Entanglement distillation between solid-state quantum network nodes

N. Kalb, A. A. Reiserer, P. C. Humphreys, J. J. W. Bakermans, S. J. Kamerling, N. H. Nickerson, S. C. Benjamin, D. J. Twitchen, M. Markham, R. Hanson

The impact of future quantum networks hinges on high-quality quantum entanglement shared between network nodes. Unavoidable imperfections necessitate a means to improve remote entanglement by local quantum operations. We realize entanglement distillation on a quantum network primitive of distant electron-nuclear two-qubit nodes. The heralded generation of two copies of a remote entangled state is demonstrated through single-photon-mediated entangling of the electrons and robust storage in the nuclear spins. After applying local two-qubit gates, single-shot measurements herald the distillation of an entangled state with increased fidelity that is available for further use. The key combination of generating, storing, and processing entangled states should enable the exploration of multiparticle entanglement on an extended quantum network.

Quantum network nodes

Our quantum network nodes comprise an NV electron spin in diamond as a communication qubit and a nearby carbon-13 nuclear spin as a memory qubit. The diamond chips holding these qubits reside in individual closed-cycle cryostats ($T = 4$ K) that are separated by 2 m (30). The electron spin state is manipulated by means of amplitude-shaped microwave pulses. Electron spin decoherence occurs on time scales exceeding a millisecond and has negligible impact on the presented results. Spin-selective resonant optical excitation enables high-fidelity initialization and single-shot nondemolition readout of the electron spin (31), as well as generation of spin-photon entanglement for connecting distant nodes (20) (see also fig. S4). We use nuclear spins with intrinsic dephasing times $T_2^*$ of 3.4(1) ms and 16.2(3) ms for node A and B, respectively (30). We implement universal control on each of these nuclear spin qubits by exploiting its hyperfine coupling to the electron spin through recently developed dynamical-decoupling-based gate sequences (32). This complete quantum toolbox enables the implementation of all four steps in the distillation protocol.

Figure IC shows the compilation of the full gate circuit into the quantum control operations of our platform. This compilation maximizes the repetition rate and minimizes the number of local quantum gates after the generation of the first remote state. In particular, by initializing the memory qubit at the start of the protocol, we can implement the SWAP operation with just two conditional quantum gates instead of the three that would be required for arbitrary input states (30). Note that our SWAP implementation maps the communication qubit energy eigenstates onto memory superposition states $|+X\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$.

To benchmark the performance of the local quantum logic, we execute a combination of the SWAP (orange box in Fig. IC) and the gates of the distillation step (purple box in Fig. IC) to generate a maximally entangled Bell state between the communication and memory qubits (see Fig. 2A). The full density matrix of the resulting two-qubit state is reconstructed via quantum state tomography [QST; see (30) for further details]. We find a fidelity with the ideal Bell state of 0.96(1) [0.98(1)] for node A [B],...
indicating high-quality operations in both nodes (Fig. 2B).

**Robust storage of quantum information**

A critical capability for the network nodes is the robust storage of quantum information in the memories while the communication qubits are used to generate remote entangled states. This requires the memory qubits to have long coherence times and be resilient to operations on the communication qubit. The generation of remote entanglement, in particular, poses two challenges as its probabilistic nature means that an a priori unknown number of attempts is required.

First, each failed entangling attempt leaves the communication qubits in an unknown state, which necessitates a reset by optical pumping. This reset is a stochastic process which, in combination with the always-on hyperfine interaction between communication and memory qubit, causes dephasing of stored memory states (27, 33).

Memories with a small parallel hyperfine coupling are used so that the precession frequency of these memories exhibits only a weak dependency $\Delta \omega$ on the state of the communication qubit during the repumping process of a few hundred nanoseconds [$\Delta \omega = 2\pi \times 22.4(1) \text{kHz}$ and $\Delta \omega = 2\pi \times 26.3(1) \text{kHz}$]. Decoherence via the perpendicular hyperfine component is suppressed by an applied magnetic field of about 40 mT.

Second, the interaction between communication and memory qubits leads to a deterministic phase shift $\varphi_{A/B}$ on the memory per entangling attempt. Because it is unknown which entangling attempt will herald success, real-time feedback on the memory is required to compensate for these phase shifts before the final two-qubit gate of Fig. 1C is applied. In addition, the feedback must preserve the coherence of the communication qubit as it holds the second copy of the raw entangled state. We realize such real-time feedback through dynamical decoupling of the electron spin synchronized with the nuclear spin precession frequency that induces an electron-state–independent phase gate on the memory (32). At each node, the number of entangling attempts until success $N$ is tracked by a microprocessor that terminates the subsequent decoupling sequence when the desired rotation $R_\pi(\varphi_{A/B})^N$ has been applied. Ideally, this leaves the memory with the desired phase relation regardless of the number of entangling attempts. We calibrate and verify this feedback at each node separately (Fig. 3, A and B) and measure a negligible effect on the memory state fidelity while the state of the communication qubit is preserved as desired.

