The Uncertainty Principle Determines the Nonlocality of Quantum Mechanics

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Two central concepts of quantum mechanics are Heisenberg's uncertainty principle and a subtle form of nonlocality that Einstein famously called "spooky action at a distance." These two fundamental features have thus far been distinct concepts. We show that they are inextricably and quantitatively linked: Quantum mechanics cannot be more nonlocal with measurements that respect the uncertainty principle. In fact, the link between uncertainty and nonlocality holds for all physical theories. More specifically, the degree of nonlocality of any theory is determined by two factors: the strength of the uncertainty principle and the strength of a property called "steering," which determines which states can be prepared at one location given a measurement at another.

A measurement allows us to gain information about the state of a physical system. For example, when measuring the position of a particle, the measurement outcomes correspond to possible locations. Whenever the state of the system is such that we can predict this position with certainty, then there is only one measurement outcome that can occur. Heisenberg (*1*) observed that quantum mechanics imposes strict restrictions on what we can hope to learn—there are incompatible measurements such as position and momentum whose results cannot be simultaneously predicted with certainty. These restrictions are known as uncertainty relations. For example, uncertainty relations tell us that if we were able to predict the momentum of a particle with certainty, then, when measuring its position, all measurement outcomes would occur with equal probability. That is, we would be completely uncertain about its location.

Nonlocality can be exhibited when measurements on two or more distant quantum systems are performed; that is, the outcomes can be correlated in a way that defies any local classical description (*2*). This is why we know that quantum theory will never be superseded by a local classical theory. Nonetheless, even quantum correlations are restricted to some extent: Measurement results cannot be correlated so strongly that they would allow signaling between two distant systems. However, quantum correlations are still weaker than what the no-signaling principle demands (*3*).

So why are quantum correlations strong enough to be nonlocal, yet not as strong as they could be? Is there a principle that determines the degree of this nonlocality? Information theory (*4*, *5*), communication complexity (*6*), and local quantum mechanics (*7*) provided us with some rationale for why limits on quantum theory may exist. But evidence suggests that many of these attempts provide only partial answers. Here, we take a very different approach and relate the limitations of nonlocal correlations to two inherent properties of any physical theory.

At the heart of quantum mechanics lies Heisenberg's uncertainty principle (*1*). Traditionally, uncertainty relations have been expressed in terms of commutators

$$\Delta A \Delta B \geq \frac{1}{2}|\langle\psi|[A, B]|\psi\rangle| \qquad (1)$$

with standard deviations $\Delta X = \sqrt{\langle\psi|X^2|\psi\rangle - \langle\psi|X|\psi\rangle^2}$ for $X \in \{A, B\}$. However, the more modern approach is to use entropic measures. Let $p(x^{(t)}|t)_\sigma$ denote the probability that we obtain outcome $x^{(t)}$ when performing a measurement labeled t when the system is prepared in the state σ. In quantum theory, σ is a density operator, whereas for a general physical theory, we assume that σ is simply an abstract representation of a state. The well-known Shannon entropy $H(t)_\sigma$ of the distribution over measurement outcomes of measurement t on a system in state σ is thus

$$H(t)_\sigma := -\sum_{x^{(t)}} p(x^{(t)}|t)_\sigma \log p(x^{(t)}|t)_\sigma \qquad (2)$$

In any uncertainty relation, we wish to compare outcome distributions for multiple measurements. In terms of entropies, such relations are of the form

$$\sum_t p(t) H(t)_\sigma \geq c_{\mathcal{T},\mathcal{D}} \qquad (3)$$

where $p(t)$ is any probability distribution over the set of measurements $\mathcal{T}$, and $c_{\mathcal{T},\mathcal{D}}$ is some positive constant determined by $\mathcal{T}$ and the distribution $\mathcal{D} = \{p(t)\}_t$. To see why Eq. 3 forms an uncertainty relation, note that whenever $c_{\mathcal{T},\mathcal{D}} > 0$ we cannot predict the outcome of at least one of the measurements t with certainty [i.e., $H(t)_\sigma > 0$]. Such relations have the great advantage that the lower bound $c_{\mathcal{T},\mathcal{D}}$ does not depend on the state σ (*8*). Instead, $c_{\mathcal{T},\mathcal{D}}$ depends only on the measurements and hence quantifies their inherent incompatibility. It has been shown that for two measurements, entropic uncertainty relations do in fact imply Heisenberg's uncertainty relation (*9*), providing us with a more general way of capturing uncertainty [see (*10*) for a survey].

