susceptible to hosting interesting topological effects (see the Commentary by Kraus and Zilberberg[24]).

Once again, the underlying root of all these phenomena is the nontrivial topology of the bands in the energy spectrum. By the same mechanism topological phases of matter can also arise in pure classical systems. For instance, 2D families of coupled gyroscopes and pendula can display nontrivial topological band structures (see the Commentary by Huber[25]).

There are many other phases of matter in which different topological characteristics play a fundamental role, such as Dirac, Weyl and nodal line semimetals, topological crystalline insulators and topological fluids. The subject is in full effervescence.

It is also worth noting that in gauge theories, the topology of the bundle in which the gauge fields are defined is not the only topological factor that is relevant for physical effects; the topological structure of the actual space of gauge fields also plays a role in physical phenomena like gauge anomalies[26,27]. It is therefore possible that it could also play a relevant role in the analysis of new topological phases of matter.

In the dawn of the topological matter revolution, one of the major challenges of pure topology — the famous Poincaré conjecture — has finally been proved[28–30]: any 3D topological space X with $\pi_1(X) = 0$ is topologically equivalent to the S^3 sphere. Recalling Gamow's remark about the unforeseeable applications of topology, there is no doubt that, in one way or another, this result will turn out to be helpful in understanding the riddles of nature. ❐

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A road to reality with topological superconductors

Carlo Beenakker and Leo Kouwenhoven

Topological matter can host low-energy quasiparticles, which, in a superconductor, are Majorana fermions described by a real wavefunction. The absence of complex phases provides protection for quantum computations based on topological superconductivity.

Quantum mechanics is complex. The only way Erwin Schrödinger could get his equation $i\hbar d\psi/dt = H\psi$ to work was to multiply the time derivative of the wavefunction ψ by the imaginary unit i. He complained about that in a 1926 letter to Lorentz (as quoted in ref. 1): "What is unpleasant here, and indeed directly to be objected to, is the use of complex numbers — ψ is surely fundamentally a real function." But the i was there to stay. Freeman Dyson called this apparently illogical step "one of the most profound jokes of nature"[2]: "Schrödinger put the square root of minus one into the equation, and suddenly it made sense. Suddenly it became a wave equation instead of a heat conduction equation … and that square root of minus one means that nature works with complex numbers and not with real numbers."

Topological superconductivity provides a road to reality; topological superconductors and topological insulators both combine a gapped bulk with gapless surface excitations, which are governed by a relativistic wave equation. But while the wavefunction ψ is complex in an insulator, ψ is real in a topological superconductor. A real ψ means that scattering phase shifts are limited to ±1, which profoundly changes the way quantum interference operates and promises a robustness of phase coherence that a complex ψ lacks.

Bogoliubov meets Majorana

The mathematics that allows for a real wavefunction is simple. If an electron at energy E has a time-dependent $\psi \propto e^{-iEt/\hbar}$, then its antiparticle (a 'hole') has $\psi \propto e^{+iEt/\hbar}$ and a linear superposition would give a real ψ. In a physical system, this superposition is produced by transitions between states of charge $+e$ and $-e$ that are normally forbidden by charge conservation. This is where the superconductor enters, by providing a reservoir of Cooper pairs of charge $2e$ that absorbs the charge difference. The electron–hole superposition, a so-called Bogoliubov quasiparticle[3], has a Hamiltonian $H = iA$ that turns out to be purely imaginary. The i then cancels with the i in front of $d\psi/dt$ to produce a purely real wave equation, $\hbar d\psi/dt = A\phi$.