With this feedback realized, we investigate the robustness of the memory as a function of the elapsed entangling attempts. The memory is initialized in one of the six cardinal states of the Bloch sphere ($|0\rangle$, $|1\rangle$, $|X\rangle$, and $|Y\rangle = (|0\rangle + i|1\rangle)/\sqrt{2}$); a number of entangling attempts are then executed, followed by phase feedback and measurement of the relevant memory expectation value (Fig. 3C). Dephasing-sensitive states $|X\rangle$, $|Y\rangle$ decay with $1/e$ values of $273.5(4)$ $272.4(4)$ entangling attempts in node A (node B), whereas the energy eigenstates $|0\rangle$, $|1\rangle$ are preserved with high fidelity, as expected. The memories thus provide faithful storage during remote entangling attempts.

**Experimental entanglement distillation**

With local control and storage in place, we now turn toward the execution of the full distillation protocol (Figs. S1 and S2). Following Campbell’s protocol (34), we generate the remote states that provide the resources for distillation by first initializing both communication qubits in a superposition with variable angle $\theta$, $|0\rangle = \sin \theta |0\rangle - i \cos \theta |0\rangle$–$|0\rangle$. Subsequent optical excitation for state $|0\rangle$ and overlap of the emission of both communication qubits on a beam splitter (see steps 1 and 3 of Fig. 1C and histograms in fig. S3) generate the raw remote state $|\psi_{\text{raw}}\rangle$ if a single photon is detected (34). For equal and small detection probabilities for both nodes and negligible dark counts, $|\psi_{\text{raw}}\rangle$ reads

$$|\psi_{\text{raw}}\rangle = (1 - \sin \theta) |\psi_{+}\rangle + \sin \theta |0\rangle/0.0$$

The states $|\psi_{+}\rangle = (|0\rangle + e^{i\phi}|1\rangle)/\sqrt{2}$ are entangled states, with a relative phase depending on which detector clicked ($\pm$) and an additional
internal phase $\phi$ due to the unknown path length between both emitters and the beam splitter. The fraction of the nonentangled admixture $|0,0 \rangle$, $|0,1 \rangle$, and $|1,0 \rangle$ can be directly controlled through the choice of the initial communication qubit state $|\theta\rangle$; note that the choice of $|\theta\rangle$ also affects the probability of successful entanglement generation (scaling as $\sin^2\theta$). We next swap the raw state onto the memories such that the communication qubit is free for another round of remote state generation (step 2 in Fig. 1C). Once a second state is successfully generated (step 3), we apply a conditional quantum gate within each node and read out the communication qubits in a single shot. Owing to the quantum nondemolition nature of this readout, the memory qubits do not experience additional dephasing during this step (3I). Readout of the communication qubit projects the memories into one of four states depending on the readout results (3O):

$$
(0_A, 0_B) : \frac{1}{2} \cos^4\theta U |\psi^{+}\rangle |\psi^{+}\rangle U^\dagger |0,0\rangle + |1,1\rangle |1,1\rangle U^\dagger
$$

$$
(0_A, 1_B) : \frac{1}{2} \sin^2\theta \cos^2\theta U |0,1\rangle |0,1\rangle + |1,0\rangle |1,0\rangle U^\dagger
$$

$$
(1_A, 0_B) : \frac{1}{2} \sin^2\theta \cos^2\theta U |0,1\rangle |0,1\rangle + |1,0\rangle |1,0\rangle U^\dagger
$$

$$
(1_A, 1_B) : \frac{1}{2} \sin^2\theta \cos^2\theta U |0,1\rangle |0,1\rangle + |1,0\rangle |1,0\rangle U^\dagger + \frac{1}{2} \cos^4\theta U |\psi^{+}\rangle |\psi^{+}\rangle U^\dagger
$$

with the relative phase of the final Bell state given by the photon detection signature, i.e., the photons in steps 1 and 3 were detected in the same (+) or in different (−) output ports. Notably, the protocol is agnostic to correlated dephasing of the raw states and is therefore only sensitive to optical path-length drifts that occur within an individual run of the protocol (see Fig. S7) (34). This is in stark contrast to probabilistic single-photon protocols (22, 23, 29, 35) that require path-length stabilization over the full course of data acquisition.