One may consider many entropic measures instead of the Shannon entropy. For example, the min-entropy

$$H_\infty(t)_\sigma := -\log \max_{x^{(t)}} p(x^{(t)}|t)_\sigma \qquad (4)$$

used in (*8*) plays an important role in cryptography and provides a lower bound on $H(t)_\sigma$. Entropic functions are, however, a rather coarse way of measuring the uncertainty of a set of measurements, as they do not distinguish the uncertainty inherent in obtaining any combination of outcomes $x^{(t)}$ for different measurements t. It is thus useful to consider much more fine-grained uncertainty relations consisting of a series of inequalities, one for each combination of possible outcomes, which we write as a string $\mathbf{x} = (x^{(1)}, \ldots, x^{(n)}) \in \mathcal{B}^{\times n}$ with $n = |\mathcal{T}|$ (*11*). That is, for each $\mathbf{x}$, a set of measurements $\mathcal{T}$, and distribution $\mathcal{D} = \{p(t)\}_t$,

$$P^{\text{cert}}(\sigma;\mathbf{x}) := \sum_{t=1}^{n} p(t)\, p(x^{(t)}|t)_\sigma \leq \zeta_{\mathbf{x}}(\mathcal{T},\mathcal{D}) \qquad (5)$$

For a fixed set of measurements, the set of inequalities

$$U = \left\{ \sum_{t=1}^{n} p(t)\, p(x^{(t)}|t)_\sigma \leq \zeta_{\mathbf{x}} | \forall \mathbf{x} \in \mathcal{B}^{\times n} \right\} \qquad (6)$$

thus forms a fine-grained uncertainty relation, as it dictates that one cannot obtain a measurement outcome with certainty for all measurements simultaneously whenever $\zeta_{\mathbf{x}} < 1$. The values of $\zeta_{\mathbf{x}}$ thus confine the set of allowed probability distributions, and the measurements have uncertainty if $\zeta_{\mathbf{x}} < 1$ for all $\mathbf{x}$. To characterize the "amount of uncertainty" in a particular physical theory, we are thus interested in the values of

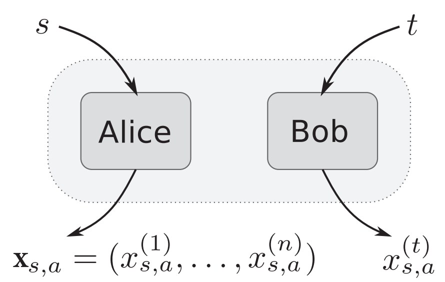

Fig. 1. Any game where, for every choice of settings s and t and answer a of Alice, there is only one correct answer b for Bob can be expressed using strings $\mathbf{x}_{s,a}$ such that $b = \mathbf{x}_{s,a}^{(t)}$ is the correct answer for Bob (*13*) for s and a. The game is equivalent to Alice outputting $\mathbf{x}_{s,a}$ and Bob outputting $\mathbf{x}_{s,a}^{(t)}$.

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$$\zeta_{\mathbf{x}} = \max_{\sigma} \sum_{t=1}^{n} p(t)\, p(x^{(t)}|t)_{\sigma} \qquad (7)$$

where the maximization is taken over all states allowed on a particular system (for simplicity, we assume it can be attained in the theory considered). We will also refer to the state $\rho_{\mathbf{x}}$ that attains the maximum as a "maximally certain state." However, we will also be interested in the degree of uncertainty exhibited by measurements on a set of states Σ quantified by $\zeta_{\mathbf{x}}^{\Sigma}$ defined with the maximization in Eq. 7 taken over $\sigma \in \Sigma$. Fine-grained uncertainty relations are directly related to the entropic ones, and they have both a physical and an information-processing appeal (*12*).

