Entanglement Renormalization, Quantum Error Correction, and Bulk Causality

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IBM → Stanford

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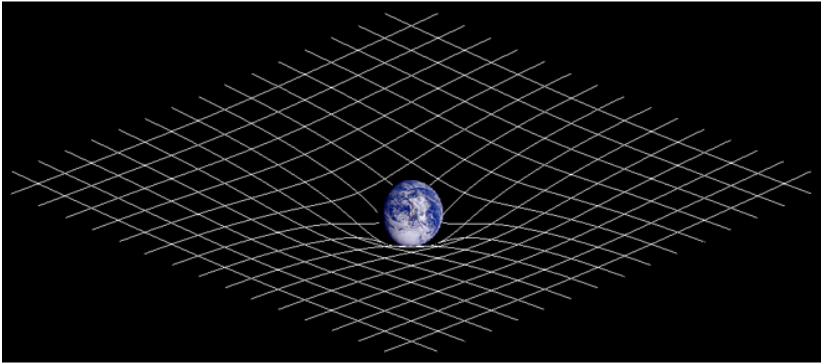
Joint work with Michael J. Kastoryano(NBIA → Cologne)

Space and Time



$$\mathcal{H} = \mathcal{H}_{\vec{x}_1} \otimes \mathcal{H}_{\vec{x}_2} \otimes \cdots$$

Causality

Einstein Causality:

$$[O_{\vec{x}_1}, O_{\vec{x}_2}] = 0 \quad \text{if} \quad (\vec{x}_1 - \vec{x}_2)^2 > 0.$$

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In the **quantum** theory of gravity, what happens? The safest assumption would be this:

$$\langle \psi | [O_{\vec{x}_1}, O_{\vec{x}_2}] | \psi' \rangle \approx 0 \quad \text{if} \quad (\vec{x}_1 - \vec{x}_2)^2 \gg 0$$

for some ψ, ψ' in some set S

AdS/CFT correspondence

The Large N Limit of Superconformal field theories and supergravity

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Abstract

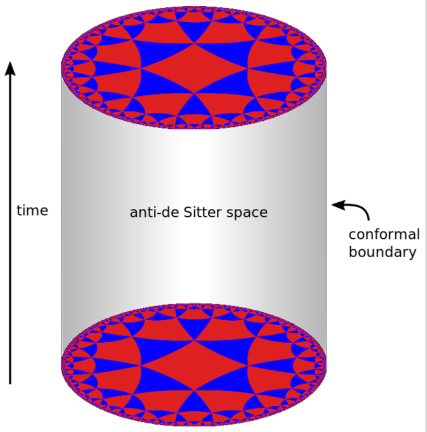
We show that the large N limit of certain conformal field theories in various dimensions include in their Hilbert space a sector describing supergravity on the product of Anti-deSitter spacetimes, spheres and other compact manifolds. This is shown by taking

But what does it mean?

Disclaimer

AdS/CFT vs Maxwell's equation

Quantum gravity in a box



$\phi(x, r, t) \rightarrow \phi'_{\text{CFT}}(x, t)$ at the boundary.
Solve Einstein's equation.

EM field in a box

$E_{\text{tangent}} = 0$ at the boundary.
Solve $\nabla^2 \vec{E} = 0$.

HKLL formula

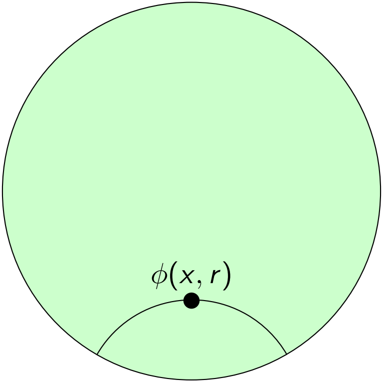
Hamilton, Kabat, Lifschytz, Lowe(2006)

$$\phi(x, r, t) = \int dx' dt' K(x', t'|r, x, t)\phi_0(x', t')$$

1. $K(x', t'|r, x, t)$: Smearing **function**
2. $\phi_0(x', t')$: **Operators**
3. This construction ensures that low-order correlators between $\phi(x, r, t)$ are exactly equal to the gravity prediction.
4. Funny fact! : There is more than one way to ensure (3).