This applies to any superconductor, but in a typical situation the physical consequences of a real ψ remain hidden because of a conspiracy of symmetries: particle–hole symmetry together with spin–rotation symmetry pairs up the Bogoliubov quasiparticles, so that they are effectively represented by a complex wavefunction — in much the same way that a complex number is represented by its real and imaginary parts. Although particle–hole symmetry is unavoidable in a superconductor, spin–rotation symmetry can be broken, allowing for an unpaired Bogoliubov quasiparticle with a manifestly real ψ. The theoretical physicists who studied this scenario at the turn of the century called it a Majorana fermion[4–7], in reference to a hypothetical elementary particle from the early days of quantum physics[8]. The theoretical models that produced Majorana fermions had an exotic superconducting order, with spin-triplet Cooper pairs in a chiral *p*-wave orbital state[9] — chirality refers to the $p_x \pm ip_y$ structure of the order parameter. These were among the first appearances on paper of topological superconductors, but it would take another decade for a breakthrough in our thinking how they might be realized in the laboratory.

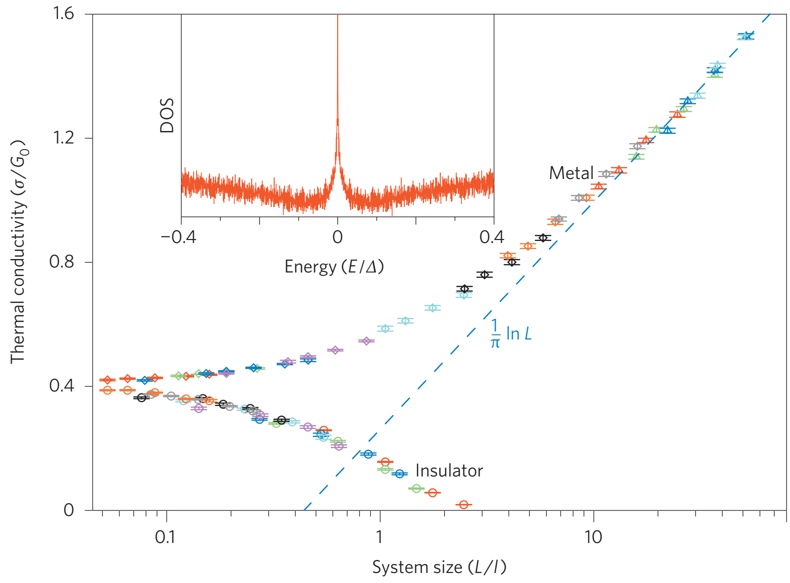

Figure 1 | Majorana metal in a computer simulation of a chiral *p*-wave superconductor. The main plot shows the thermal conductivity σ (in units of the thermal conductance, G_0) as a function of system size L (in units of the mean free path, l)[29]. The data points at different disorder strengths (indicated by different colours) all collapse onto a pair of scaling curves, designated 'metal' and 'insulator'. The ln L scaling is characteristic of a Majorana metal[5], originating from a proliferation of Majorana bound states at $E = 0$. The inset shows the corresponding mid-gap peak in the density of states (DOS)[28]. Figure reproduced from: main plot, ref. 29, APS; inset, ref. 28, APS.

Routes to topological superconductivity

There may well be materials that develop topological superconductivity on their own — strontium ruthenate is one longstanding candidate for spin-triplet pairing[10]. Spin-singlet pairing, however, is overwhelmingly more common. The breakthrough that has opened up a great variety of routes to topological superconductivity is the realization that one can start from a conventional spin-singlet superconductor and use the proximity effect to induce a topologically non-trivial superconducting state in a material with strong spin–orbit coupling[11–14].

The reasoning behind such hybrid approaches is that the mechanism that produces a topologically non-trivial state is the same for insulators and superconductors: an inversion of the excitation gap in the bulk that leaves behind a gapless surface state. Quite generally, a gap closing followed by a reopening will invert the sign of the gap and transform a topologically trivial state into a non-trivial state. So to create a topological superconductor we need two competing actors: a bad actor that seeks to kill the induced superconductivity, and a good actor that tries to revive it.