As an example, consider the binary spin-½ observables Z and X. If we can obtain a particular outcome $x^{(Z)}$ given that we measured Z with certainty—that is, $p(x^{(Z)}|Z) = 1$—then the complementary observable must be completely uncertain, that is, $p(x^{(X)}|X) = ½$. If we choose which measurement to make with probability ½, then this notion of uncertainty is captured by the relations

$$\frac{1}{2}p(x^{(X)}|X) + \frac{1}{2}p(x^{(Z)}|Z) \leq \zeta_{\mathbf{x}} = \frac{1}{2} + \frac{1}{2\sqrt{2}}$$

$$\text{for all } \mathbf{x} = (x^{(X)}, x^{(Z)}) \in \{0,1\}^2 \qquad (8)$$

where the maximally certain states are given by the eigenstates of $(X+Z)/\sqrt{2}$ and $(X-Z)/\sqrt{2}$.

We now introduce the concept of nonlocal correlations. Instead of considering measurements on a single system, we consider measurements on two (or more) space-like separated systems traditionally named Alice and Bob. We use $t \in \mathcal{T}$ to label Bob's measurements, and $b \in \mathcal{B}$ to label his measurement outcomes. For Alice, we use $s \in \mathcal{S}$ to label her measurements, and $a \in \mathcal{A}$ to label her outcomes (*11*). When Alice and Bob perform measurements on a shared state σ_{AB}, the outcomes of their measurements can be correlated. Let $p(a,b|s,t)_{\sigma_{AB}}$ be the joint probability that they obtain outcomes a and b for measurements s and t. We can now again ask ourselves: What probability distributions are allowed?

Quantum mechanics as well as classical mechanics obeys the no-signaling principle, meaning that information cannot travel faster than light.

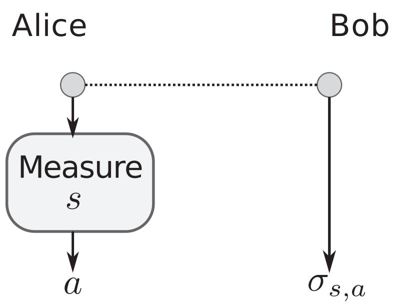

Fig. 2. When Alice performs a measurement labeled s and obtains outcome a with probability $p(a|s)$, she effectively prepares the state $\sigma_{s,a}$ on Bob's system. This is known as steering.

This means that the probability that Bob obtains outcome b when performing measurement t cannot depend on which measurement Alice chooses to perform (and vice versa). More formally, $\sum_a p(a,b|s,t)_{\sigma_{AB}} = p(b|t)_{\sigma_B}$ for all s, where $\sigma_B = \mathrm{tr}_A(\sigma_{AB})$. Curiously, however, this is not the only constraint imposed by quantum mechanics (*3*). Here we find that the uncertainty principle imposes the other limitation.

To do so, let us recall the concept of so-called Bell inequalities (*2*). These are most easily explained in their more modern form in terms of a game played between Alice and Bob. Let us choose questions $s \in \mathcal{S}$ and $t \in \mathcal{T}$ according to some distribution $p(s,t)$ and send them to Alice and Bob, respectively, where we take for simplicity $p(s,t) = p(s)p(t)$. The two players now return answers $a \in \mathcal{A}$ and $b \in \mathcal{B}$. Every game comes with a set of rules that determines whether a and b are winning answers given questions s and t. Again for simplicity, we thereby assume that for every s, t, and a, there exists exactly one winning answer b for Bob (and similarly for Alice). That is, for every setting s and outcome a of Alice, there is a string $\mathbf{x}_{s,a} = (x_{s,a}^{(1)}, ..., x_{s,a}^{(n)}) \in \mathcal{B}^{\times n}$ of length $n = |\mathcal{T}|$ that determines the correct answer $b = x_{s,a}^{(t)}$ for question t for Bob (Fig. 1). We say that s and a determine a "random-access coding" (*13*), meaning that Bob is not trying to learn the full string $\mathbf{x}_{s,a}$ but only the value of one entry. The case of non-unique games is a straightforward but cumbersome generalization.

As an example, the Clauser-Horne-Shimony-Holt (CHSH) inequality (*14*), one of the simplest Bell inequalities whose violation implies nonlocality, can be expressed as a game in which Alice and Bob receive binary questions $s,t \in \{0,1\}$ respectively, and similarly their answers $a,b \in \{0,1\}$ are single bits. Alice and Bob win the CHSH game if their answers satisfy $a \oplus b = s \cdot t$. We can label Alice's outcomes using string $\mathbf{x}_{s,a}$, and Bob's goal is to retrieve the tth element of this string. For $s = 0$, Bob will always need to give the same answer as Alice in order to win, and hence we have $\mathbf{x}_{0,0} = (0,0)$, and $\mathbf{x}_{0,1} = (1,1)$. For $s = 1$, Bob needs to give the same answer for $t = 0$, but the opposite answer if $t = 1$. That is, $\mathbf{x}_{1,0} = (0,1)$ and $\mathbf{x}_{1,1} = (1,0)$.