The experimental implementation of entanglement generation requires that the communication qubits’ optical transitions are kept on resonance despite shot-to-shot fluctuations and long-term drifts of the respective local charge environments. An automatic feedback loop and resonance search routine is used to compensate for charge jumps such that the experiment is push-button and runs without human intervention. To further optimize the data rate, the number of remote entangling attempts is bounded to 1000 for step 1 and up to 500 rounds for step 3, leading to event rates (i.e., two remote states were successfully generated) of around 10 Hz. These bounds are a compromise between maximizing the success probability (favoring more attempts) and minimizing effects of drifts and of memory decoherence (favoring fewer attempts).

**Distillation results**

We start by running the complete protocol using $\theta = \pi/6$ and perform full quantum state tomography on the distilled state. This way, using the complete information obtained on the resulting output, we can verify whether the protocol works as desired. Figure 4A shows the resulting data for a maximum of 50 entangling attempts in the second round of state generation. This truncation yields optimal state storage during each run of the protocol (30). Quantitatively, the measured fidelity with the ideal Bell state of $0.65(3) > 0.5$ proves entanglement of the distilled state. Furthermore, the density matrix has high populations in the Bell-state subspace only, showing that the distillation successfully diminishes the separable admixture.

To gain further insight into the performance of the protocol, we measure the fidelity of the distilled state for different amounts of the separable admixture in the raw states; i.e., for different $\theta$ (see Fig. 4B, blue dots). The results are again truncated after a maximum of 50 entangling attempts in the

---

**Fig. 3. Quantum state storage during entangling operations.**

(A) Real-time feedback circuit for memory qubits. We initialize memory A/B, which then experiences a phase shift of $-\phi_{A/B}$ per executed entangling attempt. After reinitialization of the communication qubit, the distillation step of the protocol is performed (Fig. 1C). Blue-rimmed gate indicates the feedback. (B) Memory state stabilization over the full course of data acquisition. The oscillation observed without feedback (orange) is successfully compensated (blue) by the feedback. Solid lines are fits to the data. (C) Memory lifetime of node A (triangles) and node B (circles). We initialize the memory in one of the six cardinal states of the Bloch sphere (see bottom panel), sweep the number of entangling attempts, apply feedback, and read out the expectation value that is relevant for the estimation of the state fidelity with the initial cardinal state. All fidelity-irrelevant expectation values are set to zero, i.e., we depict projections of the state onto the respective axis in the Bloch sphere picture. The average state fidelities are separately plotted for phase-sensitive superposition states (blue) and phase-insensitive eigenstates (green). Blue solid lines depict a generalized exponential fit (30). The decay is limited by the stochastic repumping process, microwave pulse errors, and/or environmental dephasing. The color gradient of the top and bottom panels matches to facilitate comparisons. Error bars represent one standard deviation (SD).

---

Kalb et al., Science 356, 928–932 (2017) 2 June 2017
The nuclei) (Fig. 4B, solid purple line for raw state on the perfectly known initial path-length difference determined parameters under the assumption of the distillation step (step 4) using independently measured fidelity with the ideal state as a function of $\theta$ for a maximum of 50 entangling attempts in the second round. Blue data are the measured raw state fidelity. Dashed lines are derived from our model (30). Purple data are the measured raw state fidelity on the communication qubits. Solid orange (purple) line is the modeled fidelity of the raw state on the memories (communication qubits) that would be obtained if the initial internal phase was known. The memory state is calculated for the average number of entangling attempts until success (25). The orange shaded region is the modeled memory fidelity for minimal (0 attempts) and maximal (50 attempts) dephasing. The $\theta$-dependence of the distilled state fidelity is explained by the finite probability of misidentifying a separable state (the presence of which scales with $\theta$) as a successfully distilled entangled state owing to imperfect quantum control and decoherence of the stored state. Fidelities were obtained by measuring the expectation values $\langle XX \rangle$, $\langle YY \rangle$, and $\langle ZZ \rangle$. We denote the Pauli operators as $\hat{X}$, $\hat{Y}$, and $\hat{Z}$.

For a more detailed understanding of the different error sources contributing to the measured fidelity, we develop an extensive model of the full protocol using independently measured quantities and two free parameters: one factor accounting for additional memory control errors and the second for phase fluctuations of the raw states (30). We find good agreement between the modeled fidelity and the data for each of the different separable admixtures (see Fig. 4B, blue dashed line, and (30)) and for the evolution of the correlations with the number of entangling attempts (Fig. 4C). The model indicates that the state fidelities are mainly limited by memory qubit dephasing and control errors, as well as nonzero two-photon distinguishability. The latter effect, quantified by a measured two-photon interference visibility of 0.73(3) (see fig. S6), is especially harmful in the above comparison with the raw state, as this occurs twice for the distillation protocol but only once for the raw state generation. A visibility of 0.9 as observed on different NV center pairs (28) would thus yield an even stronger entanglement enhancement.