In several early implementations[15–18], an indium antimonide or indium arsenide nanowire is covered by a niobium or aluminium superconductor. The proximity effect pairs electrons of opposite spin in the nanowire, producing a superconducting gap at the Fermi level. A magnetic field tends to align the electron spins and close the gap, whereas spin–orbit coupling counteracts the alignment and reopens it. This competition creates regions in parameter space where the gap is inverted. Because the nanowire is effectively a 1D system, the surface is limited to the end points, where a gapless Majorana state is predicted to appear[19,20]. In an alternative 1D implementation[21], the semiconductor nanowire is replaced by a chain of iron atoms on a lead substrate. In such a system the atomic magnetization can play the roles of both the bad and the good actor[22], aligning the spins locally while disrupting the alignment by a rotation of the magnetic moment from one atom to the next.

These implementations have in common that the gap inversion is tuned by the variation of some parameter (typically the magnetic field or electron density). An alternative route to a 1D or 2D topological superconductor starts from the inverted bandgap of a 2D or 3D topological insulator and induces superconductivity in the edge or surface states[11,23]. Experiments in this direction are reported in refs 24–27.

No single implementation has yet emerged as the 'ideal' platform for the study of topological superconductivity, but the great variety of options holds promise for rapid experimental developments.

In what follows, we give an overview of some of the manifestations of a real Majorana wavefunction that are waiting to be observed.

Majorana metal

Although a superconductor is a perfect conductor of electricity, it is typically a poor thermal conductor. In a normal metal, the addition of disorder would only make things worse, but a 2D topological superconductor will start to conduct heat if enough defects are introduced[5]. This unusual state of matter is called a thermal metal or Majorana metal, because Majorana fermions bound to defects are responsible for the heat conduction.

Defects create bound states within the superconducting gap. In a conventional superconductor, these will only rarely align in energy E (measured relative to the middle of the gap, Δ), so they are not an effective transport channel. An isolated Majorana bound state must have $E = 0$, otherwise its wavefunction would not be real. It is this mid-gap alignment of Majorana bound states that allows for resonant conduction if the density of defects is sufficiently large. The disorder-driven phase transition from a thermal insulator to a thermal metal has not yet been observed experimentally, but it is evident in computer simulations[28,29], for instance as seen in Fig. 1.

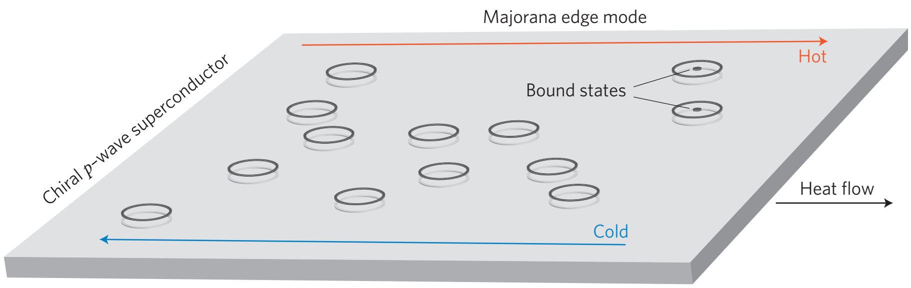

Figure 2 | Schematic of the thermal quantum Hall effect. In a 2D topological superconductor, a transverse temperature difference drives a longitudinal heat current, carried by chiral Majorana edge modes. This heat conduction mechanism dominates at low disorder strengths, when the Majorana bound states (ellipses) in the interior are sufficiently far apart that the system has not yet reached the Majorana metal phase depicted in Fig. 1.

Thermal quantum Hall effect

In the thermal insulating phase, the Majorana bound states are too far apart to allow for heat conduction in the interior of the system. What remains possible is conduction along the edge. The Majorana edge modes of a chiral *p*-wave superconductor produce the thermal analogue of the quantum Hall effect (Fig. 2).

We recall that the quantum Hall effect in a semiconductor 2D electron gas is associated both with a quantized electrical conductance and with a quantized thermal conductance. The quantization units are e^2/h and LTe^2/h, respectively, with T the temperature and $L = (1/3)(\pi k_B/e)^2$ the Lorenz number. The superconducting counterpart is called the thermal quantum Hall effect, because only the thermal conductance is quantized. The fact that the wavefunction of a Majorana fermion is real rather than complex reduces the quantization unit by a factor of two[5]: an unpaired Majorana mode has a thermal conductance of $G_0 = (1/2)LTe^2/h$.