Alice and Bob may agree on any strategy ahead of time, but once the game starts, their physical distance prevents them from communicating. For any physical theory, such a strategy consists of a choice of shared state σ_{AB}, as well as measurements, where we may without loss of generality assume that the players perform a measurement depending on the question they receive and return the outcome of said measurement as their answer. For any particular strategy, we may hence write the probability that Alice and Bob win the game as

$$P^{\mathrm{game}}(\mathcal{S},\mathcal{T},\sigma_{AB}) = \sum_{s,t} p(s,t) \sum_a p(a, b = x_{s,a}^{(t)}|s,t)_{\sigma_{AB}} \qquad (9)$$

To characterize what distributions are allowed, we are generally interested in the winning probability maximized over all possible strategies for Alice and Bob:

$$P_{\max}^{\mathrm{game}} = \max_{\mathcal{S},\mathcal{T},\sigma_{AB}} P^{\mathrm{game}}(\mathcal{S},\mathcal{T},\sigma_{AB}) \qquad (10)$$

which, in the case of quantum theory, we refer to as a Tsirelson's type bound for the game (*15*). For the CHSH inequality (*14*), we have $P_{\max}^{\mathrm{game}} = 3/4$ classically, $P_{\max}^{\mathrm{game}} = 1/2 + 1/(2\sqrt{2})$ quantum mechanically, and $P_{\max}^{\mathrm{game}} = 1$ for a theory allowing any nonsignaling correlations. $P_{\max}^{\mathrm{game}}$ quantifies the strength of nonlocality for any theory, with the understanding that a theory possesses genuine nonlocality when it differs from the value that can be achieved classically. However, the connection we will demonstrate between uncertainty relations and nonlocality holds even before the optimization.

The final concept we need in our discussion is steerability, which determines what states Alice can prepare on Bob's system remotely. Imagine that Alice and Bob share a state σ_{AB}, and consider the reduced state $\sigma_B = \mathrm{tr}_A(\sigma_{AB})$ on Bob's side. In quantum mechanics, as well as many other theories in which Bob's state space is a convex set $\mathcal{S}$, the state $\sigma_B \in \mathcal{S}$ can be decomposed in many different ways as a convex sum,

$$\sigma_B = \sum_a p(a|s)\sigma_{s,a} \text{ with } \sigma_{s,a} \in \mathcal{S} \qquad (11)$$

corresponding to an ensemble $\mathcal{E}_s = \{p(a|s), \sigma_{s,a}\}_a$. Schrödinger (*16*, *17*) noted that in quantum mechanics, for all s there exists a measurement on Alice's system that allows her to prepare $\mathcal{E}_s = \{p(a|s), \sigma_{s,a}\}_a$ on Bob's site (Fig. 2). That is, for measurement s, Bob's system will be in the state $\sigma_{s,a}$ with probability $p(a|s)$. Schrödinger called this steering to the ensemble $\mathcal{E}_s$, and it does not violate the no-signaling principle, because for each of Alice's settings, the state of Bob's system is the same once we average over Alice's measurement outcomes.

Even more, he observed that for any set of ensembles $\{\mathcal{E}_s\}_s$ that respect the no-signaling constraint (i.e., for which there exists a state σ_B such that Eq. 11 holds), we can in fact find a bipartite quantum state σ_{AB} and measurements that allow Alice to steer to such ensembles. We can imagine theories in which our ability to steer is more restricted, or perhaps even less restricted (some amount of signaling is permitted). Our notion of steering thus allows forms of steering not considered in quantum mechanics (*16*–*19*) or other restricted classes of theories (*20*). Our ability to steer, however, is a property of the set of ensembles we consider, and not a property of one single ensemble.