HKLL formula

For a fixed t...



HKLL formula

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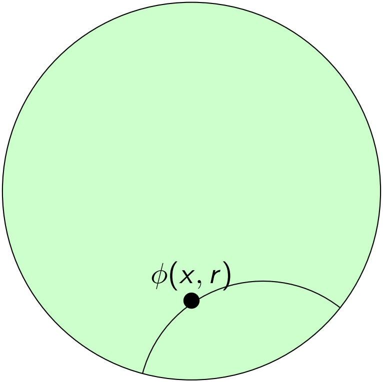
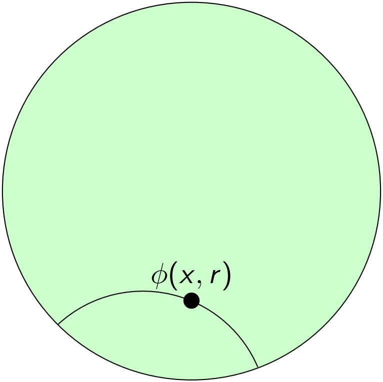


HKLL formula

For a fixed t...



Resolution : quantum error correcting code

Almheiri, Dong, Harlow(2015) proposed that this formula

$$\phi(x,r,t) = \int dx' dt' K(x',t'|r,x,t)\phi_0(x',t'))$$

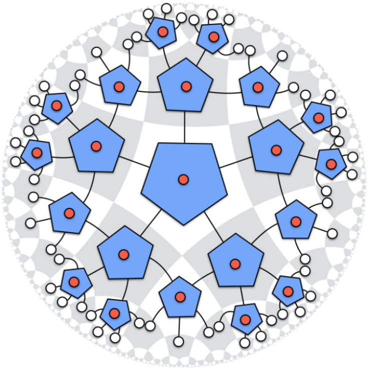
- should not be thought as an operator equality,
- but rather as an equality that holds in a certain subspace.
- In particular, there must be a family of subspaces associated to each (x, r) (at fixed t).

Subspace = Quantum error correcting code

- ADH specified exactly what kind of error correction properties these codes must satisfy.

Holographic quantum error correcting codes

Pastawski, Yoshida, Harlow, Preskill (2015)



- For each bulk site/sites, one can assign a QECC.
- Deeper you go, better protected against boundary erasure.
- Aspects of entanglement wedge reconstruction realized.

- Pluperfect tensor network(2015), Random tensor network (2016), Subalgebra holographic code(2016), **Dynamics for holographic codes (2017)**, Space-time random tensor network(2018)...

But where do they come from?

Main result

Colloquially

Holographic quantum error correcting codes emerge in the ground state of CFT.

- As a corollary, we can specify a set of operators and states such that their causal relation resembles that of a physical space in one higher dimension.

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A bit more accurately

There is a family of QECCs, defined in terms of the CFT data, that approximately reproduces the error correction properties of the holographic QECCs.

- As a corollary, we have an emergent Lieb-Robinson type bound at low energy. It establishes the locality of the dynamics in one higher dimension.

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A bit more accurately

There is a family of QECCs, defined in terms of the CFT data, that approximately reproduces the error correction properties of the holographic QECCs.

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Assumptions

1. Ground states of CFTs are described by the multi-scale entanglement renormalization ansatz(MERA).
2. (Unnecessary) Simplifying assumption : MERA is translationally invariant, and scale invariant.
3. Lowest scaling dimension is positive.