The complexity of heat measurements at low temperatures is an obstacle to the detection of the thermal quantum Hall effect, but there is a purely electrical alternative[30]. Although the Majorana edge mode carries no charge on average, it is not in an eigenstate of charge, so there are quantum fluctuations. These produce a quantized shot noise power of $(1/2)e^2/h$ per eV of voltage bias, where the factor 1/2 has the same origin as in the quantized thermal conductance.

Majorana qubits

Although widely separated Majorana bound states are not useful for transport properties, they promise to be very useful for the storage of quantum information[7]. Because they are all pinned to $E = 0$, they introduce a degeneracy in the ground state of the topological superconductor. The degeneracy factor 2^N is exponential in the number N of pairs of bound states — Majorana qubits — so a massive amount of information can be stored in the ground state. The same information can be stored in quantum superpositions of the states of N electron spins, but such superpositions suffer from dephasing. An isolated Majorana has no phase; hence, as long as the bound states remain far apart, the quantum information should be protected from dephasing.

An elementary operation on the Majorana qubits is the pairwise exchange (braiding) of two Majoranas. If the operation is carried out very slowly, adiabatically, the superconductor remains in the ground state. For a non-degenerate state, this would amount to multiplication by a phase factor, but a degenerate state is transformed by a unitary operation. This is the celebrated non-Abelian exchange statistics of Majoranas[6] — non-Abelian because unitary operations do not commute. Not all unitary operations can be obtained by exchanging Majorana qubits, but a hybrid design[31,32] that also includes some well-developed superconducting electronics[33,34] is a promising road towards a fault-tolerant quantum computer.

From 2D to 3D

The central new insight of topological insulators is that topologically non-trivial band structures are not limited to 2D systems, such as the quantum Hall insulator: a 3D bulk insulator can have an electrically conducting surface if time-reversal symmetry is not broken[35,36]. This insight carries over to topological superconductors: a 3D superconductor can be thermally insulating in the bulk with a thermally conducting surface. The surface conduction is topologically protected in the absence of a magnetic field or magnetic impurities. A promising route to 3D topological superconductivity, followed in copper-doped bismuth selenide[37,38], is to start from a 3D topological insulator and dope it to induce a transition into a superconducting state. The transition brings some remarkable new physics into play.

The quasiparticles on the surface of a 3D topological insulator are massless Dirac fermions, familiar from graphene. The superconducting counterpart has massless Majorana fermions on its surface. Both quasiparticles have the same relativistic band structure, $E^2 = v^2(p_x^2 + p_z^2)$, with energy-independent velocity v and momentum (p_x, p_z) in the xz plane. The Dirac Hamiltonian $H_0 = -i\hbar v(\sigma_x\partial/\partial x + \sigma_z\partial/\partial z)$ that produces this band structure (with Pauli matrices σ_x and σ_z) is purely imaginary. For Dirac fermions we may add a disorder potential $V(x,z)$, but this is forbidden for Majorana fermions because $H_0 + V$ is then no longer imaginary and the real wave equation would become complex. The physical implication is that Majorana fermions transport heat ballistically over the surface of the topological superconductor, unscattered by disorder. This is a fundamental difference from topological insulators, in which disorder cannot localize the surface electrons, but it does scatter them and degrades the ballistic motion to diffusion.

Outlook

Thinking ahead about applications of topological insulators, one looks at spintronics, because of the spin–momentum locking of the conducting surface electrons. The same helicity applies to Majorana fermions, but their charge neutrality makes applications in that context less natural. Much of the present research aims at the integration of topological superconductivity into superconducting electronics, with the aim of improving the robustness of a quantum computation by storing information in Majorana bound states. Mobile Majorana fermions, either in edge states or in surface states, have thermal conduction properties that may or may not find applications. What is evident at this time is that topological superconductors provide a laboratory for the study of the remarkable complexity of quantum mechanics without complex numbers. ❐

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Topological mechanics

Sebastian D. Huber

Electronic topological insulators have inspired the design of new mechanical systems that could soon find real-life applications.