We are now in a position to derive the relation between how nonlocal a theory is and how uncertain it is. We may express the probability (Eq. 9) that Alice and Bob win the game by using a particular strategy (Fig. 1) as

$$P^{\mathrm{game}}(\mathcal{S},\mathcal{T},\sigma_{AB}) = \sum_s p(s) \sum_a p(a|s) P^{\mathrm{cert}}(\sigma_{s,a}; \mathbf{x}_{s,a}) \qquad (12)$$

where $\sigma_{s,a}$ is the reduced state of Bob's system for setting s and outcome a of Alice, and $p(t)$ in $P^{\text{cert}}(\cdot)$ is the probability distribution over Bob's questions $\mathcal{T}$ in the game. This immediately suggests that there is indeed a close connection between games and our fine-grained uncertainty relations. In particular, every game can be understood as giving rise to a set of uncertainty relations, and vice versa. It is also clear from Eq. 5 for Bob's choice of measurements $\mathcal{T}$ and distribution $\mathcal{D}$ over $\mathcal{T}$ that the strength of the uncertainty relations imposes an upper bound on the winning probability,

$$P^{\text{game}}(\mathcal{S},\mathcal{T},\sigma_{AB}) \leq \sum_s p(s)\sum_a p(a|s)\ \zeta_{\mathbf{x}_{s,a}}(\mathcal{T},\mathcal{D}) \leq \max_{\mathbf{x}_{s,a}} \zeta_{\mathbf{x}_{s,a}}(\mathcal{T},\mathcal{D}) \qquad (13)$$

where we have made explicit the functional dependence of $\zeta_{\mathbf{x}_{s,a}}$ on the set of measurements. This seems curious given that we know from (*21*) that any two incompatible observables lead to a violation of the CHSH inequality, and that to achieve Tsirelson's bound Bob must measure maximally incompatible observables (*22*) that yield stringent uncertainty relations. However, from Eq. 13 we may be tempted to conclude that for any theory it would be in Bob's best interest to perform measurements that are very compatible and have weak uncertainty relations in the sense that the values $\zeta_{\mathbf{x}_{s,a}}$ can be very large. But can Alice prepare states $\sigma_{s,a}$ that attain $\zeta_{\mathbf{x}_{s,a}}$ for any choice of Bob's measurements?

The other important ingredient in understanding nonlocal games is thus steerability. We can think of Alice's part of the strategy $\mathcal{S}$, σ_{AB} as preparing the ensemble $\{p(a|s), \sigma_{s,a}\}_a$ on Bob's system whenever she receives question s. Thus, when considering the optimal strategy for nonlocal games, we want to maximize $P^{\text{game}}(\mathcal{S}, \mathcal{T}, \sigma_{AB})$ over all ensembles $\mathcal{E}_s = \{p(a|s), \sigma_{s,a}\}_a$ that Alice can steer to, and use the optimal measurements $\mathcal{T}_{\text{opt}}$ for Bob. That is,

$$P^{\text{game}}_{\max} = \max_{\{\mathcal{E}_s\}_s} \sum_s p(s)\sum_a p(a|s)\zeta^{\sigma_{s,a}}_{\mathbf{x}_{s,a}}(\mathcal{T}_{\text{opt}},\mathcal{D}) \qquad (14)$$

and hence the probability that Alice and Bob win the game depends only on the strength of the uncertainty relations with respect to the sets of steerable states. To achieve the upper bound given by Eq. 13, Alice needs to be able to prepare the ensemble $\{p(a|s),\rho_{\mathbf{x}_{s,a}}\}_a$ of maximally certain states on Bob's system. In general, it is not clear that the maximally certain states for the measurements that are optimal for Bob in the game can be steered to.

It turns out that in quantum mechanics, this can be achieved in cases where we know the optimal strategy. For all XOR games (*23*)—that is, correlation inequalities for two outcome observables (which include CHSH as a special case)—as well as other games where the optimal measurements are known, we find that the states that are maximally certain can be steered to (*12*). The uncertainty relations for Bob's optimal measurements thus give a tight bound,

$$P^{\text{game}}_{\max} = \sum_s p(s)\sum_a p(a|s)\ \zeta_{\mathbf{x}_{s,a}}(\mathcal{T}_{\text{opt}},\mathcal{D}) \qquad (15)$$

where we recall that $\zeta_{\mathbf{x}_{s,a}}$ is the bound given by the maximization over the full set of allowed states on Bob's system. It is an open question whether this holds for all games in quantum mechanics.