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2.
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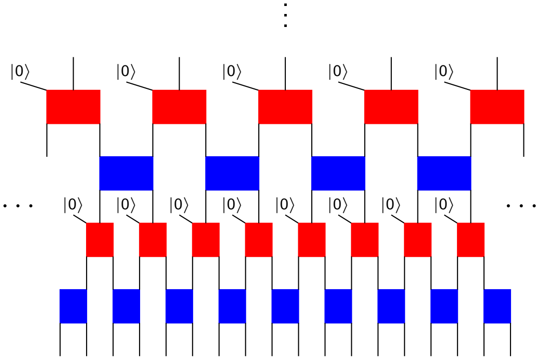

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Operators	Δ MERA($\chi = 22$)	Δ Correct value
σ	0.124997	0.125
ϵ	1.0001	1

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σ_1	0.1339	$2/15 = 0.1\hat{3}$
σ_2	0.1339	$2/15 = 0.1\hat{3}$
ϵ	0.8204	0.8
Z_1	1.3346	$4/3 = 1.\hat{3}$
Z_2	1.3351	$4/3 = 1.\hat{3}$

Pfeifer, Evenbly, Vidal(2009)

Also, see a related talk by Volkher Scholz yesterday...

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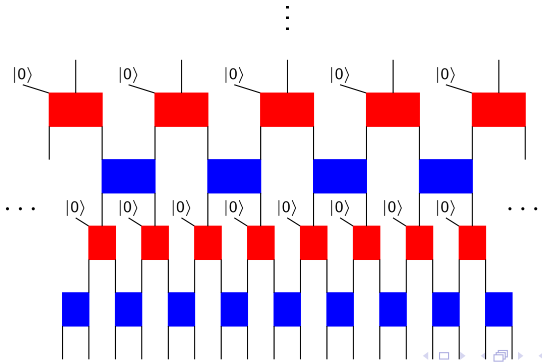

Assumptions

1. The ground state can be described by the multi-scale entanglement renormalization ansatz(MERA).
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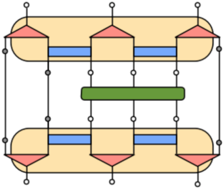


$= \Phi(O)$

- Φ is unital by construction.
- $-\log \lambda_1 > 0$, where λ_1 is the second largest modulus of the eigenvalues of Φ.

How do we show it?

Colloquially

Holographic quantum error correcting code emerges in the ground state of CFT. As a corollary, we can specify a set of operators and states such that their causal relation resembles that of a physical space in one higher dimension.

Key logical steps

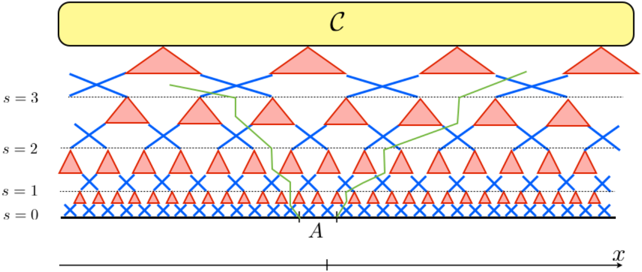
1. Subspace is defined. (This is formally a code.)
2. Correctability condition formulated.
3. Positive scaling dimension $\rightarrow$ correctability of certain regions.

Subspace defined

Promote the isometry at (x, r) to a unitary.

- Before: $\cdots V_{(x,r)} \cdots$ is a state.
- After: $\cdots U_{(x,r)} \cdots$ is an isometry from a bounded-d Hilbert space to the physical Hilbert space.

* For simplification, we just change every $V_{(x,r)}$ at fixed r to some $U_{(x,r)}$.



Quantum error correction duality

Local correctability $\leftrightarrow$ Local decoupling

Colloquially

For a subspace of $\mathcal{H}_A \otimes \mathcal{H}_B \otimes \mathcal{H}_C$, erasure of A can be corrected from a channel acting on B if and only if A is decoupled from CR.
* R: purifying space

More specifically, these two things are **equal.**

$$\text{Local correctability: } \inf_{\mathcal{R}_B^{AB}} \sup_{\rho^{ABCR}} \mathfrak{B}(\mathcal{R}_B^{AB}(\rho^{BCR}), \rho^{ABCR})$$

- $\mathfrak{B}(\cdot, \cdot)$: Bures distance
- $\mathcal{R}_B^{AB}$: local correction channel