Acoustic metamaterials are artificially designed mechanical structures with the purpose of obtaining emergent functionalities such as vibration isolation, acoustic cloaking or adaptive behaviour. In this Commentary, I review how a recently established bridge between the phenomenology of electrons in topological insulators[1–3] and the world of classical mechanical systems might lead to new design principles for such metamaterials.

At first sight, there seems to be an unbridgeable gap between the quantum mechanical description of electrons in solids on the one hand and Newton's equations of motion describing mechanical modes, or phonons, on the other. When it comes to geometric or topological properties, however, this need not be the case.

To understand this connection, let us remind ourselves of the Foucault pendulum. In 1851, Léon Foucault demonstrated the rotation of the Earth by showing that the plane of swing of a pendulum rotates throughout the day. This rotation obtained a beautiful geometric description with the introduction of the concept of parallel transport more than 60 years later[4]: the angle of rotation does not depend on the precise details of the pendulum, but only on the solid angle that the pendulum's point of support traces out in a day.

While the example of the Foucault pendulum is rooted in classical mechanics, research on geometric phases only really took off in the framework of quantum mechanics in the early 1980s. Sir Michael Berry demonstrated the geometric nature of phases appearing in adiabatic quantum evolution[5], David Thouless and co-workers[6] described the quantum Hall effect in terms of a topological invariant called the Chern number, and Barry Simon uncovered the mathematical connection between the two[7].

Only in 2005, with the prediction[8] of a 'topological insulator', was a whole new world of topology-dominated free-electron physics unravelled. But how is this world related to classical mechanics?

It had already been discovered in 1985 that cousins of Berry's phases also appear in classical systems[9]. Unlike in the context of metamaterials, which typically involve an extensive number of degrees of freedom, Hannay[9] focused on systems with single (or few) modes. Catalysed by the birth of the field of electronic topological insulators, topology found a new path back to classical systems[10–13].

To appreciate this new analogy, let us consider the equations of motion for a set of coupled oscillators.

$$\ddot{x}_i = -D_{ij}\,x_j + A_{ij}\,\dot{x}_j \qquad (1)$$

Here, the real, symmetric and positive-definite dynamical matrix D encodes the forces between the oscillators x_i. The skew-symmetric matrix A describes the conservative (non-dissipative) coupling between positions and velocities. The second-order time derivative in Newton's equations seems to be incompatible with the Schrödinger equation. However, one can rewrite equation (1) in a form that offers more insight[14]

$$i\frac{\partial}{\partial t}\begin{pmatrix}\sqrt{D}^{\mathrm{T}}x\\ i\dot{x}\end{pmatrix} = \begin{pmatrix}0 & \sqrt{D}^{\mathrm{T}}\\ \sqrt{D} & iA\end{pmatrix}\begin{pmatrix}\sqrt{D}^{\mathrm{T}}x\\ i\dot{x}\end{pmatrix} \qquad (2)$$

which casts Newton's equations into a Hermitian eigenvalue problem for the frequencies ω, akin to the Schrödinger equation. Besides being first-order in time, this equation makes one important symmetry explicit: owing to the reality of the coefficients in equation (1), for any solution of the equations of motion with frequency ω there is a corresponding solution with $-\omega$. The 'supersymmetric'[10] block form of the matrix in equation (2) encodes this structure in a 'particle–hole' symmetry usually known from the problem of superconductivity.

Given this set-up, three routes to topological mechanical systems present themselves. First, one can directly capitalize on the intrinsic particle–hole symmetry. In this case, topological boundary modes arise in the gap around zero frequency. This route is particularly appealing, as it seems to be the only way to connect topology to the thermodynamic or low-energy properties of a metamaterial (Fig. 1). (Remember that there is no Pauli principle for phonons to fill Bloch bands.)

A second route would be to engineer a topological dynamical matrix D directly. In this case, the stable surface states lie at finite frequencies (Fig. 1). Although they are not of