An important consequence of this is that any theory that allows Alice and Bob to win with a probability exceeding $P^{\text{game}}_{\max}$ requires measurements that do not respect the fine-grained uncertainty relations given by $\zeta_{\mathbf{x}_{s,a}}$ for the measurements used by Bob (the same argument can be made for Alice). Even more, it can lead to a violation of the corresponding min-entropic uncertainty relations (*12*). For example, if quantum mechanics were to violate CHSH using the same measurements for Bob, it would need to violate the min-entropic uncertainty relations (*8*). This relation holds even if Alice and Bob were to perform altogether different measurements when winning the game with a probability exceeding $P^{\text{game}}_{\max}$: For these new measurements, there exist analogous uncertainty relations on the set Σ of steerable states, and a higher winning probability thus always leads to a violation of such an uncertainty relation. Conversely, if a theory allows any states violating even one of these fine-grained uncertainty relations for Bob's (or Alice's) optimal measurements on the sets of steerable states, then Alice and Bob are able to violate the Tsirelson's type bound for the game.

Although the connection between nonlocality and uncertainty is more general, we examine the example of the CHSH inequality in detail to gain some intuition on how uncertainty relations of various theories determine the extent to which the theory can violate Tsirelson's bound (*12*). To briefly summarize, in quantum theory, Bob's optimal measurements are X and Z, which have uncertainty relations given by $\zeta_{\mathbf{x}_{s,a}} = 1/2 + 1/(2\sqrt{2})$ of Eq. 8. Thus, if Alice could steer to the maximally certain states for these measurements, they would be able to achieve a winning probability given by $P^{\text{game}}_{\max} = \zeta_{\mathbf{x}_{s,a}}$—that is, the degree of nonlocality would be determined by the uncertainty relation. This is the case. If Alice and Bob share the singlet state, then Alice can steer to the maximally certain states by measuring in the basis given by the eigenstates of $(X+Z)/\sqrt{2}$ or of $(X-Z)/\sqrt{2}$. For quantum mechanics, our ability to steer is only limited by the no-signaling principle, but we encounter strong uncertainty relations. On the other hand, for a local hidden variable theory, we can also have perfect steering, but only with an uncertainty relation given by $\zeta_{\mathbf{x}_{s,a}} = 3/4$, and thus we also have the degree of nonlocality given by $P^{\text{game}}_{\max} = 3/4$. This value of nonlocality is the same as deterministic classical mechanics, where we have no uncertainty relations on the full set of deterministic states but our abilities to steer to them are severely limited. In the other direction are theories that are maximally nonlocal, yet remain no-signaling (*3*). These have no uncertainty (i.e., $\zeta_{\mathbf{x}_{s,a}=1}$), but unlike in the classical world, we still have perfect steering, so they win the CHSH game with probability 1.

For any physical theory, we can thus consider the strength of nonlocal correlations to be a tradeoff between two aspects: steerability and uncertainty. In turn, the strength of nonlocality can determine the strength of uncertainty in measurements. However, it does not determine the strength of complementarity of measurements (*12*). The concepts of uncertainty and complementarity are usually linked, but we find that it is possible to have theories that are just as nonlocal and uncertain as quantum mechanics, but that have less complementarity. This suggests a rich structure relating these quantities.

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23. In an XOR game, the answers $a \in \{0,1\}$ and $b \in \{0,1\}$ of Alice and Bob, respectively, are single bits; whether Alice and Bob win or not depends only on the XOR of the answers $a \oplus b = a + b \bmod 2$.

24. The retrieval game used was discovered in collaboration with A. Doherty. Supported by the Royal Society (J.O.) and by NSF grants PHY-04056720 and PHY-0803371, the National Research Foundation of Singapore, and the Singapore Ministry of Education (S.W.). Part of this work was done while J.O. was visiting California Institute of Technology, and while J.O. and S.W. were visiting the Kavli Institute for Theoretical Physics (Santa Barbara, CA), which is funded by NSF grant PHY-0551164.

Supporting Online Material

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Figs. S1 to S4
References

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Editor's Summary

Quantum Connection

A system that is quantum mechanically entangled with another distant system can be predicted by measuring the distant system. This form of "action-at-a-distance," or nonlocality, seemingly contradicts Heisenberg's uncertainty principle, which is one of the fundamental aspects of quantum mechanics. **Oppenheim and Wehner** (p. 1072) show that the degree of nonlocality in quantum mechanics is actually determined by the uncertainty principle. The unexpected connection between nonlocality and uncertainty holds true for other physical theories besides quantum mechanics.