [SODA, 2017] says that if $Tr(\Pi_1 \rho)$ or $Tr(\Pi_2 \rho)$ is ≈ 1, then $Tr(\Pi^* \rho)$ is a constant $\approx 1/7$.

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- We need stronger guarantee for one-shot purpose, but do not need efficient construction.
- Use Jordan's lemma to construct projector Π^* such that if $Tr(\Pi_1\rho)$ or $Tr(\Pi_2\rho)$ is ≈ 1, then $Tr(\Pi^*\rho)$ is ≈ 1.

Results

- We show the following achievability with error $\varepsilon + \delta$

$$\max_{\Psi_{AA'}} \min_i \mathrm{I}_H^{\varepsilon}(B : A')_{\mathcal{N}^i_{A\to B}(\Psi_{AA'})} - O(\log s \cdot \log(\log s/\delta)).$$

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$$\max_{\Psi_{AA'}} \min_i \mathrm{I}_H^{\varepsilon}(B : A')_{\mathcal{N}^i_{A\to B}(\Psi_{AA'})}.$$

- Additive factor of $O(\log s)$ is present in classical case as well.

Conclusion

- We have discussed techniques that allows one-shot source compression and channel coding in large class of quantum networks.
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- Convex-split technique allows for near optimal characterization of expected communication cost of distributed tasks (A., Garg, Harrow, Yao [2016]).
- Applicable to resource theory (yesterday's talk) and connected to port-based teleportation (earlier talk).

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- Application to complex quantum networks will require a solution to the problem of satisfying max-entropy constraints on overlapping registers.

Last slide

Thank you!

Outline for section 6

Proof of convex-split lemma

- A simple fact:
  - Let $\rho = \sum_i p_i \rho_i$. Then

$$\mathrm{D}(\rho \| \theta) = \sum_i p_i (\mathrm{D}(\rho_i \| \theta) - \mathrm{D}(\rho_i \| \rho)).$$

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$$\mathrm{D}(\rho\|\theta) = \sum_i p_i(\mathrm{D}(\rho_i\|\theta) - \mathrm{D}(\rho_i\|\rho)).$$

- Recall: $\tau = \frac{1}{N}\sum_{j=1}^{N} \Psi_{RB_j} \otimes \sigma_{-j}$.
  - $\sigma_{-j} := \sigma_{B_1} \otimes \sigma_{B_2} \ldots \otimes \sigma_{B_{j-1}} \otimes \sigma_{B_{j+1}} \ldots \otimes \sigma_{B_N}$.
  - $\sigma := \sigma_{B_1} \otimes \sigma_{B_2} \ldots \otimes \sigma_{B_N}$

Proof

- $\mathrm{D}(\tau \| \Psi_R \otimes \sigma) =$
  - $\frac{1}{N}\sum_i (\mathrm{D}(\Psi_{RB_j} \otimes \sigma_{-j} \| \Psi_R \otimes \sigma) - \mathrm{D}(\Psi_{RB_j} \otimes \sigma_{-j} \| \tau))$

Proof

- $\mathrm{D}(\tau \| \Psi_R \otimes \sigma) =$
  - $\frac{1}{N}\sum_i (\mathrm{D}(\Psi_{RB_j} \| \Psi_R \otimes \sigma_{B_j}) - \mathrm{D}(\Psi_{RB_j} \otimes \sigma_{-j} \| \tau))$

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- $\mathrm{D}(\tau \| \Psi_R \otimes \sigma) =$
  - $\mathrm{D}(\Psi_{RB} \| \Psi_R \otimes \sigma_B) - \frac{1}{N} \sum_i \mathrm{D}(\Psi_{RB_j} \otimes \sigma_{-j} \| \tau)$

Proof

- $\mathrm{D}(\tau \| \Psi_R \otimes \sigma) \leq$
    - $\mathrm{D}(\Psi_{RB} \| \Psi_R \otimes \sigma_B) - \frac{1}{N} \sum_i \mathrm{D}(\Psi_{RB_j} \| \tau_{RB_j})$

Proof

- $\mathrm{D}(\tau \| \Psi_R \otimes \sigma) \leq$
    - $\mathrm{D}(\Psi_{RB} \| \Psi_R \otimes \sigma_B) - \mathrm{D}(\Psi_{RB_1} \| \tau_{RB_1})$

Proof

- $\mathrm{D}(\tau \| \Psi_R \otimes \sigma) \leq$
  - $\mathrm{D}(\Psi_{RB} \| \Psi_R \otimes \sigma_B) - \mathrm{D}(\Psi_{RB_1} \| \tau_{RB_1})$
- $\tau_{RB_1} =$
  - $\frac{1}{N}\Psi_{RB_1} + (1 - \frac{1}{N})\Psi_R \otimes \sigma_{B_1}$.

Proof

- $\mathrm{D}(\tau \| \Psi_R \otimes \sigma) \leq$
  - $\mathrm{D}(\Psi_{RB} \| \Psi_R \otimes \sigma_B) - \mathrm{D}(\Psi_{RB_1} \| \tau_{RB_1})$
- $\tau_{RB_1} \preceq$
  - $\frac{2^k}{N} \Psi_R \otimes \sigma_{B_1} + (1 - \frac{1}{N}) \Psi_R \otimes \sigma_{B_1}$.

Proof

- $\mathrm{D}(\tau \| \Psi_R \otimes \sigma) \leq$
  - $\mathrm{D}(\Psi_{RB} \| \Psi_R \otimes \sigma_B) - \mathrm{D}(\Psi_{RB_1} \| \tau_{RB_1})$
- $\tau_{RB_1} \preceq$
  - $(1 + \frac{2^k}{N})\Psi_R \otimes \sigma_{B_1}$.

Proof

- $\mathrm{D}(\tau \| \Psi_R \otimes \sigma) \leq$
  - $\mathrm{D}(\Psi_{RB} \| \Psi_R \otimes \sigma_B) - \mathrm{D}(\Psi_{RB_1} \| \Psi_R \otimes \sigma_B) + \log(1 + \frac{2^k}{N})$
- Done.