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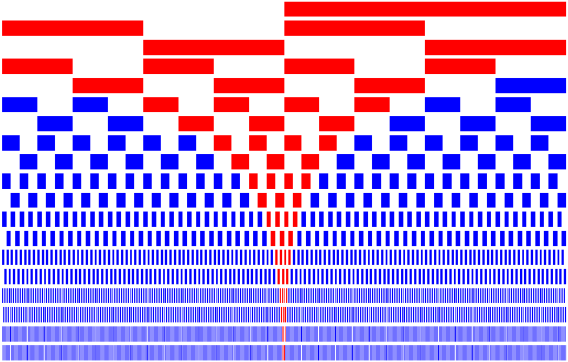
$$\textbf{Local decoupling: } \min_{\omega^A} \sup_{\rho^{ABCR}} \mathfrak{B}(\omega^A \otimes \rho^{CR}, \rho^{ACR}))$$

[Flammia, Haah, Kastoryano, K](2017)

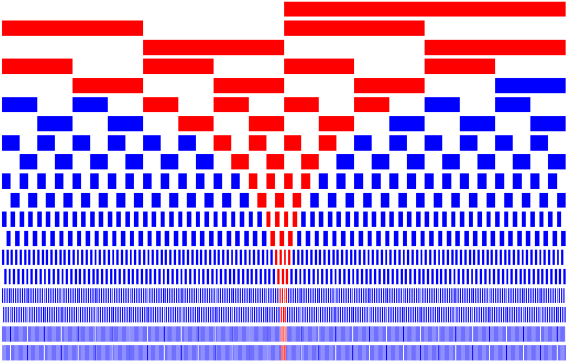
Decoupling Condition : Picture

Support Size: 2



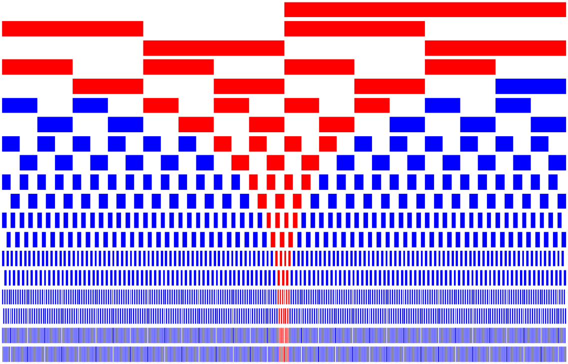
Decoupling Condition : Picture

Support Size: 4



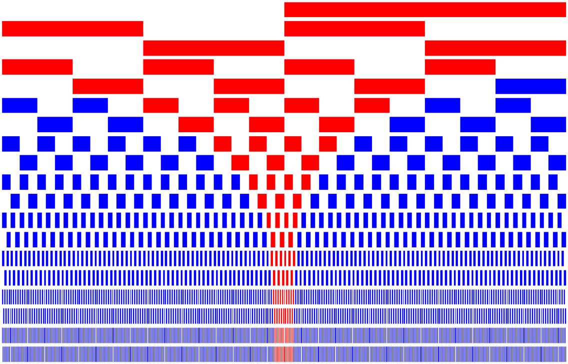
Decoupling Condition : Picture

Support Size: 8



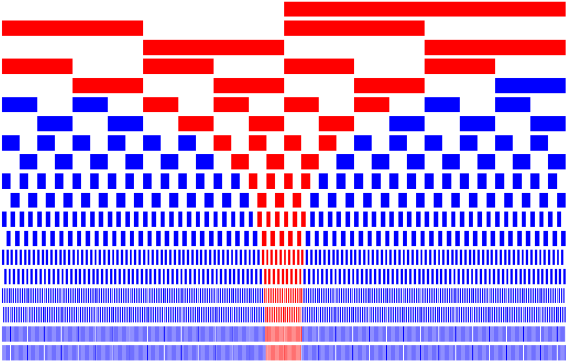
Decoupling Condition : Picture

Support Size: 16



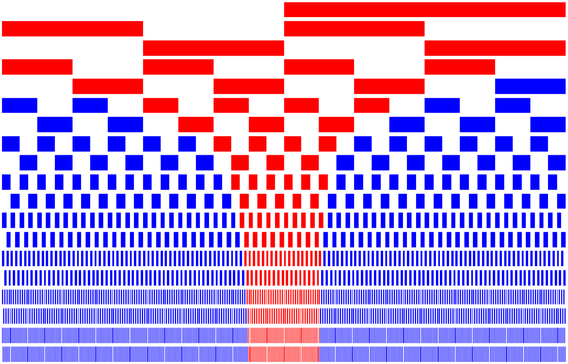
Decoupling Condition : Picture

Support Size: 32



Decoupling Condition : Picture

Support Size: 64



Decoupling Condition : Words

- Schrödinger : $\langle O \rangle = \left(\langle 0 | V_1^\dagger U_1^\dagger \cdots V_n^\dagger U_n^\dagger \right) O \left(U_n V_n \cdots U_1 V_1 | 0 \rangle \right)$
- Heisenberg : $\langle O \rangle = \langle 0 | \left(V_1^\dagger U_1^\dagger \cdots V_n^\dagger U_n^\dagger O U_n V_n \cdots U_1 V_1 \right) | 0 \rangle$

In the Heisenberg picture,

1. The support of the operator contracts exponentially.
2. Then, the norm of the non-unital part of the operator contracts exponentially.
3. Eventually, the operator is mapped to an approximate identity operator.

Decoupling Condition : Equations

Step 1: def. of trace norm

$$\|\rho^{ACR} - \rho^{A} \otimes \rho^{CR}\|_1 = \sup_{\|M_{ACR}\| \leq 1} \mathrm{Tr}[M_{ACR}(\rho^{ACR} - \rho^{A} \otimes \rho^{CR})]$$

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Step 2: MERA identity

$$\mathrm{Tr}[M_{ACR}(\rho_r^{ACR} - \rho_r^{A} \otimes \rho_r^{CR})] = \mathrm{Tr}[\Phi_r(M_{ACR})(\rho_{r-1}^{ACR} - \rho_{r-1}^{A} \otimes \rho_{r-1}^{CR})]$$

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Step 3: Under Φ_r, the support of M_{ACR} **strictly** contracts until its support on the physical space becomes $\mathcal{O}(1)$. (Follows from the circuit geometry.)