### Ebit rate

![Ebit rate comparison](http://science.sciencemag.org/)

Previously demonstrated entangling protocols based on two-photon coincidences (25, 28) require steps 1 and 3 to succeed in subsequent attempts, leading to a success probability scaling with the square of the photon detection probability $p_{d\text{-}}$. By contrast, the distillation protocol allows step 3 to succeed in many of many attempts following success in step 1, leading to a...
success probability scaling linearly with \( p_{\text{det}} \) in the ideal case. Given that in a typical quantum network setting \( p_{\text{det}} \) will be small (in our case, \( p_{\text{det}} \approx 10^{-3} \)), the distillation protocol can provide a distinct rate advantage despite the overhead of the additional local quantum logic.

To quantitatively compare our results with two-photon–coincidence protocols, we set an upper bound on the rate \( r \) of entangled bit (e bit) generation for each protocol using \( r = N E_2 \) with the logarithmic negativity \( E_2 \) and the rate of success \( \nu \). Figure 5 compares the e bit rate of the presented distillation protocol to the modeled rate of the Barrett-Kok two-photon–coincidence protocol (36) used in earlier experiments on NV centers (20, 28). We find that the distillation protocol (blue dots) outperforms the two-photon–coincidence protocol for identical experimental conditions, not only when assuming the measured two-photon indistinguishability (orange solid line) but even for the case in which the two-photon–coincidence protocol would be able to access perfect two-photon indistinguishability (orange dashed line).

**Conclusion and outlook**

The combination of generating, storing, and processing remote entangled qubits as demonstrated in the current distillation experiment provides a universal primitive for realizing extended quantum networks. The distillation itself is a powerful method to counteract unavoidable decoherence as entanglement is distributed throughout the network. Also, the protocol enables a speedup of entanglement generation that can be harnessed in related platforms such as other solid-state defect centers (37) and trapped ions (38). Improvements can be expected by encoding qubits into decoherence-protected subspaces (27), by using isotopically purified materials with longer qubit dephasing times (39–41), by implementing a faster reset or a measurement-based reset of the communication qubit, and by increasing the entangling rates through photonic cavities (42, 43). Furthermore, the techniques used in recent demonstrations of multi-qubit control and quantum error correction on a four-qubit node (26, 44) are fully compatible with the current experiment, thus highlighting the potential for scaling to more qubits and extending network functionality in the near future. Finally, the methods developed provide the toolkit to explore and utilize many-particle entanglement on a multinode quantum network.

**REFERENCES AND NOTES**

30. Materials and methods are available as supplementary materials.
42. A. Faraon, P. E. Barclay, C. Santori, K.-M. C. Fu, R. G. Beausoleil, Nat. Photonics 3, 301–305 (2009).

**ACKNOWLEDGMENTS**

We thank T. H. Taminiau, E. T. Campbell, D. Vincenzo, M. Lukin, T. Northup, L. M. K. Vandersypen, and S. Wehner for fruitful discussions. We acknowledge support from the Engineering and Physical Sciences Research Council National Quantum Technology Hub, the Netherlands Organisation for Scientific Research (NWO) through a VICI grant (R.H.), and the European Research Council through a Starting Grant (R.H.). Data contained in this paper are archived at doi:10.4121/uuid:7cf9f14-1a8c-4c50-9ac3-b0612b86c5e.

**SUPPLEMENTARY MATERIALS**

www.sciencemag.org/content/356/6341/928/suppl/DC1

Materials and Methods

Tables S1 to S6

Figs. S1 to S10

References (45–48)

2 March 2017, accepted 9 May 2017

10.1126/science.aan0070
Entanglement distillation between solid-state quantum network nodes
N. Kalb, A. A. Reiserer, P. C. Humphreys, J. J. W. Bakermans, S. J. Kamerling, N. H. Nickerson, S. C. Benjamin, D. J. Twitchen, M. Markham and R. Hanson (June 1, 2017)
Science 356 (6341), 928-932. [doi: 10.1126/science.aan0070]

Editor's Summary

Entangle, swap, purify, repeat

The key to a successful quantum internet will be the ability to generate robust entanglement between distant quantum memories. Unavoidable interactions with the environment, however, generally result in the loss of entanglement. Kalb et al. describe an entanglement distillation protocol that could be used to enhance the purity and robustness of entanglement between quantum nodes of a primitive quantum network.

Science, this issue p. 928