Step 4: Under Φ_r, norm of non-identity local operator in the physical space **strictly** contracts. (Positive scaling dimension)

Application : Emergent lightcone in "scale"

Lieb-Robinson bound

In a locally interacting system, there is a constant "speed of light," such that correlations outside the lightcone decays exponentially.

$$\|[O_A, O_B(t)]\| \leq ce^{(v|t| - \mathrm{dist}(A,B)/\xi)}$$

Application : Emergent lightcone in "scale"

Theorem

For any two states $|\psi\rangle, |\psi'\rangle \in \mathcal{C}_s$, for a local operator O_1 and a logical operator O_2,

$$|\langle\psi| [O_1(t), O_2] |\psi'\rangle| \leq c'(v|t| + \mathcal{O}(s))e^{-\alpha s}$$

Interesting facts

- Generally $[O_1, O_2] \neq 0$.
- The Hamiltonian is assumed to be locally interacting in the physical Hilbert space, but it is generically nonlocal with respect to the subfactors labeled by (x, r).
- Only holds in a **subspace**, not in the entire Hilbert space.

* See also arXiv:1705.01728 [Qi and Yang (2017)]

Application : Tradeoff bounds

In the $|\lambda_2| \to 0$ limit,

$$d \leq C \left(\frac{n}{k} \right)^{\alpha},$$

- In our case, $\alpha \approx 0.63$.
- In holography, in a "similar" limit, $\alpha \approx 0.78$. [Pastawski and Preskill (2016)]

Summary

Summary

1. Holographic quantum error correcting codes (in some sense) appear naturally in low energy states of CFT.
2. Local dynamics can emerge out of nonlocal Hamiltonian, once restricted to a particular subspace.
3. Tradeoff bound derived.

Outlook

Outlook

- More quantum error correcting codes that were there in hindsight?
  - Brandao et al. (2017)
- Other mechanisms for causality?
- Constraining holography from quantum error correction?
  - Pastawski and Preskill(2016)
- A more refined bound in terms of OPE coefficients?
- Time evolution by a non-integrable Hamiltonian can be interpreted as a black hole with a long throat. Can we understand how causality arises in this throat?